Example 0 :-

$V = \{0,1\}$, $P = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$, $\pi(0) = \beta/\alpha+\beta$, $\pi(1) = \frac{\alpha}{\alpha+\beta}$

$$\| P(X_n \in \cdot) - \pi(\cdot) \|_{TV} = |(1-\alpha-\beta)^n \, | \mu(0) - \frac{\beta}{\alpha+\beta} |$$

(eigenvalue of P)

Mixing times :-

- classical
- modern

— get rates — large state space — fix a target distance to reach within the stationary distribution.

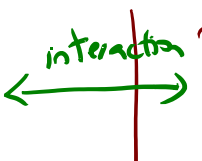
Mixing time :- # of steps (time) to reach the target distance

(depend on $|V|$)

[Sugar in water - mix it]

Statistical physics — Spin Systems — Monte Carlo Simulation : [Boltzmann vats]

how long do you run the simulation?



Theoretical Computer Science

- Sampling
- approximate Counting algorithm

"mixing time" $= O(\log |V|^{\alpha})$, $\alpha > 0$

Mathematics

- Random shuffling of cards

Spectral theory — Representation Theory — number theory

Random walk on graphs

Discrete Differential Geometry of graphs

$\frac{PI}{N}$ / $\frac{LS}{\text{Inequalities}}$

Discrete Potential Theory

C-Analysis / Differential Equation