

Intro to Reinforcement Learning Part I

11-777 Multimodal Machine Learning Fall 2021

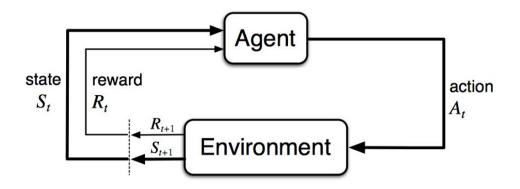
Amir Zadeh Slides from Paul Liang

Used Materials

Acknowledgement: Some of the material and slides for this lecture were borrowed from the Deep RL Bootcamp at UC Berkeley organized by Pieter Abbeel, Yan Duan, Xi Chen, and Andrej Karpathy, as well as Katerina Fragkiadaki and Ruslan Salakhutdinov's 10-703 course at CMU, who in turn borrowed much from Rich Sutton's class and David Silver's class on Reinforcement Learning.

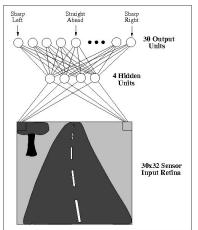
Contents

- Introduction to RL
- Markov Decision Processes (MDPs)
- Solving known MDPs using value and policy iteration
- Solving unknown MDPs using function approximation and Q-learning

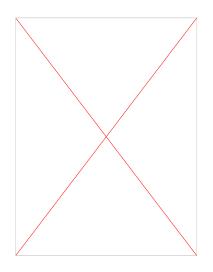


Reinforcement Learning









ALVINN, 1989

AlphaGo, 2016

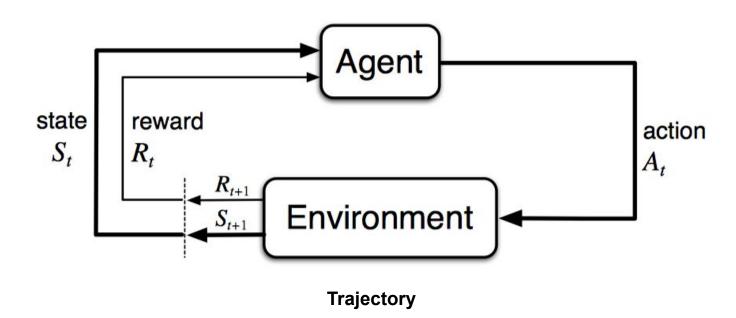
DQN, 2015







Reinforcement Learning

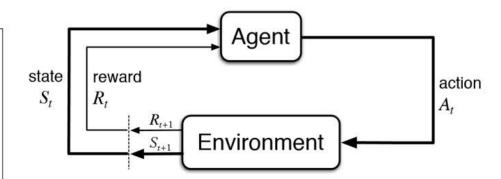


 $s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \dots$

Markov Decision Process (MDPs)

An MDP is defined by:

- Set of states S
- Set of actions A
- Transition function P(s' | s, a)
- Reward function R(s, a, s')
- Start state s₀
- Discount factor γ
- Horizon H



Trajectory

$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \dots$$

Markov assumption + Fully observable

A state should summarize all past information and have the **Markov property.**

$$\mathbb{P}[R_{t+1} = r, S_{t+1} = s' | S_0, A_0, R_1, ..., S_{t-1}, A_{t-1}, R_t, S_t, A_t] = \mathbb{P}[R_{t+1} = r, S_{t+1} = s' | S_t, A_t]$$

for all $s' \in \mathcal{S}, r \in \mathcal{R}$, and all histories

We should be able to throw away the history once state is known

If some information is only partially observable: Partially Observable MDP (POMDP)

Return

We aim to maximize total discounted reward:

$$G_t = R_{t+1} + \gamma R_{t+2} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

Discount factor

 γ close to 0 leads to "myopic" evaluation γ close to 1 leads to "far-sighted" evaluation

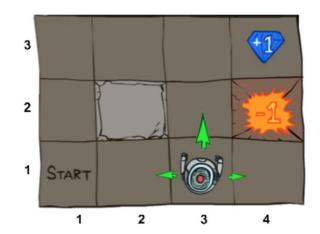
Policy

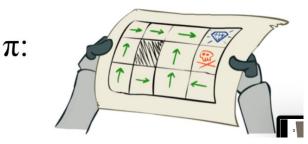
Definition: A policy is a distribution over actions given states

$$\pi(a \mid s) = \mathbf{Pr}(A_t = a \mid S_t = s), \forall t$$

- A policy fully defines the behavior of an agent
- The policy is stationary (time-independent)
- During learning, the agent changes its policy as a result of experience

Special case: deterministic policies

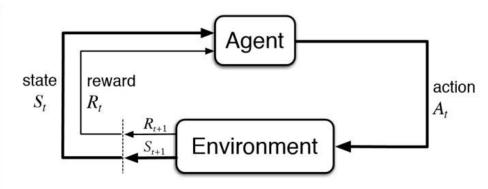




Learn the optimal policy to maximize return

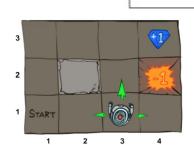
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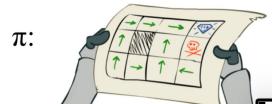
- Set of states S
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Return:

$$G_t = R_{t+1} + \gamma R_{t+2} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$





Goal:
$$\underset{\pi}{\operatorname{arg\,max}} \mathbb{E} \left[\sum_{t=0}^{H} \gamma^t R_t | \pi \right]$$

Reinforcement Learning

Sequential decision making

Supervised Learning

One-step decision making

Reinforcement Learning

- Sequential decision making
- Maximize cumulative reward

Supervised Learning

- One-step decision making
- Maximize immediate reward

Reinforcement Learning

- Sequential decision making
- Maximize cumulative reward
- Sparse rewards



Supervised Learning

- One-step decision making
- Maximize immediate reward
- Dense supervision

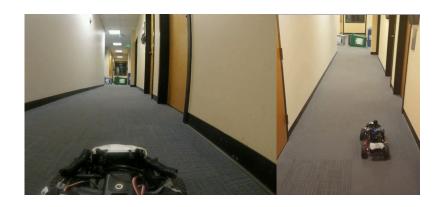
Reinforcement Learning

- Sequential decision making
- Maximize cumulative reward
- Sparse rewards
- Environment maybe unknown



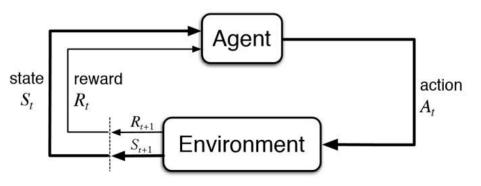
Supervised Learning

- One-step decision making
- Maximize immediate reward
- Dense supervision
- Environment always known



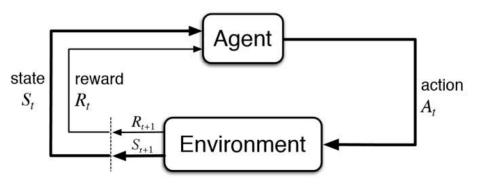
Imitation learning!





Imitation learning!



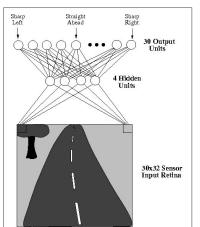


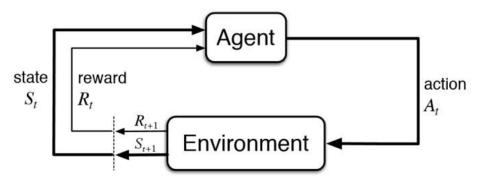
Obtain expert trajectories (e.g. human driver/video demonstrations):

$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \dots$$

Imitation learning!







Obtain expert trajectories (e.g. human driver/video demonstrations):

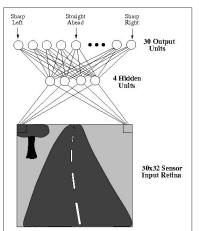
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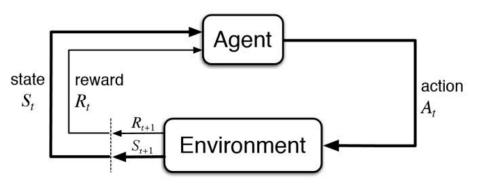
Perform supervised learning by predicting expert action

$$D = \{(s0, a*0), (s1, a*1), (s2, a*2), ...\}$$

Imitation learning!







Obtain expert trajectories (e.g. human driver/video demonstrations):

$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \dots$$

Perform supervised learning by predicting expert action

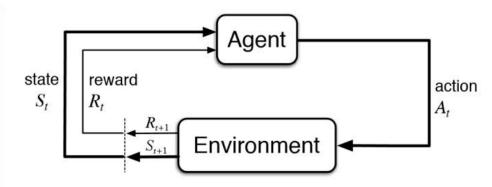
$$D = \{(s0, a*0), (s1, a*1), (s2, a*2), ...\}$$

But: distribution mismatch between training and testing
Hard to recover from sub-optimal states
Sometimes not safe/possible to collect expert trajectories

Learn the optimal policy to maximize return

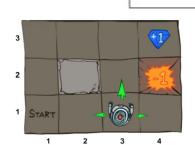
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Return:

$$G_t = R_{t+1} + \gamma R_{t+2} + \dots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$



π:



Goal:
$$\underset{\pi}{\operatorname{arg\,max}} \mathbb{E} \left[\sum_{t=0}^{H} \gamma^t R_t | \pi \right]$$

State and action value functions

- Definition: the **state-value function** $V^{\pi}(s)$ of an MDP is the expected return starting from state s, and following policy

$$V^{\pi}(s) = \mathbb{E}_{\pi}\left[G_t \middle| S_t = s
ight]$$
 Captures long term reward

- Definition: the **action-value function** $Q^{\pi}(s,a)$ is the expected return starting from state s, taking action a, and then following policy

$$Q^{\pi}(s,a) = \mathbb{E}_{\pi}\left[G_t \middle| S_t = s, A_t = a
ight]$$
 Captures long term reward

Optimal state and action value functions

- Definition: the **optimal state-value function** $V^{st}(s)$ is the maximum value function over all policies

$$V^*(s) = \max_{\pi} V^{\pi}(s)$$

- Definition: the **optimal action-value function** $Q^*(s,a)$ is the maximum action-value function over all policies

$$Q^*(s, a) = \max_{\pi} Q^{\pi}(s, a)$$

Solving MDPs

Prediction: Given an MDP (S, A, T, r, γ) and a policy

$$\pi(a|s) = \mathbb{P}[A_t = a|S_t = s] \quad V^{\pi}(s) \quad Q^{\pi}(s, a)$$

find the state and action value functions.

Solving MDPs

• **Prediction**: Given an MDP (S, A, T, r, γ) and a policy

$$\pi(a|s) = \mathbb{P}[A_t = a|S_t = s] \quad V^{\pi}(s) \quad Q^{\pi}(s, a)$$

find the state and action value functions.

• **Optimal control**: given an MDP (S, A, T, r, γ) , find the optimal policy (aka the planning problem). Compare with the learning problem with missing information about rewards/dynamics.

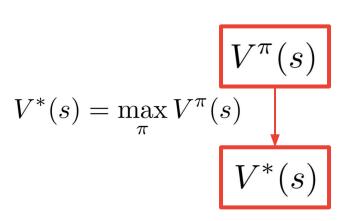
$$V^*(s) \qquad Q^*(s,a)$$

Value functions

- Value functions measure the goodness of a particular state or state/action pair: how good is it for an agent to be in a particular state or execute a particular action at a particular state, for a given policy
- Optimal value functions measure the best possible states or state/action pairs under all possible policies

	state values	action values
prediction	${ m v}_{\pi}$	q_{π}
control	V_*	q_*

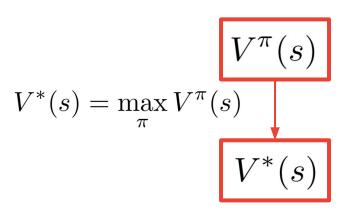
State value functions

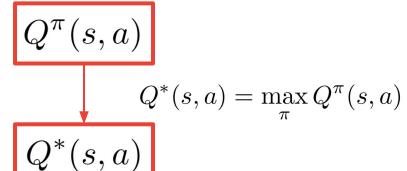


$$Q^{\pi}(s,a)$$

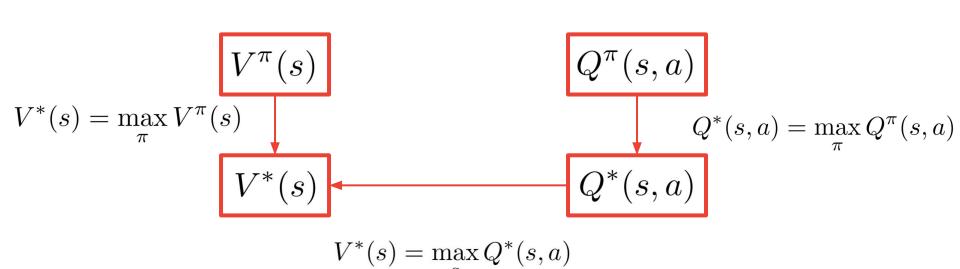
$$Q^*(s,a)$$

State value functions

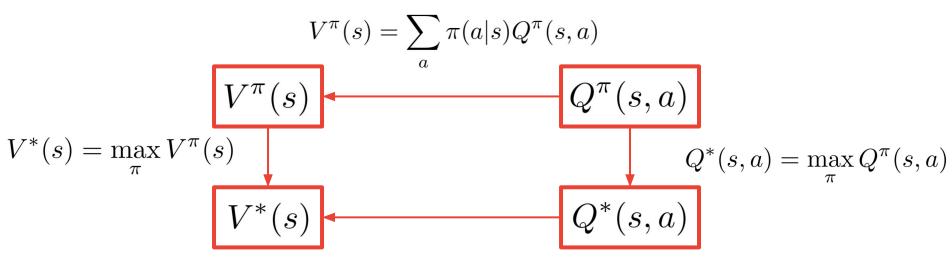




State value functions



State value functions



$$V^*(s) = \max_{a} Q^*(s, a)$$

Obtaining the optimal policy

Optimal policy can be found by maximizing over Q*(s,a)

$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \ Q^*(s, a) \\ 0, & \text{else} \end{cases}$$

Obtaining the optimal policy

Optimal policy can be found by maximizing over Q*(s,a)

$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \ Q^*(s, a) \\ 0, & \text{else} \end{cases}$$

Optimal policy can also be found by maximizing over V*(s') with one-step look ahead

$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \mathbb{E}_{s'} \left[r(s, a, s') + \gamma V^*(s') \right] \\ 0, & \text{else} \end{cases}$$

$$\pi^*(a|s) = \begin{cases} 1, & \text{if } a = \arg\max_a \left[\sum_{s'} p(s'|s, a) (r(s, a, s') + \gamma V^*(s')) \right] \\ 0, & \text{else} \end{cases}$$

Recursively:
$$G_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \dots$$

$$= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \dots \right)$$

$$= r_{t+1} + \gamma G_{t+1}$$

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$$= r_{t+1} + \gamma G_{t+1}$$

By taking expectations:
$$V^\pi(s) = \mathbb{E}_\pi \left[G_t | S_t = s \right]$$

$$= \mathbb{E}_\pi \left[r_{t+1} + \gamma G_{t+1} | S_t = s \right]$$

$$= \mathbb{E}_\pi \left[r_{t+1} + \gamma V^\pi(S_{t+1}) | S_t = s \right]$$

Recursively:
$$G_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \dots$$

$$= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \dots \right)$$

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$$= \mathbb{E}_\pi \left[r_{t+1} + \gamma V^\pi(S_{t+1}) | S_t = s \right]$$

$$= \sum_{\sigma} \pi(a|s)$$

Recursively:
$$G_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \dots$$

$$= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \dots \right)$$

$$= r_{t+1} + \gamma G_{t+1}$$

By taking expectations:
$$\begin{split} V^\pi(s) &= \mathbb{E}_\pi \left[G_t | S_t = s \right] \\ &= \mathbb{E}_\pi \left[r_{t+1} + \gamma G_{t+1} | S_t = s \right] \\ &= \mathbb{E}_\pi \left[r_{t+1} + \gamma V^\pi(S_{t+1}) | S_t = s \right] \\ &= \sum \pi(a|s) \mathbb{E}_{s'} \left[r(s,a,s') + \gamma V^\pi(s') \right] \end{split}$$

Recursively:
$$G_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \gamma^3 r_{t+4} \dots$$

$$= r_{t+1} + \gamma \left(r_{t+2} + \gamma r_{t+3} + \gamma^2 r_{t+4} \dots \right)$$

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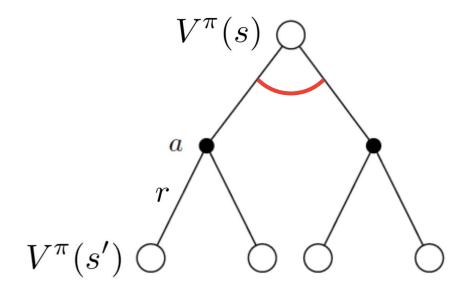
$$= \mathbb{E}_{\pi} \left[r_{t+1} + \gamma G_{t+1} | S_t = s \right]$$

$$= \mathbb{E}_{\pi} \left[r_{t+1} + \gamma V^{\pi}(S_{t+1}) | S_t = s \right]$$

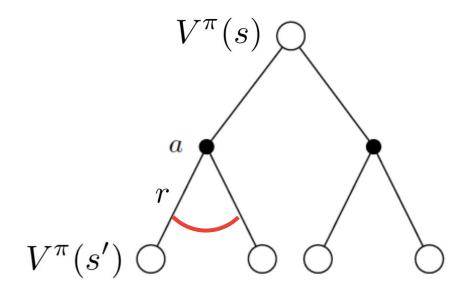
$$= \sum_{a} \pi(a|s) \mathbb{E}_{s'} \left[r(s,a,s') + \gamma V^{\pi}(s') \right]$$

$$= \sum_{a} \pi(a|s) \sum_{a} p(s'|s,a) \left[r(s,a,s') + \gamma V^{\pi}(s') \right]$$

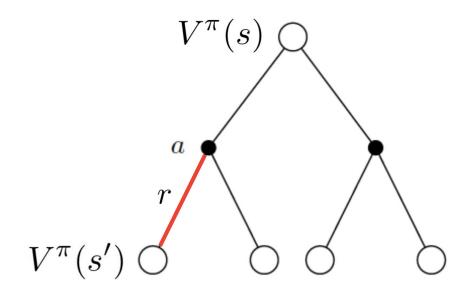
Bellman expectation for state value functions



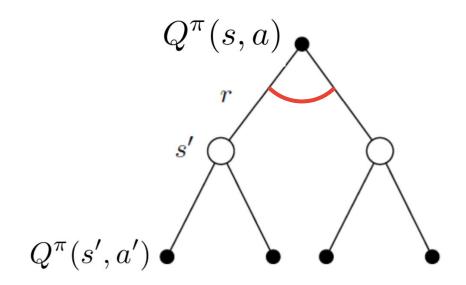
$$V^{\pi}(s) = \sum_{s} \pi(a|s)$$



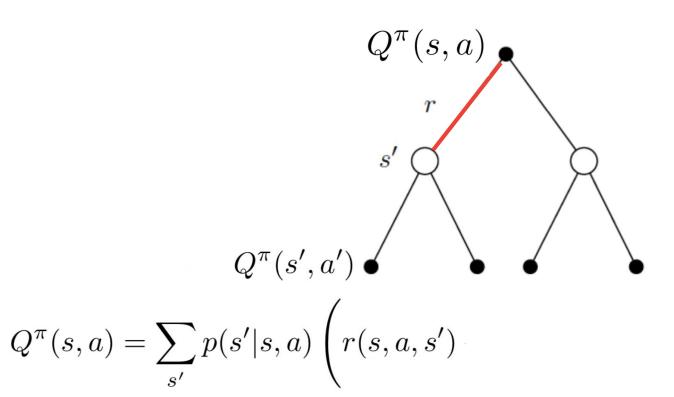
$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s, a)$$

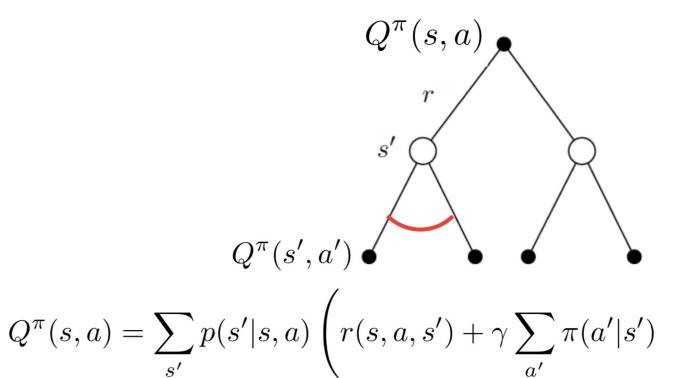


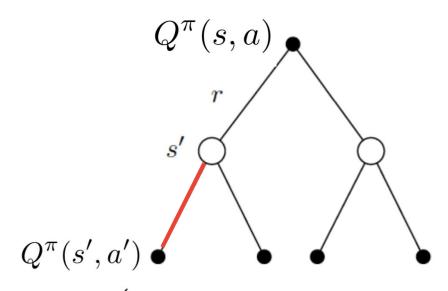
$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V^{\pi}(s') \right]$$



$$Q^{\pi}(s, a) = \sum_{s'} p(s'|s, a)$$







$$Q^{\pi}(s, a) = \sum_{s'} p(s'|s, a) \left(r(s, a, s') + \gamma \sum_{a'} \pi(a'|s') Q^{\pi}(s', a') \right)$$

Solving the Bellman expectation equations

$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V^{\pi}(s') \right]$$

Solving the Bellman expectation equations

$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V^{\pi}(s') \right]$$

Solve the linear system

variables: $V^{\pi}(s)$ for all s

constants: p(s'|s,a), r(s,a,s')

Solving the Bellman expectation equations

$$V^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma V^{\pi}(s') \right]$$

Solve the linear system

variables: $V^{\pi}(s)$ for all s

constants: p(s'|s,a), r(s,a,s')

Solve by iterative methods

$$V_{[k+1]}^{\pi}(s) = \sum_{a} \pi(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

Policy Evaluation

1. Policy evaluation

Iterate until convergence:

$$V_{[k+1]}^{\pi}(s) = \sum_{s} \pi_{[k]}(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

Policy Iteration

1. Policy evaluation

Iterate until convergence:

$$V_{[k+1]}^{\pi}(s) = \sum_{a} \pi_{[k]}(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

2. Policy Improvement

Find the best action according to one-step look ahead

$$\pi_{[k+1]}(a|s) = \arg\max_{a} \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

Policy Iteration

1. Policy evaluation

Iterate until convergence:

$$V_{[k+1]}^{\pi}(s) = \sum_{a} \pi_{[k]}(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

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Policy Iteration

1. Policy evaluation

Iterate until convergence:

$$V_{[k+1]}^{\pi}(s) = \sum_{a} \pi_{[k]}(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a,s') + \gamma V_{[k]}^{\pi}(s') \right]$$

2. Policy Improvement

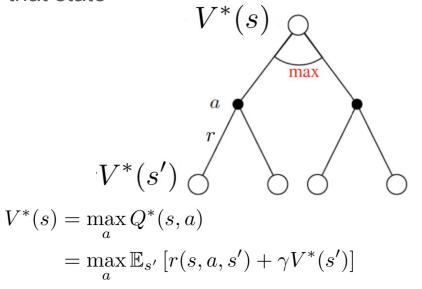
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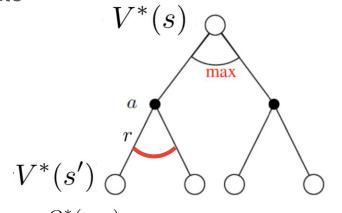
Bellman optimality for state value functions

The value of a state under an optimal policy must equal the expected return for the best action from that state



Bellman optimality for state value functions

The value of a state under an optimal policy must equal the expected return for the best action from that state

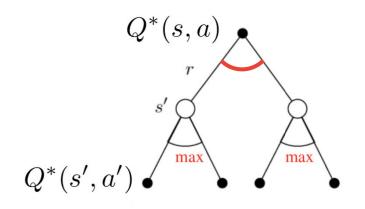


$$V^{*}(s) = \max_{a} Q^{*}(s, a)$$

$$= \max_{a} \mathbb{E}_{s'} [r(s, a, s') + \gamma V^{*}(s')]$$

$$= \max_{a} \left[\sum_{s'} p(s'|s, a) (r(s, a, s') + \gamma V^{*}(s')) \right]$$

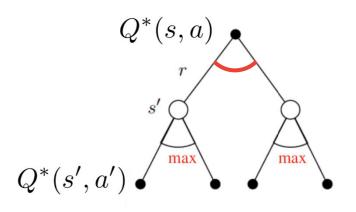
Bellman optimality for action value functions



$$Q^{*}(s, a) = \mathbb{E}_{s'} [r(s, a, s') + \gamma V^{*}(s')]$$

= $\mathbb{E}_{s'} [r(s, a, s') + \gamma \max_{a'} Q^{*}(s', a')]$

Bellman optimality for action value functions



$$Q^{*}(s, a) = \mathbb{E}_{s'} \left[r(s, a, s') + \gamma V^{*}(s') \right]$$

$$= \mathbb{E}_{s'} \left[r(s, a, s') + \gamma \max_{a'} Q^{*}(s', a') \right]$$

$$= \sum_{s'} p(s'|s, a) \left(r(s, a, s') + \gamma \max_{a'} Q^{*}(s', a') \right)$$

Solving the Bellman optimality equations

$$V^*(s) = \max_{a} \left[\sum_{s'} p(s'|s, a) (r(s, a, s') + \gamma V^*(s')) \right]$$

Solving the Bellman optimality equations

$$V^*(s) = \max_{a} \left[\sum_{s'} p(s'|s, a) (r(s, a, s') + \gamma V^*(s')) \right]$$

Solve by iterative methods

$$V_{[k+1]}^*(s) = \max_{a} \left[\sum_{s'} p(s'|s, a) (r(s, a, s') + \gamma V_{[k]}^*(s')) \right]$$

Value Iteration

Algorithm:

Start with $V_0^*(s) = 0$ for all s.

For k = 1, ..., H:

For all states s in S:

$$V_k^*(s) \leftarrow \max_{a} \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s') \right)$$

Value Iteration

Algorithm:

Start with $V_0^*(s) = 0$ for all s.

For k = 1, ..., H:

For all states s in S:

$$V_k^*(s) \leftarrow \max_{a} \sum_{s'} P(s'|s, a) \left(R(s, a, s') + \gamma V_{k-1}^*(s') \right)$$

$$\pi_k^*(s) \leftarrow \arg\max_{a} \sum_{s'} P(s'|s, a) \left(R(s, a, s') + \gamma V_{k-1}^*(s') \right)$$

Find the best action according to one-step look ahead

This is called a value update or Bellman update/back-up

Value Iteration Repeat until policy converges. Guaranteed to converge to optimal policy.

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For k = 1, ..., H:

For all states s in S:

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Find the best action according to one-step look ahead

This is called a value update or Bellman update/back-up

Q-Value Iteration

 $Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

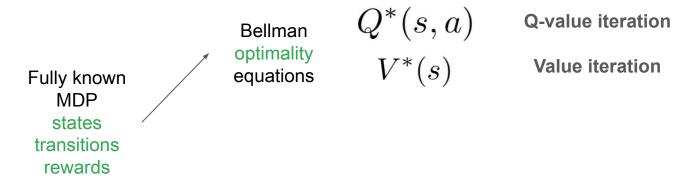
Bellman Equation:

$$Q^*(s, a) = \sum_{s'} P(s'|s, a) (R(s, a, s') + \gamma \max_{a'} Q^*(s', a'))$$

Q-Value Iteration:

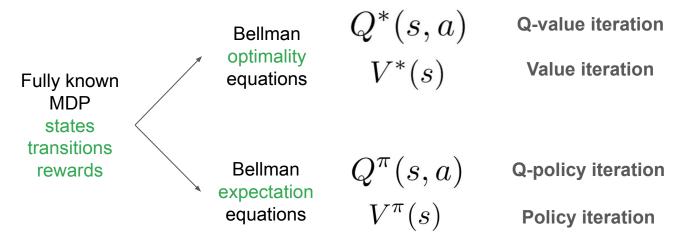
$$Q_{k+1}^*(s,a) \leftarrow \sum_{s'} P(s'|s,a) (R(s,a,s') + \gamma \max_{a'} Q_k^*(s',a'))$$

Summary: Exact methods



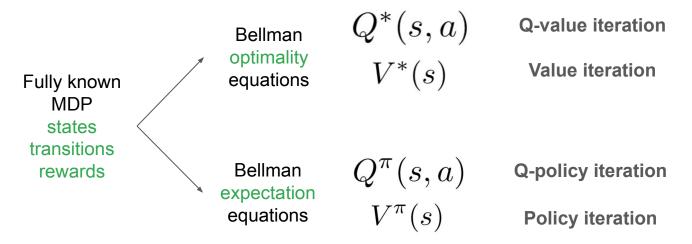
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Summary: Exact methods



Repeat until policy converges. Guaranteed to converge to optimal policy.

Summary: Exact methods



Repeat until policy converges. Guaranteed to converge to optimal policy.

Limitations:

Iterate over and storage for all states and actions: requires small, discrete state and action space

Update equations require fully observable MDP and known transitions

Solving unknown MDPs using function approximation

Recap: Q-value iteration

 $Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

Bellman Equation:

$$Q^*(s, a) = \sum_{s'} P(s'|s, a) (R(s, a, s') + \gamma \max_{a'} Q^*(s', a'))$$

Q-Value Iteration:

$$Q_{k+1}^*(s,a) \leftarrow \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma \max_{a'} Q_k^*(s',a') \right)$$

This is problematic when do not know the transitions

- Q-value iteration: $Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a) (R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$ Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$

- $\begin{array}{l} \bullet \quad \text{Q-value iteration:} \quad Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a) (R(s,a,s') + \gamma \max_{a'} Q_k(s',a')) \\ \bullet \quad \text{Rewrite as expectation:} \quad Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right] \\ \end{array}$
- (Tabular) Q-Learning: replace expectation by samples
 - For an state-action pair (s,a), receive: $s' \sim P(s'|s,a)$ simulation and exploration
 - Consider your old estimate: $Q_k(s,a)$
 - Consider your new sample estimate:

$$target(s') = r(s, a, s') + \gamma \max_{a'} Q_k(s', a')$$

$$\operatorname{error}(s') = \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a)\right)$$

Tabular Q-learning update

learning rate

$$Q_{k+1}(s, a) = Q_k(s, a) + \alpha \operatorname{error}(s')$$

$$= Q_k(s, a) + \alpha \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$$

Key idea: implicitly estimate the transitions via simulation

Algorithm:

Start with $\,Q_0(s,a)\,$ for all s, a.

Get initial state s

For k = 1, 2, ... till convergence

Sample action a, get next state s'

If s' is terminal:

target = r(s, a, s')

Sample new initial state s'

else:

$$target = r(s, a, s') + \gamma \max_{a'} Q_k(s', a')$$

$$Q_{k+1}(s,a) = Q_k(s,a) + \alpha \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a) \right)$$

 $s \leftarrow s'$

Bellman optimality

$$Q^{*}(s, a) = \mathbb{E}_{s'} \left[r(s, a, s') + \gamma \max_{a'} Q^{*}(s', a') \right]$$

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Bellman optimality

$$Q^*(s, a) = \mathbb{E}_{s'} \left[r(s, a, s') + \gamma \max_{a'} Q^*(s', a') \right]$$

$$r \max_{a'} Q_k(s', a')$$

$$r \max_{a'} Q_k(s', a') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$$

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$$s \leftarrow s'$$

- Choose random actions?
- ullet Choose action that maximizes $Q_k(s,a)$ (i.e. greedily)?
- ε-Greedy: choose random action with prob. ε, otherwise choose action greedily

Epsilon-greedy

Poor estimates of Q(s,a) at the start:

Bad initial estimates in the first few cases can drive policy into sub-optimal region, and never explore further.

$$\pi(s) = \left\{ \begin{array}{ll} \max_a \hat{Q}(s,a) & \text{with probability } 1 - \epsilon \\ \text{random action} & \text{otherwise} \end{array} \right.$$

Gradually decrease epsilon as policy is learned.

Algorithm:

```
Start with Q_0(s,a) for all s, a.
```

Get initial state s

For k = 1, 2, ... till convergence

Sample action a, get next state s'

If s' is terminal:

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$$target = r(s, a, s') + \gamma \max_{a'} Q_k(s', a')$$

$$Q_{k+1}(s, a) = Q_k(s, a) + \alpha \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$$

$$s \leftarrow s'$$

action greedily

ε-Greedy: choose random action with prob. ε, otherwise choose

Convergence

- Amazing result: Q-learning converges to optimal policy -even if you're acting suboptimally!
- This is called off-policy learning
- Caveats:
 - You have to explore enough
 - You have to eventually make the learning rate small enough
 - ... but not decrease it too quickly



Tabular Q-learning

Algorithm:

Start with $Q_0(s,a)$ for all s, a.

Get initial state s

For k = 1, 2, ... till convergence

Sample action a, get next state s'

If s' is terminal:

$$target = r(s, a, s')$$

Sample new initial state s'

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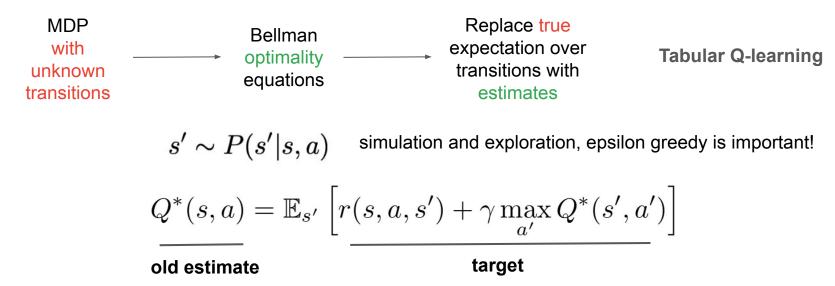
$$target = r(s, a, s') + \gamma \max_{a'} Q_k(s', a')$$

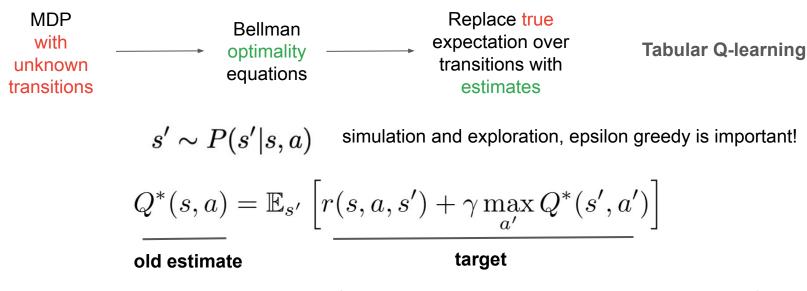
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Tabular: keep a $|S| \times |A|$ table of Q(s,a) Still requires small and discrete state and action space How can we generalize to unseen states?

 ε-Greedy: choose random action with prob. ε, otherwise choose action greedily





$$Q_{k+1}(s,a) \leftarrow Q_k(s,a) + \alpha \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a) \right)$$

MDP
with
unknown
transitions

Bellman
optimality
equations

Replace true
expectation over
transitions with
estimates

Tabular Q-learning

$$s' \sim P(s'|s,a)$$

simulation and exploration, epsilon greedy is important!

$$Q^{*}(s,a) = \mathbb{E}_{s'} \left[r(s,a,s') + \gamma \max_{a'} Q^{*}(s',a') \right]$$

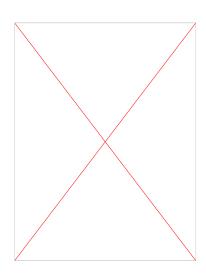
old estimate

target

$$Q_{k+1}(s,a) \leftarrow Q_k(s,a) + \alpha \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a) \right)$$

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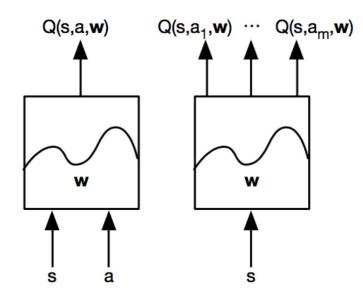
Q-learning with function approximation to **extract informative features** from **high-dimensional** input states.



DQN, 2015

Represent value function by Q network with weights w

$$Q(s,a,\mathbf{w})\approx Q^*(s,a)$$



- + high-dimensional, continuous states
- + generalization to new states

Optimal Q-values should obey Bellman equation

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q(s',a')^* \mid s,a
ight]$$

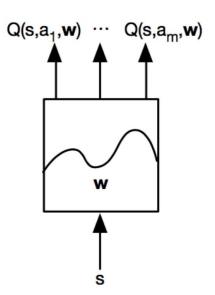
- Treat right-hand $r + \gamma \max_{a'} Q(s', a', \mathbf{w})$ as a target
- Minimize MSE loss by stochastic gradient descent

$$I = \left(r + \gamma \max_{a} Q(s', a', \mathbf{w}) - Q(s, a, \mathbf{w})\right)^{2}$$

Minimize MSE loss by stochastic gradient descent

$$I = \left(r + \gamma \max_{a} Q(s', a', \mathbf{w}) - Q(s, a, \mathbf{w})\right)^{2}$$

- Converges to Q* using table lookup representation
- But diverges using neural networks due to:
 - Correlations between samples
 - Non-stationary targets



Experience replay

- To remove correlations, build data-set from agent's own experience

s_1, a_1, r_2, s_2	
s_2, a_2, r_3, s_3	\rightarrow s, a, r, s'
s_3, a_3, r_4, s_4	exploration, epsilon greedy is important!
	exploration, epsilon greedy is important:
$s_t, a_t, r_{t+1}, s_{t+1}$	

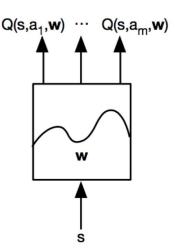
Sample random mini-batch of transitions (s,a,r,s') from D

Fixed Q-targets

- Sample random mini-batch of transitions (s,a,r,s') from **D**
- Compute Q-learning targets w.r.t. old fixed parameters w-
- Optimize MSE between Q-network and Q-learning targets

s_1, a_1, r_2, s_2	
s_2, a_2, r_3, s_3	
s_3, a_3, r_4, s_4	
$s_t, a_t, r_{t+1}, s_{t+1}$	

$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s'\sim\mathcal{D}_i}\left[\left(r + \gamma \max_{a'} Q(s',a';w_i^-) - Q(s,a;w_i)\right)^2
ight]$$
Q-learning target Q-network



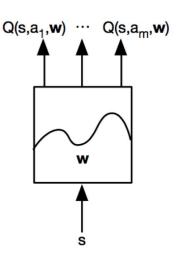
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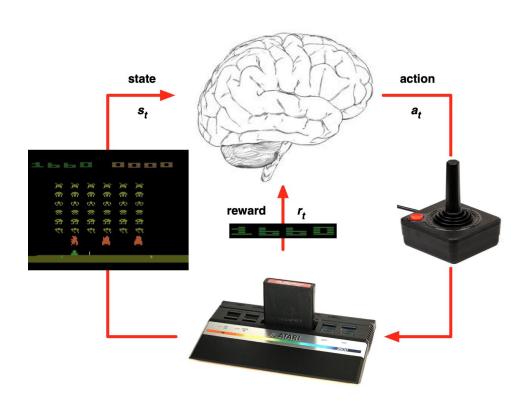
s_1, a_1, r_2, s_2	
s_2, a_2, r_3, s_3	
s_3, a_3, r_4, s_4	
$s_t, a_t, r_{t+1}, s_{t+1}$	

$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}_i} \left[\left(r + \gamma \max_{a'} Q(s',a';w_i^-) - Q(s,a;w_i) \right)^2 \right]$$
Q-learning target Q-network

- Use stochastic gradient descent
- Update w- with updated w every ~1000 iterations

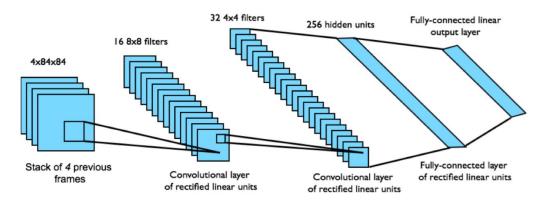


Deep Q-learning for Atari



Deep Q-learning for Atari

- End-to-end learning of values Q(s,a) from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Output is Q(s,a) for 18 joystick/button positions
- Reward is change in score for that step

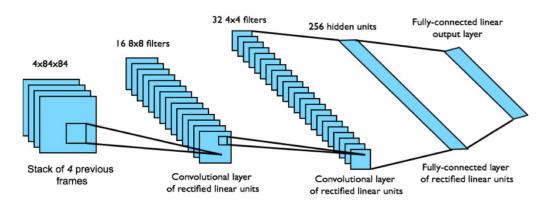


Network architecture and hyperparameters fixed across all games

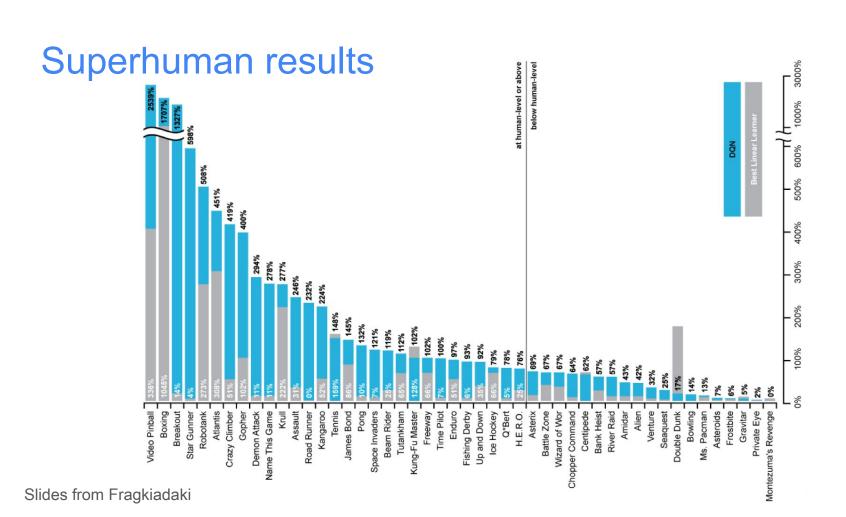
Deep Q-learning for Atari

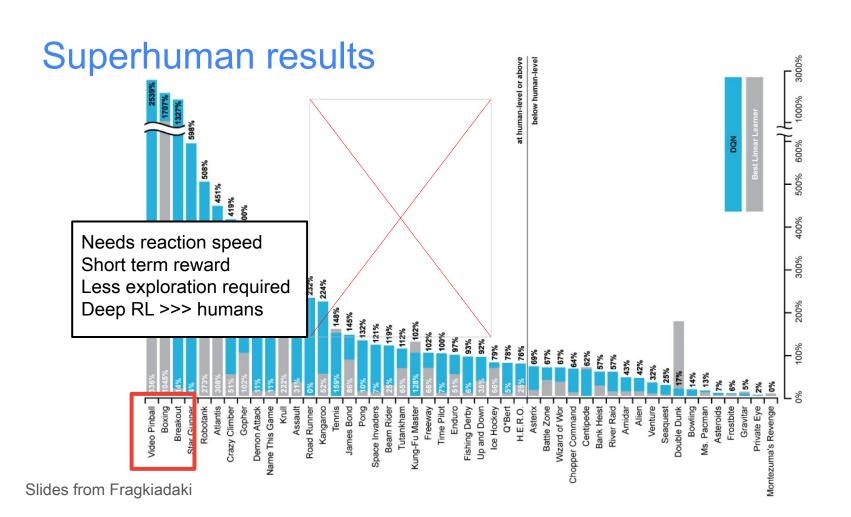
- End-to-end learning of values Q(s,a) from pixels s
- Input state s is stack of raw pixels from last 4 frames
- Encourage Markov property

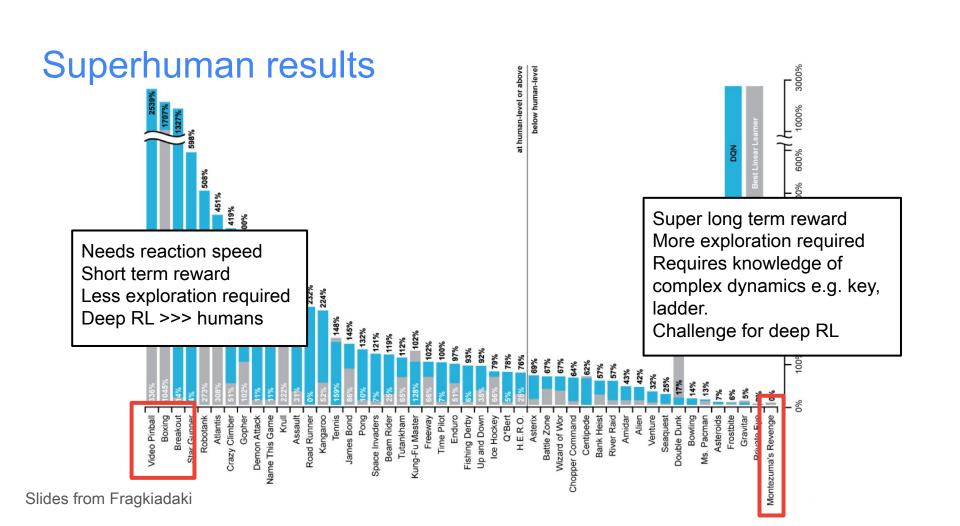
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Network architecture and hyperparameters fixed across all games







Superhuman results on Montezuma's Revenge

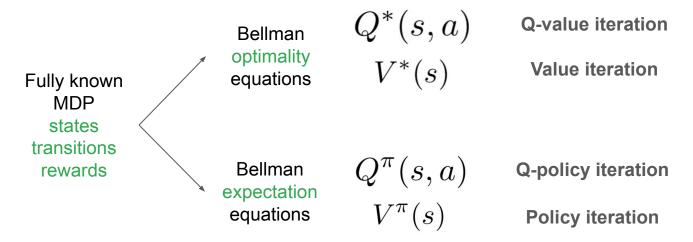


Encourages agent to explore its environment by maximizing **curiosity**.

- I.e. how well can I predict my environment?
- 1. less training data
- 2. stochastic
- 3. unknown dynamics So I should explore more.

Burda et. al., ICLR 2019

Summary: Exact methods



Repeat until policy converges. Guaranteed to converge to optimal policy.

Limitations:

Iterate over and storage for all states and actions: requires small, discrete state and action space

Update equations require fully observable MDP and known transitions

MDP
with
unknown
transitions

Bellman
optimality
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Replace true
expectation over
transitions with
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$$s' \sim P(s'|s,a)$$

simulation and exploration, epsilon greedy is important!

$$Q^*(s,a) = \mathbb{E}_{s'} \left[r(s,a,s') + \gamma \max_{a'} Q^*(s',a') \right]$$

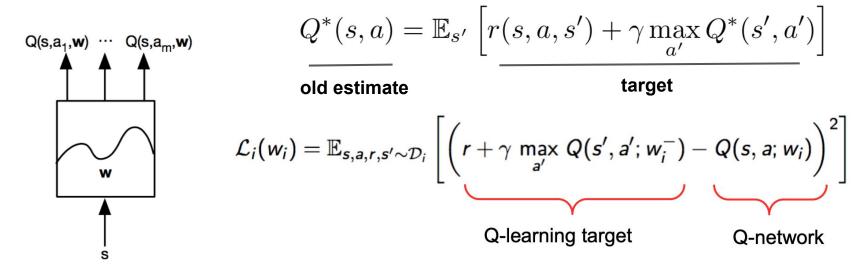
old estimate

target

$$Q_{k+1}(s,a) \leftarrow Q_k(s,a) + \alpha \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a) \right)$$

Tabular: keep a $|S| \times |A|$ table of Q(s,a) Still requires small and discrete state and action space How can we generalize to unseen states?

Summary: Deep Q-learning

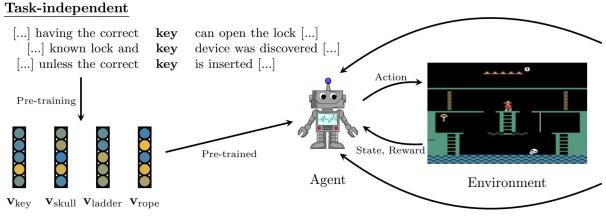


Stochastic gradient descent + Experience replay + Fixed Q-targets

Works for high-dimensional state and action spaces Generalizes to unseen states

Applications: RL and Language

RL and Language



Task-dependent

Language-assisted

Key Opens a door of the same color as the key.

Skull They come in two varieties, rolling skulls and bouncing skulls ... you must jump over rolling skulls and walk under bouncing skulls.

Language-conditional

Go down the ladder and walk right immediately to avoid falling off the conveyor belt, jump to the yellow rope and again to the platform on the right.

Language-conditional RL

- Instruction following
- Rewards from instructions
- Language in the observation and action space

Navigation via instruction following



Train

Go to the short red torch Go to the blue keycard Go to the largest yellow object Go to the green object



Test

Go to the tall green torch Go to the red keycard Go to the smallest blue object

Chaplot et. al., AAAI 2018 Misra et. al., EMNLP 2017

Navigation via instruction following



Train

Go to the short red torch Go to the blue keycard Go to the largest yellow object Go to the green object



Test

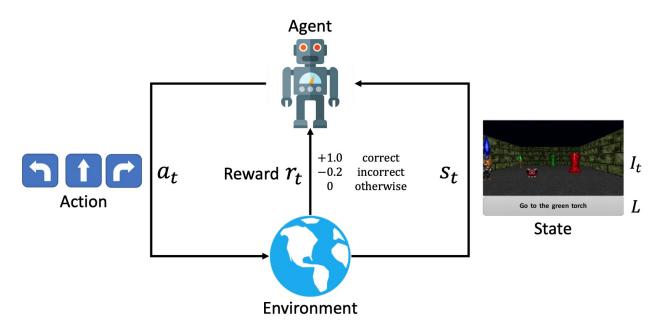
Go to the tall green torch Go to the red keycard Go to the smallest blue object

Fusion Alignment

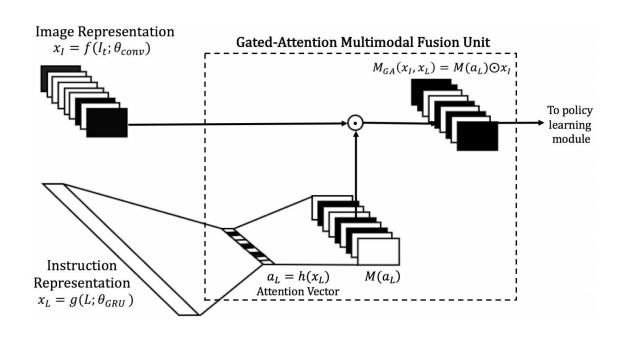
Ground language Recognize objects Navigate to objects Generalize to unseen objects

> Chaplot et. al., AAAI 2018 Misra et. al., EMNLP 2017

Interaction with the environment

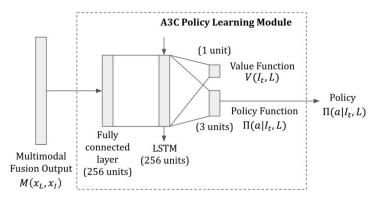


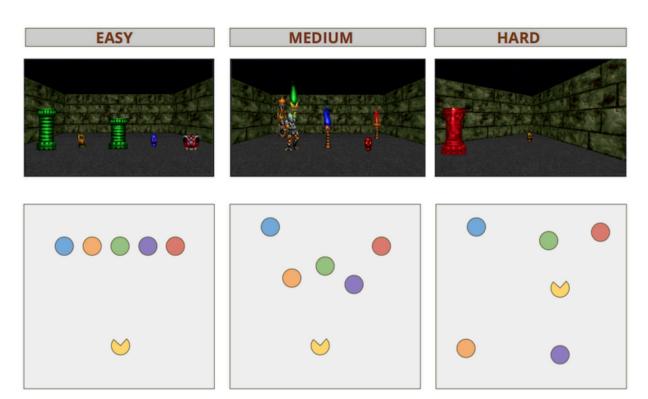
Gated attention via element-wise product



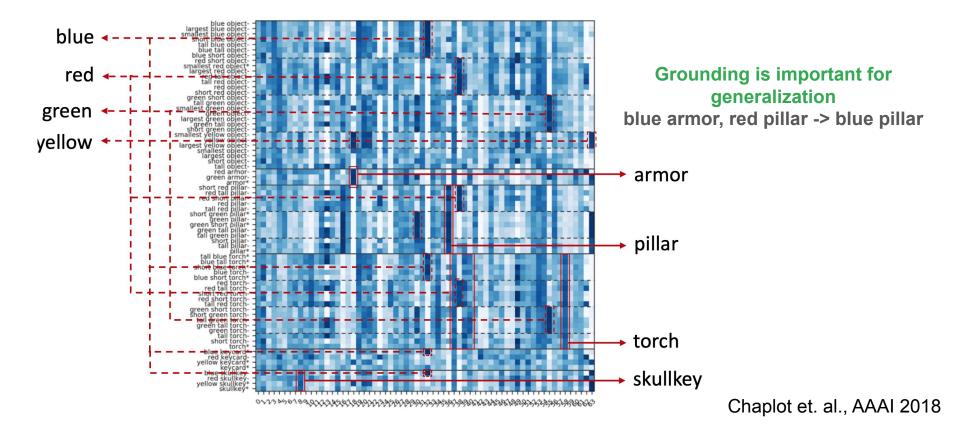
Fusion
Alignment
Ground language
Recognize objects

- Policy learning
 - Asynchronous Advantage Actor-Critic (A3C) (Mnih et al.)
 - uses a deep neural network to parametrize the policy and value functions and runs multiple parallel threads to update the network parameters.
 - use **entropy regularization** for improved exploration
 - use **Generalized Advantage Estimator** to reduce the variance of the policy gradient updates (Schulman et al.)











Montezuma's revenge

Sparse, long-term reward problem
General solution: reward shaping via auxiliary rewards



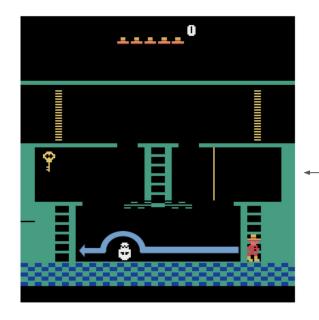
Montezuma's revenge

Sparse, long-term reward problem
General solution: reward shaping via auxiliary rewards

Encourages agent to explore its environment by maximizing **curiosity**. How well can I **predict** my environment?

- 1. Less training data
- 2. Stochastic
- 3. Unknown dynamics So I should **explore more**.

Pathak et. al., ICML 2017 Burda et. al., ICLR 2019



Montezuma's revenge

Sparse, long-term reward problem
General solution: reward shaping via auxiliary rewards

Natural language for reward shaping

"Jump over the skull while going to the left"

from Amazon Mturk :-(asked annotators to play the game and describe entities

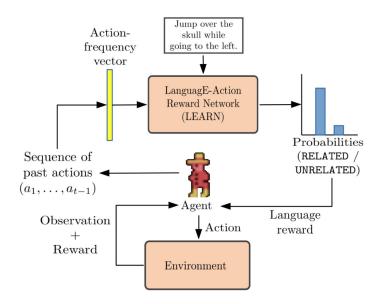
Intermediate rewards to speed up learning



Montezuma's revenge

Natural language for reward shaping

Encourages agent to take actions related to the instructions



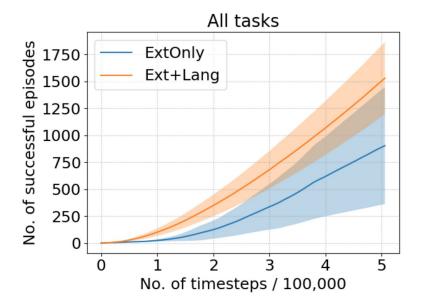
Language-conditional RL: Rewards from instructions



Montezuma's revenge

Natural language for reward shaping

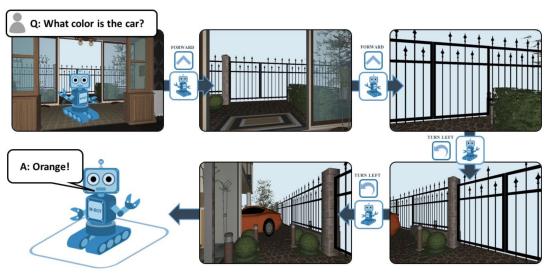
Encourages agent to take actions related to the instructions



Language-conditional RL: Language in S and A

Embodied QA: Navigation + QA



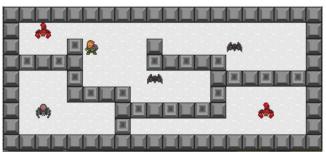


Language-assisted RL

- Language for communicating domain knowledge
- Language for structuring policies

Properties of entities in the environment are annotated by language







is an enemy who chases you



is a stationary collectible



is a randomly moving enemy

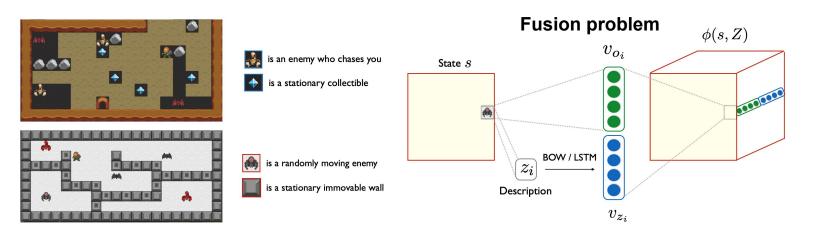


is a stationary immovable wall

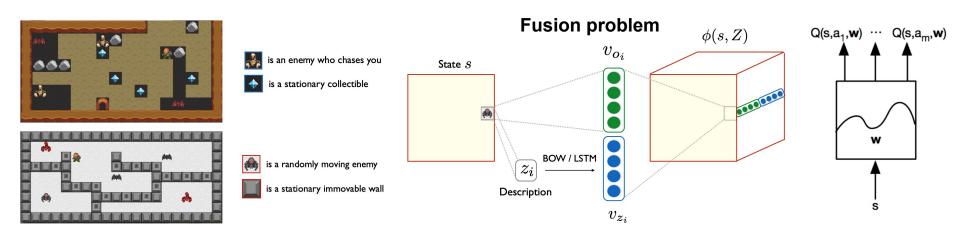
from Amazon Mturk :-(asked annotators to play the game and describe entities

Narasimhan et. al., JAIR 2018

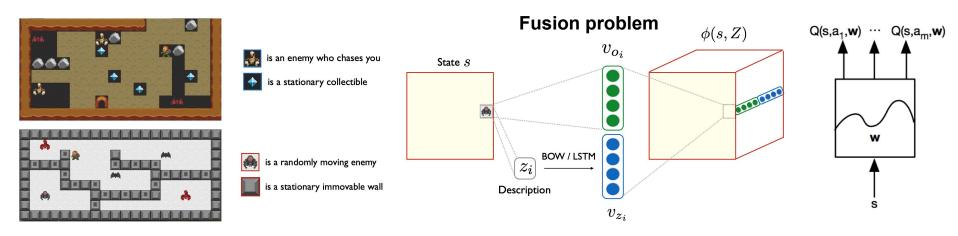
Properties of entities in the environment are annotated by language



Properties of entities in the environment are annotated by language



Properties of entities in the environment are annotated by language



Grounded language learning

Helps to ground the meaning of text to the dynamics, transitions, and rewards Language helps in multi-task learning and transfer learning

Learning to read instruction manuals



The natural resources available where a population settles affects its ability to produce food and goods. Build your city on a plains or grassland square with a river running through it if possible.

Figure 1: An excerpt from the user manual of the game Civilization II.

Learning to read instruction manuals



The natural resources available where a population settles affects its ability to produce food and goods. Build your city on a plains or grassland square with a river running through it if possible.

- Choose relevant sentences
- 2. Label words into action-description, state-description, or background

Learning to read instruction manuals



The natural resources available where a population settles affects its ability to produce food and goods. Build your city on a plains or grassland square with a river running through it if possible.

Map tile attributes:

- Terrain type (e.g. grassland, mountain, etc)
- Tile resources (e.g. wheat, coal, wildlife, etc)

City attributes:

- City population
- Amount of food produced

Unit attributes:

- Unit type (e.g., worker, explorer, archer, etc)
- Is unit in a city?

- Choose relevant sentences
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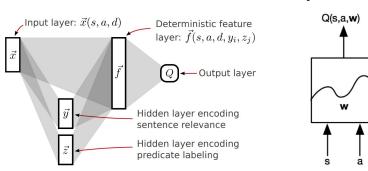
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Branavan et. al., JAIR 2012

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• Phalanxes are twice as effective at defending cities as warriors.



• Build the city on plains or grassland with a river running through it.



- You can rename the city if you like, but we'll refer to it as washington.
- There are many different strategies dictating the order in which advances are researched

Relevant sentences

- After the road is built, use the settlers to start improving the terrain.
- When the settlers becomes active, chose build road.
- Use settlers or engineers to improve a terrain square within the city radius 5 *

A: action-description

S: state-description

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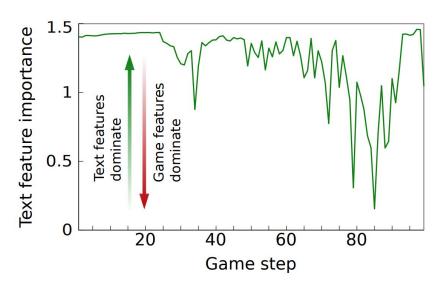


Method	% Win	% Loss	Std. Err.
Random	0	100	
Built-in AI	0	0	
Game only	17.3	5.3	± 2.7
Sentence relevance	46.7	2.8	\pm 3.5
Full model	53.7	5.9	\pm 3.5
Random text	40.3	4.3	± 3.4
Latent variable	26.1	3.7	\pm 3.1

Grounded language learning
Ground the meaning of text to the dynamics, transitions, and rewards
Language helps in learning

Learning to read instruction manuals

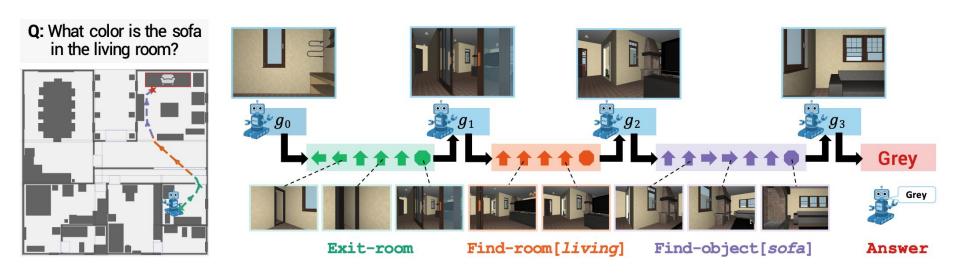




Language is most important at the start when you don't have a good policy Afterwards, the model relies on game features

Language for structuring policies

Composing modules for Embodied QA



Language for structuring policies

Composing modules for Embodied QA

