Random Walks on Graphs : Assignment 6

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April 19, 2021

Submit solutions to Q3, Q4, Q5 and Q9 on Moodle by 27th April 10:00 PM.

For the following problems (G, μ) be a locally finite, connected weighted graph on a infinite vertex set V.

1. Show that finitely generated Cayley graphs on the same group V are roughly isometric.
2. Show the following
   (a) $\mathbb{Z}^d$ satisfies (I_d).
   (b) Any infinite connected graph satisfies (I_1).
   (c) Binary tree $\mathbb{T}^2$ satisfies (I_∞) with $C_0 = 3$.
   (d) If $(I_{\alpha+\delta})$ holds then (I_α) holds.
3. Let G_i for $i = 1, 2$ be graphs, with natural weights, which satisfy (I_{α_i}) respectively. Show that the join of G_1 and G_2 satisfies $(I_{\alpha_1 \wedge \alpha_2})$. Show that if G_i satisfy (N_{α_i}) respectively, the join of G_1 and G_2 satisfies $(N_{\alpha_1 \wedge \alpha_2})$.
4. Let (G, μ) be a weighted graph. Let $\lambda > 0$ and $\mu^\lambda = \lambda\mu$. If (G, μ) satisfies (N_α) with constant C_N then (G, μ^λ) satisfies (N_α) with $\lambda^{\frac{2}{\alpha}} C_N$
5. Show that (I_α) is stable under bounded perturbations.
6. Show the co-area formula $\|\nabla f\|_1 = \int \mu(\Lambda_t(f), \Lambda_t(f)^c) \mathrm{d}t$.
7. Show that (N_α) holds for all $f \in L^1 \cap L^2$ if it holds for all $f \geq 0$ and $f \in C_0(V)$.
8. Let (G, μ) satisfy (I_α). If $\alpha \in [1, \infty)$ then $\mu(B(x, n)) \geq c_1(\alpha) n^\alpha$ for all $n \geq 1$. If $\alpha = \infty$, $\mu(B(x, n)) \geq (1 + C_0^{-1})^n \mu(x)$.
9. (a) Let $f \in H^2, a \in \mathbb{R}$. Set $g := (f - a)_+, h = f \wedge a$. Then show that $g, h \in H^2$ and $\mathcal{E}(g) \leq \mathcal{E}(f), \mathcal{E}(h) \leq \mathcal{E}(f)$.
   (b) Let $f_1, \ldots, f_n \in H^2$ and $g = \min\{f_1, \ldots, f_n\}$. Then show that
   $$\mathcal{E}(g) \leq \sum_{i=1}^{n} \mathcal{E}(f_i).$$
   (c) Let $f \in H^2$. Then
   $$\mathcal{E}(f_+) + \mathcal{E}(f_-) \leq \mathcal{E}(f) \leq 2\mathcal{E}(f_+) + 2\mathcal{E}(f_-).$$