Midterm Practice Questions[1]

16822 Geometry-based Methods for Computer Vision (Fall 2021)

https://piazza.com/cmu/fall2021/16822

OUT: Oct. 05, 2021
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START HERE: Instructions for Midterm

- **Location and date:** The midterm exam will be held on October 11, 2021 during regular class hours in REH Singleton lecture hall.
- **Open book:** This means that you may refer to any written / printed material in a book or in your notes. You may also refer to electronic copies of the book / notes. You may not however access the internet through search engines (Google, DuckDuckGo) and you may not talk to another person during the exam via messenger or any social media.
- **Number of Questions:**
  - This exam may have too many questions. You will likely not be able to finish it all. That's ok, don't worry! There's a variety of questions of different styles and topics. Just ***solve the subset*** with which you are comfortable.
  - In the midterm exam in a previous year, the max score was 182 out of 239, with mean: 136, standard deviation: 24, and median: 139. So even the best exam is far from completion, so you can relax a little and focus on what you are most comfortable with!
- **Type of Questions**
  - None of the questions require extensive algebra (some problems require a few lines of algebra and even that one is not so bad if you set up the problem properly). All of them can be answered in at most a few lines. Some of them can be answered geometrically without any equations. Space has been provided in case you need to include drawings.
  - Individual questions are independent of each other and they may be answered in any order.
  - There are no trick questions or references to obscure remarks in class or in the notes. Everything can be answered from the basic material.

[1]Compiled on Tuesday 5$^{\text{th}}$ October, 2021 at 16:17

- **Paper structure**
  - There is one-page long blank space for each problem, but it does ***not*** mean that you need that space. Some questions can be answered in a few words and/or equations. If needed, you may use the reverse side as well to answer; but make sure to specifically write the problem number you are answering.

Instructions for Specific Problem Types

For "Select One" questions, please fill in the appropriate bubble completely:

Select One: Who taught this course?

● Michael Kaess

○ Marie Curie

○ Noam Chomsky

○ Martial Hebert

For "Select all that apply" questions, please fill in all appropriate squares completely:

Select all that apply: Which are scientists?

■ Stephen Hawking

■ Albert Einstein

■ Isaac Newton

□ None of the above

For questions where you must fill in a blank, please make sure your final answer is fully included in the given space. You may cross out answers or parts of answers, but the final answer must still be within the given space.

Fill in the blank: What is the course number?

16-822

1 Assorted short questions

1. Given a pair of cameras, what motion between the two cameras would produce the essential matrix: $[0\ 0\ 0; 0\ 0\ 1; 0\ \ -1\ 0]$?

2. The transformation defined by the 4×4 matrix $H = [1\ 0\ -1\ 20; 0\ 1\ 0\ 30; 1\ 0\ 1\ 10; 10\ 0\ 0\ 1]$ leaves the conic at infinity unchanged.

   ◯ True

   ◯ False

3. Why or why not? (One sentence with less than 10 words)

4. Given a line l in an image, there exists always a plane Π such that l is the projection of Π.

   ◯ True

   ◯ False

2 Projection of lines

Definition

1. Meet operator $\wedge$ between two planes results in the intersecting line ($p \wedge q = l$)
2. Join operator $\vee$ between two points results in the connecting line ($x \vee y = l$)

1. Given a pair of cameras of projection matrices M and M' and the lines l and l' projections of a line L in 3D space. Write a formula expressing L as a function of l, l',M, and M', using linear algebra operations and one of the meet/join operators. Simply write an equation of the form $L = \ldots$. Do *not* try to make explicit the "coordinates" of L any further.

2. Given a pair of projective cameras with fundamental matrix F, let L_1 (resp. L_2) be a line in space, l_1 (resp. l_2) its projection in one of the cameras, and l'_1 (resp. l'_2) its projection in the other camera. Derive a relationship that l'_1, l'_2, l_1, l_2, and F satisfy if L_1 and L_2 are coplanar. (Hint: Think what very simple geometric property of 2 lines in $3D$ is satisfied iff they are coplanar (and, consequently, is always satisfied by any pair of lines in the 2D plane.))

3 More on line homography

Problem setup

Consider a system of two projective cameras. Assume that we selected an arbitrary line l in the first image, and l' in the second image. We define a mapping $h : l \to l'$ such that $p' = h(p)$ is the point on l' compatible with p given the epipolar geometry of the cameras.

1. We said that we choose the lines "arbitrarily". Explain why we cannot quite choose them arbitrarily and what is the condition that the lines must satisfy for h to be defined.

4 3D reconstruction from 2 images of planes

Problem setup

We tackle the problem of reconstructing the structure of a scene made of planes and simultaneously recovering the motion of a camera. A typical scenario for this would be a camera on a mobile robot observing planes at two different poses of the camera separated by a rotation R' and a translation t'. More precisely, we assume that the camera is calibrated so we can ignore K and assume $K = I$. Taking the coordinate system of the first image as the reference, the projection matrix of the second camera is $M' = [R' \; t']$.
We assume that 2 planes Π_1 and Π_2 are visible from the two positions of the robot. We do not know the position of point correspondences between the images of the planes at the 2 positions of the cameras, but we assume that we have estimated the homographies H_1 and H_2 of the 2 planes between the 2 camera poses (*i.e.,* the image p of a point of Π_1 in the first image is related to its position in the second image by: $p' = H_1 p$.)
Finally, denote the normals to Π_1 and Π_2 by n_1 and n_2, respectively, and the distance to the origin by d_1 and d_2, respectively.

1. Show that the following can be computed from H_1 and H_2.[2]

   (a) (15 points) The motion of the camera (R' and t').

[2] This classical result shows that it is possible to do structure from motion (SFM) from $\geq$ 2 planes, a useful fact in practice. Incidentally, the assumption that we know only the homographies sounds a little artificial, but in fact it corresponds to situations in which we can track the deformation of planar regions across images by correlating texture regions, for example, rather than by matching point features. In such cases, it is very natural to use the homographies directly.

(b) (5 points) The structure of the scene (n_1, n_2, d_1, d_2).

Assumptions

You will assume that:

1. The cameras are calibrated so that you can assume $K = K' = I$, and the fundamental matrix F is the essential matrix E in this case.
2. We know how to compute t' and R' from the essential matrix E. In other words, it is sufficient to show how to compute E, you can assume that t' and R' are known from E.

Hint

1. Use results derived in class on how to express planar homographies as function of the plane and of the transformation between the images, and how to relate the essential (or fundamental, same thing) matrix to a planar homography.
2. Once you've recovered E, the quantity $t^T t$ is known.

5 Plane homographies

1. For a camera with projection matrix $[A\ b]$, what is the center of projection of the camera?

2. Given two cameras with projection matrices $M = [I\ 0]$ and $M' = [A'\ b']$ and a plane of parameter $\pi = [u^T\ 1]^T$, show that the planar homography that maps the image of point on the plane in the first camera to a point in the second one is: $H = A' - b'u^T$.

3. Show that H is degenerate (singular) if the plane contains the center of the second camera.

4. Geometrically (no equations) what is the projection of the plane in the second image in that case?

6 Plane homographies under translational motion

Problem setup

In this problem we study how the image of a plane is formed in a camera undergoing a pure translational motion (for example, a camera on a mobile robot in translation). Assuming an Euclidean reference frame, let Π denote a plane of normal n and distance to the origin d. The plane is represented by the vector $\Pi = [n^T \; d]^T$. We assume that $d \neq 0$ (the plane does not pass through the origin.)

1. We assume that the first camera has projection matrix $M = K[I \; 0]$ and that the second camera has matrix $M' = K[R' \; t']$. We assume in this problem that the motion is translational only so $R' = I$. Let H be the homography from the first image to the second corresponding to the plane Π. Show that the points on the projection l_∞ of the line at infinity of Π (*i.e.*, the vanishing line of Π) remain fixed under H (*i.e.*, $Hp = p$ for $p \in l_\infty$.)

2. There is another point *not* on l_∞ that remains fixed under H. What is this point?

Note

H leaves a line invariant and one point not on the line also invariant. Such a homography is called a homology.