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SSA Form: Construction

Lecture 4: CPEN 400P

Karthik Pattabiraman, UBC

Outline

Def-Use Chains

Why do we need SSA form ?

Dominator Trees and Dominance Frontiers

Inserting Phi-Nodes

Renaming Variables

Converting Out of SSA form

Information Chains

A tuple that connects 2 data-flow events is a *chain*

| event \equiv definition
or use

- Chains express data-flow relationships directly
- Chains provide a graphical representation
- Chains jump across unrelated code, simplifying search

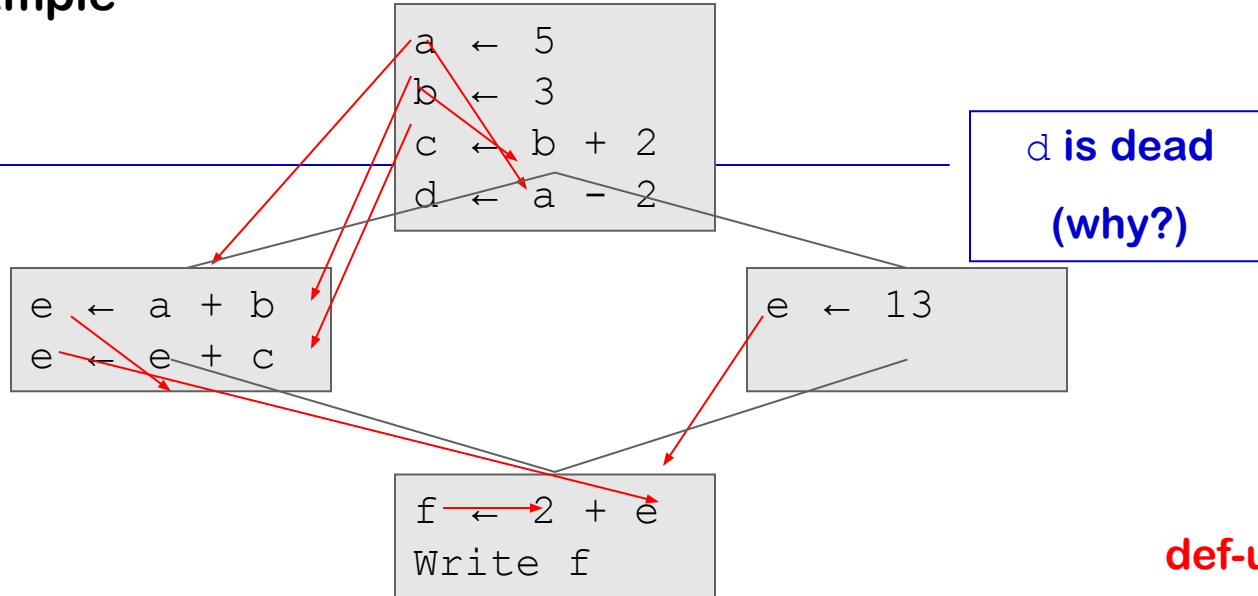
We can build chains efficiently

Def-Use chains are
the most common

Four interesting types of chain

Information Chains

Example



def-use chains

Notation

Assume that, \forall operation i and each variable v ,

- $DEFS(v,i)$ is the set of operations that may have defined v most recently before i , along some path in the CFG
- $USES(v,i)$ is the set of operations that may use the value of v computed at i , along some path in the CFG

$$x \in DEFS(A,y) \Leftrightarrow y \in USES(A,x)$$

To construct DEF-USE chains, we solve *reaching definitions* (*how?*)

Domain is |definitions|, same
as number of operations

Reaching Definitions

The equations

$$REACHES(n) = \emptyset, \forall n \in N$$

$$REACHES(n) = \bigcup_{p \in preds(n)} (DEDEF(p) \cup \overline{(REACHES(p) \cap DEFKILL(p))})$$

- $REACHES(n)$ is the set of definitions that reach block n
- $DEDEF(N)$ is the set of definitions in n that reach the end of n
- $DEFKILL(n)$ is the set of defs obscured by a new def in n

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Issues with simple def-use Chains

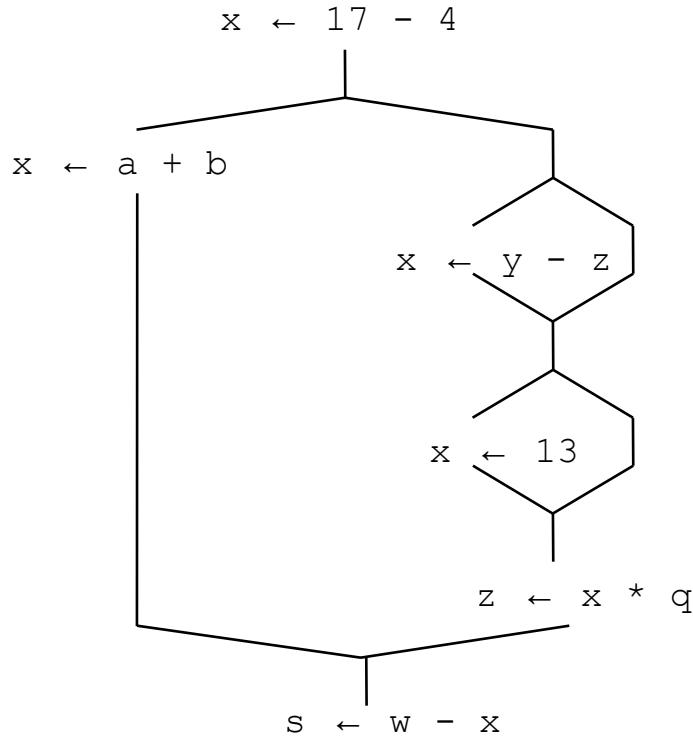
Don't encode control-flow information and data-flow in a unique way

Can't trace each value to it's definition as definition may be non-unique

Update of values is dependent on number of “meet points” in standard data-flow analysis - can take a long time to converge

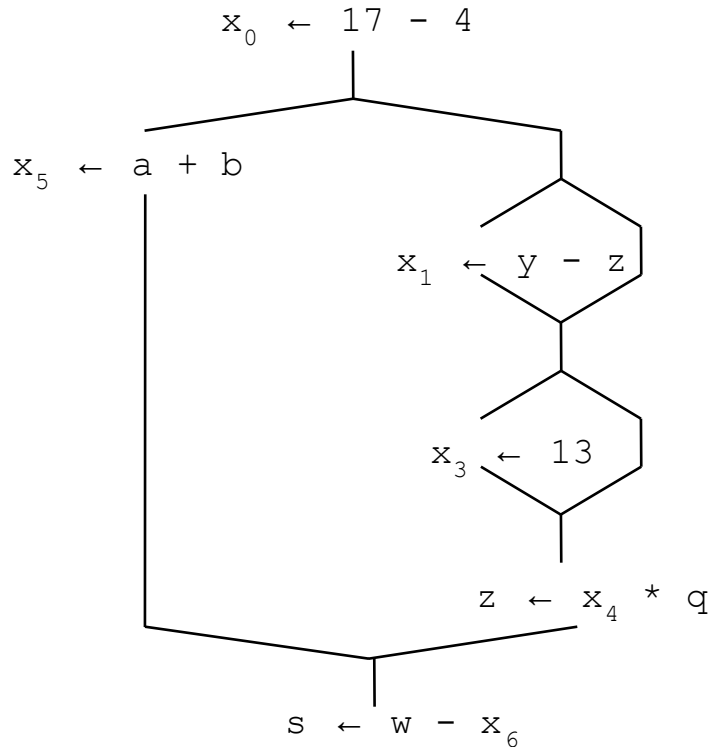
Need a way to encode both data-flow and control information efficiently to allow multiple data-flow analysis problems to be run on them

Example (without SSA form)



There are four birth points for x

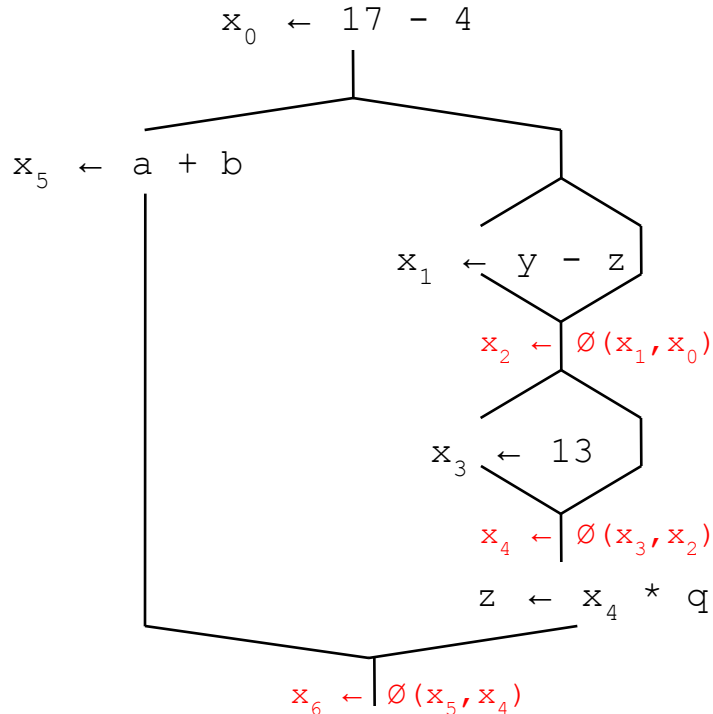
Example (without SSA form, but with renaming)



There are four birth points for x

Example (with SSA form)

Making Birth Points Explicit



Building Static Single Assignment Form

SSA-form

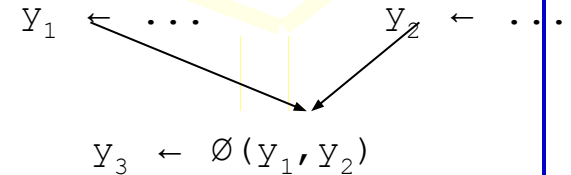
- Each name is defined exactly once
- Each use refers to exactly one name

What's hard

- Straight-line code is trivial
- Splits in the CFG are trivial
- Joins in the CFG are hard

A ϕ -function is a special kind of copy that selects one of its parameters.

The choice of parameter is governed by the CFG edge along which control reached the current block.



Few machines implement a ϕ -function in hardware.*10

Simple Algorithm For Building SSA form ?

Simple algorithm

1. Insert a ϕ at each join point for each name
2. Solve *reaching definitions*
3. Rename each name to get single definition & single use

This produces

- Correct SSA form
- More ϕ 's than any other known algorithm for SSA construction (too many to be practical)

The rest is optimization (!)

SSA Construction Algorithm - 1

(Detailed sketch)

1. Insert ϕ -functions

a.) calculate dominance frontiers

Moderately complex

b.) find global names

Compute list of blocks where each name is assigned & use as a worklist

for each name, build a list of blocks that define it

c.) insert ϕ -functions

\forall global name n

Creates the iterated dominance frontier

\forall block b in which n is assigned

\forall block d in b 's dominance frontier

insert a ϕ -function for n in d

add d to n 's list of defining blocks

This adds to the worklist !

Use a checklist to avoid putting blocks on the worklist twice; keep another checklist to avoid inserting the same ϕ -function twice.

SSA Construction Algorithm - 2

(Detailed sketch)

2. Rename variables in a pre-order walk over dominator tree
(use an array of stacks, one stack per global name)

Starting with the root block, b

1 counter per name for subscripts

- a.) generate unique names for each ϕ -function
and push them on the appropriate stacks
- b.) rewrite each operation in the block
 - i. Rewrite uses of global names with the current version
(from the stack)
 - ii. Rewrite definition by inventing & pushing new name
- c.) fill in ϕ -function parameters of successor blocks
- d.) recurse on b 's children in the dominator tree
- e.) <on exit from block b > pop names generated in b from stacks

Reset the state

Need the end-of-block name for this path

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Dominators (Recap)

Definition:

x dominates y if and only if every path from the entry of the control-flow graph to the node for y includes x

- By definition, x dominates x
- The first entry node of a procedure dominates every block in it
- We associate a DOM set with each node
- $|\text{DOM}(x)| \geq 1$

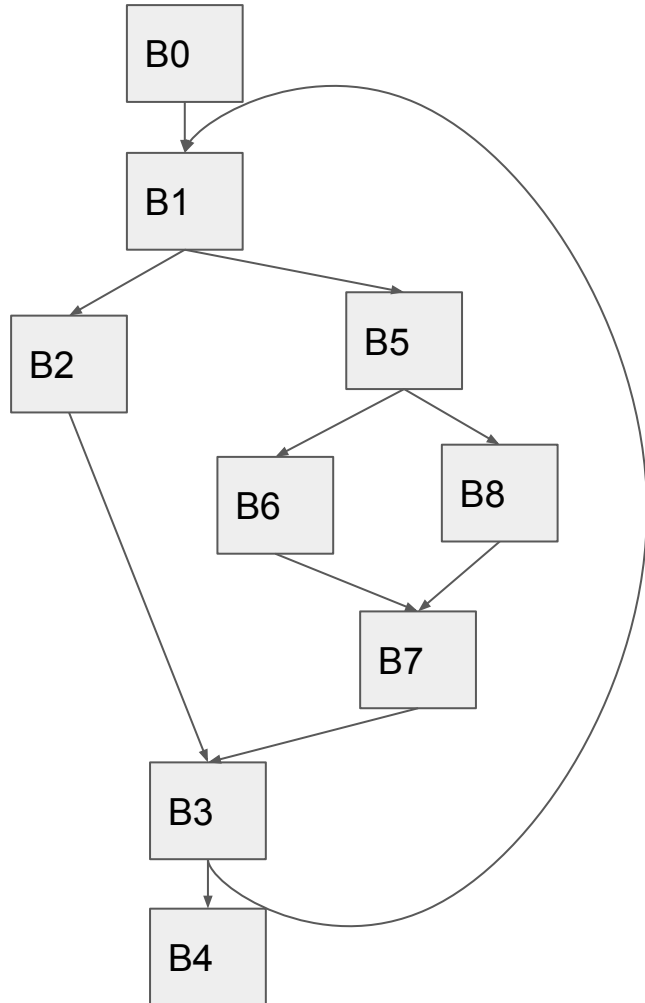
Immediate Dominators and Dominator Tree

Immediate dominator:

- For any node x , there must be a y in $\text{DOM}(x)$ closest to x (*different from x*)
- We call this y the immediate dominator of x
- Note that
- As a matter of notation, we write this as $\text{IDOM}(x)$

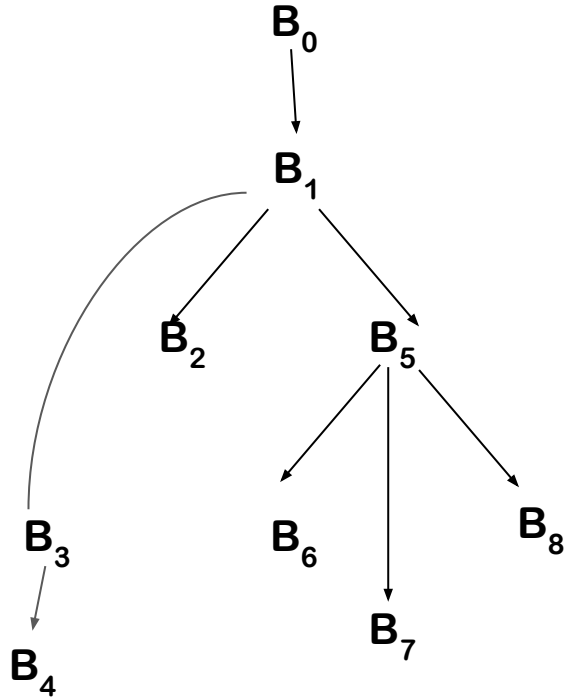
Dominator Tree: Contains every node of the flow graph and encodes IDOM set:

- If m is $\text{IDOM}(n)$, then there is an edge in the dominator tree from m to n
- Given a node in the dominator tree, its parent is $\text{IDOM}(n)$



Basic Block	Dominator Set	IDOM
B0	{ 0 }	-
B1	{ 0, 1 }	0
B2	{ 0, 1, 2 }	1
B3	{ 0, 1, 3 }	1
B4	{ 0, 1, 3, 4 }	3
B5	{ 0, 1, 5 }	1
B6	{ 0, 1, 5, 6 }	5
B7	{ 0, 1, 5, 7 }	5
B8	{ 0, 1, 5, 8 }	5

Dominator Tree



Basic Block	Dominator Set	IDOM
B0	{ 0 }	-
B1	{ 0, 1 }	0
B2	{ 0, 1, 2 }	1
B3	{ 0, 1, 3 }	1
B4	{ 0, 1, 3, 4 }	3
B5	{ 0, 1, 5 }	1
B6	{ 0, 1, 5, 6 }	5
B7	{ 0, 1, 5, 7 }	5
B8	{ 0, 1, 5, 8 }	5

Dominance Frontiers: Intuition

Where does an assignment in block n induce ϕ -functions in SSA form?

- $n \text{ DOM } m \Rightarrow$ no need for a ϕ -function in m
 - > Definition in n blocks any previous definition from reaching m
- If m has multiple predecessors, and n (strictly) dominates some of them, but not all of them, m needs a ϕ -function for each definition in n

This is also known as the dominance frontier of m - these are the locations at which phi nodes need to be inserted (with some minor optimizations)

Dominance Frontier: Formal Definition

More formally, m is in the dominance frontier of n if and only if

1. $\exists p \in \text{preds}(m)$ such that $n \in \text{DOM}(p)$, and
 2. n does not strictly dominate m ($n \notin \text{DOM}(m) - \{ m \}$)
- This notion of dominance frontier is precisely what we need to insert ϕ -functions:

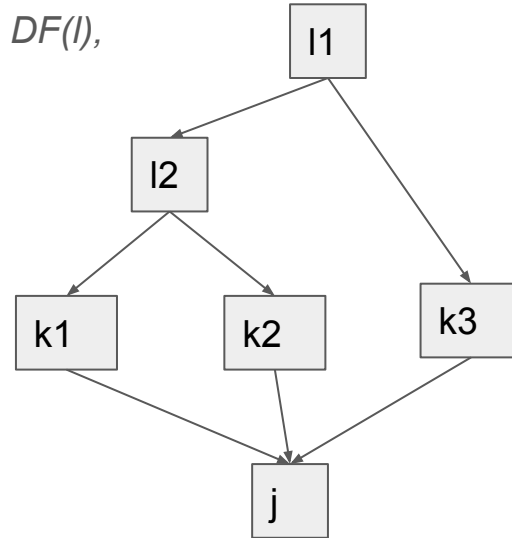
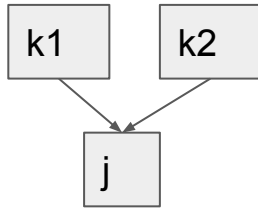
a def in block n induces a ϕ -function in each block in $\text{DF}(n)$.

Why do you need “strict” dominance in the above definition ?

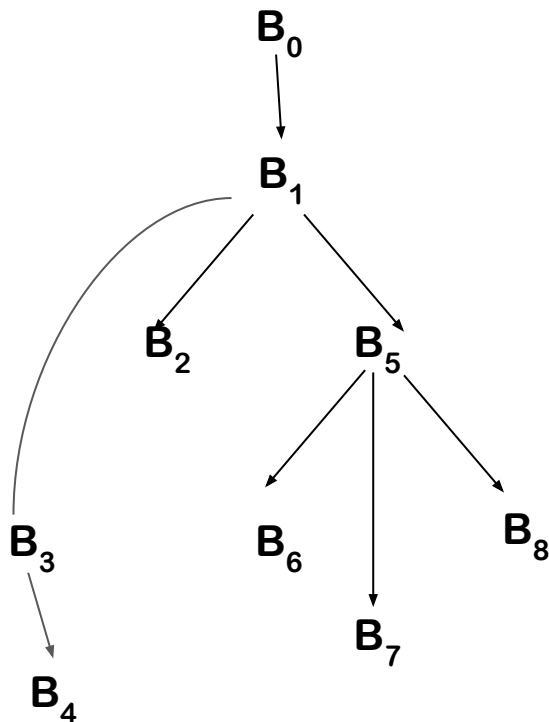
- *Single loop basic blocks (as n doesn't strictly dominate itself)*

Algorithm for computing DFs: Observations

1. Only nodes in the join points of the CFG can be in the dominance frontier
2. For a joint point j , each predecessor k of j must have j in $DF(k)$ (Why ?)
3. If j is in $DF(k)$ for some predecessor k , then
 - For each node l in $DOM(k)$, j must also be in $DF(l)$,
 - Unless l is in $DOM(j)$ (Why ?)



Algorithm for Computing DFs: Intuition



Computing Dominance Frontiers

- Only join points are in $DF(n)$ for some n
- Leads to a simple, intuitive algorithm for computing dominance frontiers

For each join point x (i.e., $|preds(x)| > 1$)

For each CFG predecessor p of x

Run from p to $IDOM(x)$ in the dominator tree, & add x to $DF(n)$ for each n from p up to but not $IDOM(x)$

Algorithm for computing DFs: Pseudo-code

For all nodes, n , in the CFG

$DF(n) \leftarrow \phi$

For all nodes, n , in the CFG

If n has multiple predecessors then

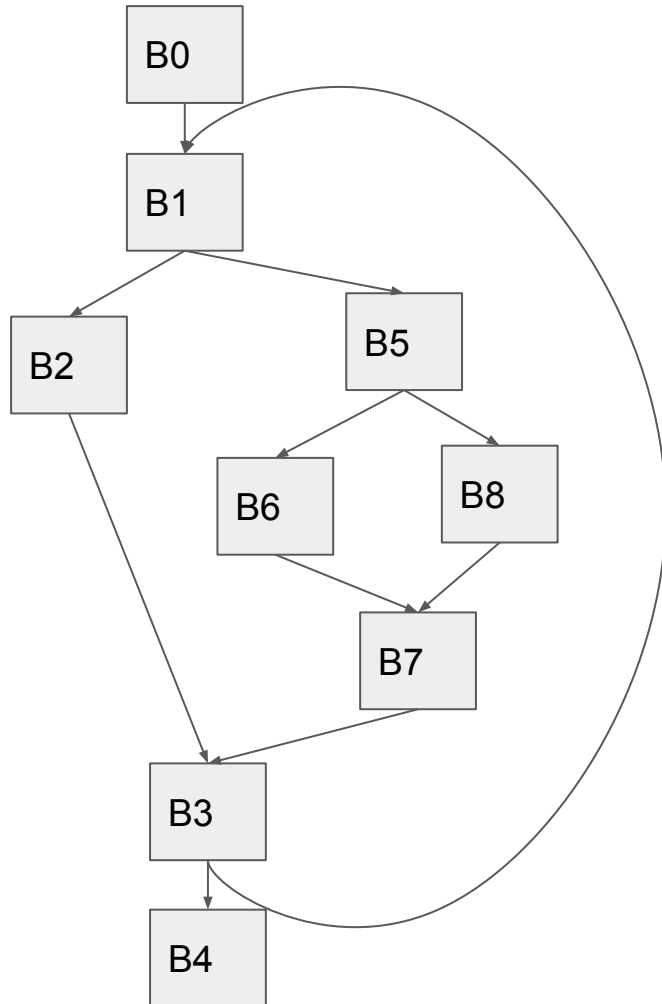
For each predecessor p of n

runner $\leftarrow p$

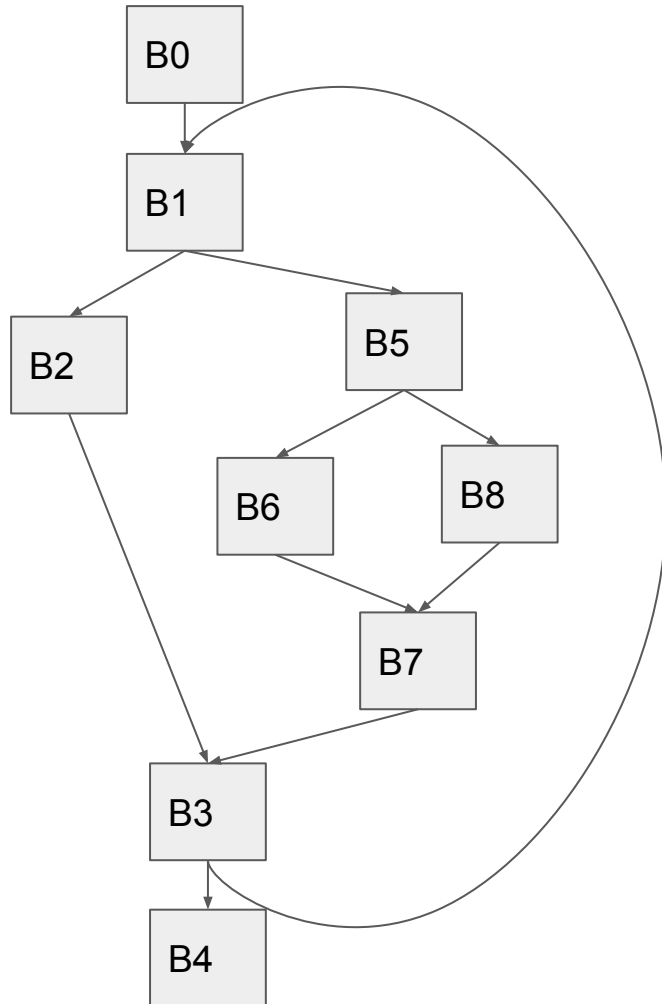
While (runner \neq IDOM(n))

$DF(\text{runner}) \leftarrow DF(\text{runner}) \cup \{ n \}$ // Add the node to the DF of the runner

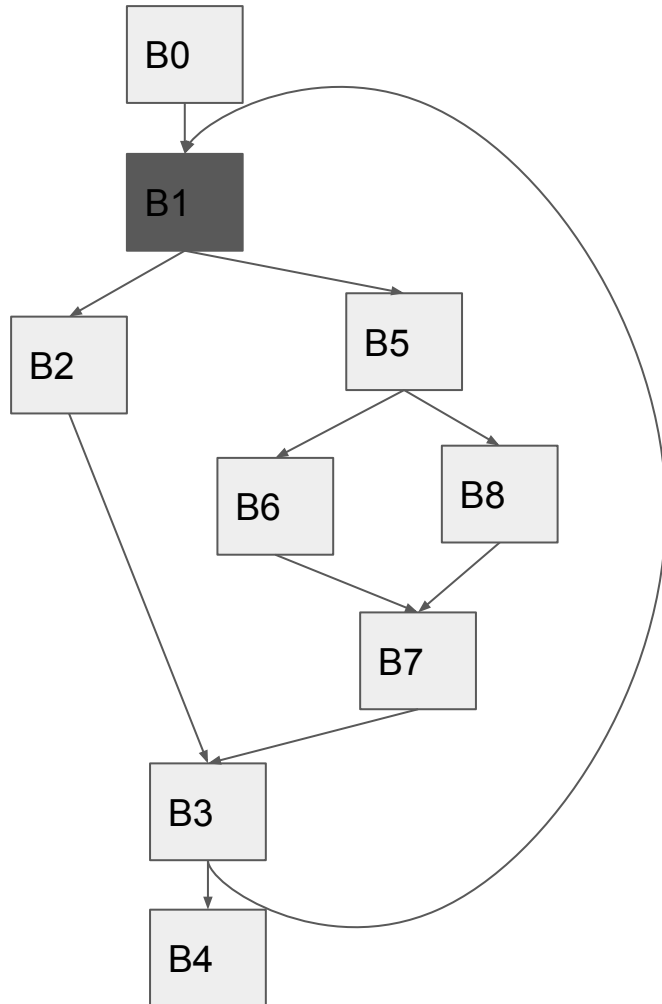
runner \leftarrow IDOM(runner) // Go to predecessor node in the DOM Tree



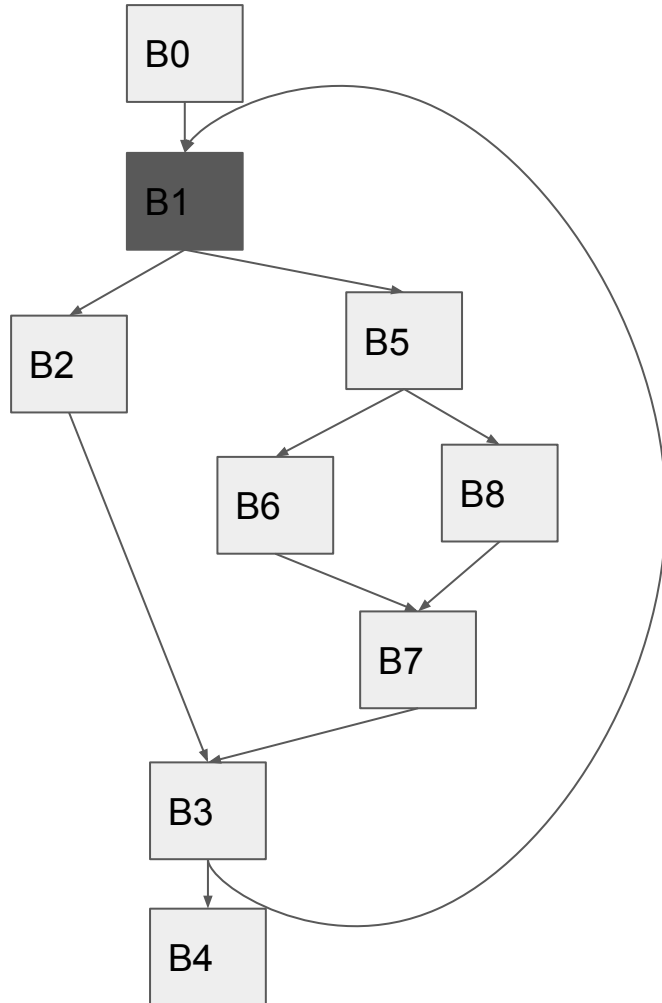
Basic Block	IDOM	DF
B0	-	
B1	0	
B2	1	
B3	1	
B4	3	
B5	1	
B6	5	
B7	5	
B8	5	



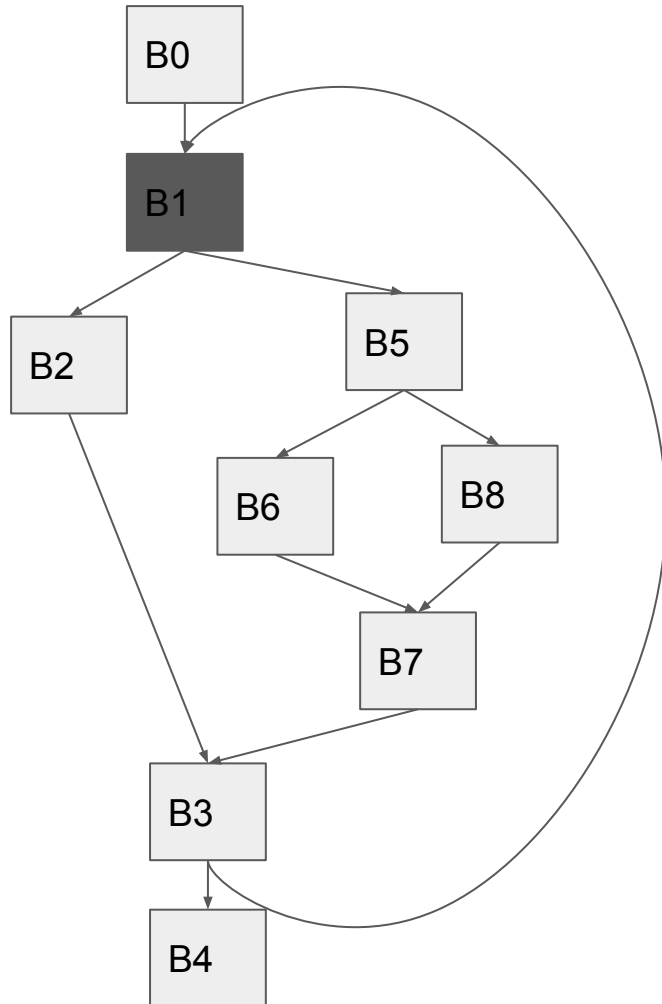
Basic Block	IDOM	DF
B0	-	
B1	0	
B2	1	
B3	1	
B4	3	
B5	1	
B6	5	
B7	5	
B8	5	



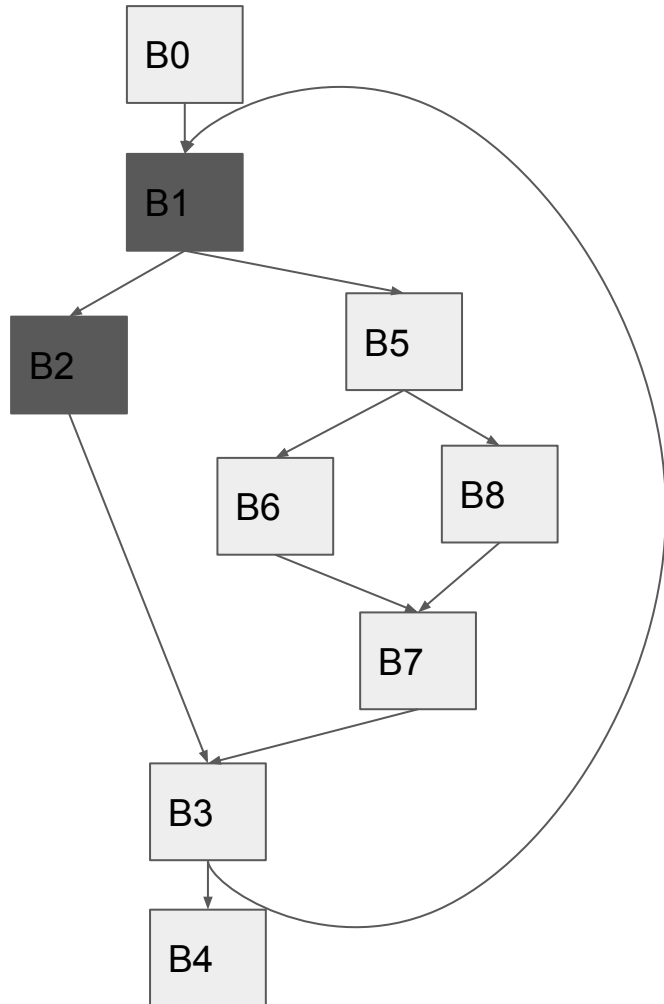
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ }
B2	1	{ }
B3	1	{ }
B4	3	{ }
B5	1	{ }
B6	5	{ }
B7	5	{ }
B8	5	{ }



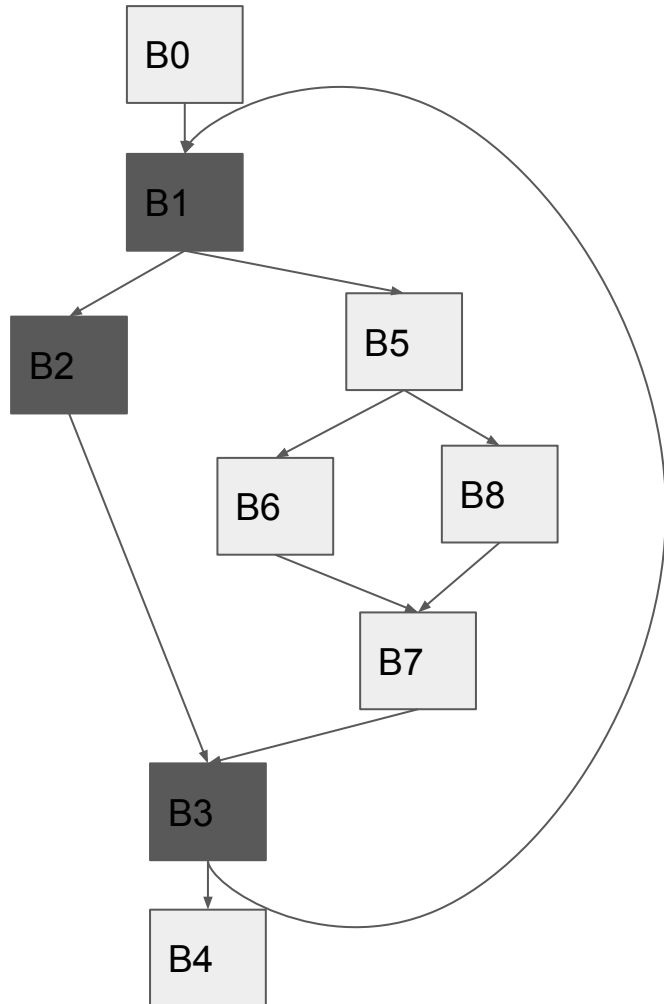
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ }
B2	1	{ }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ }
B6	5	{ }
B7	5	{ }
B8	5	{ }



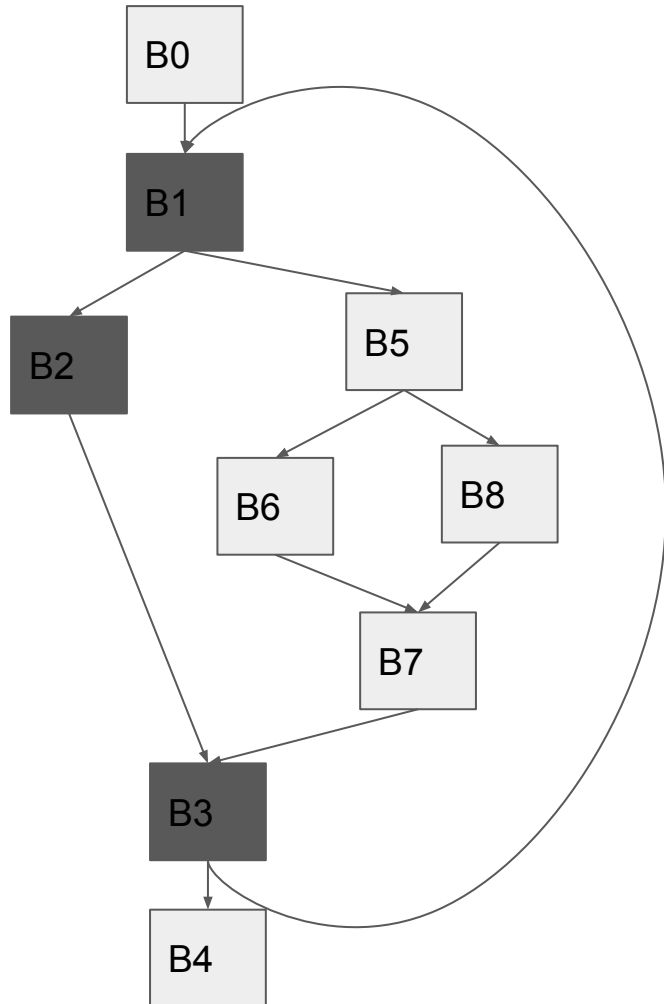
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ }
B6	5	{ }
B7	5	{ }
B8	5	{ }



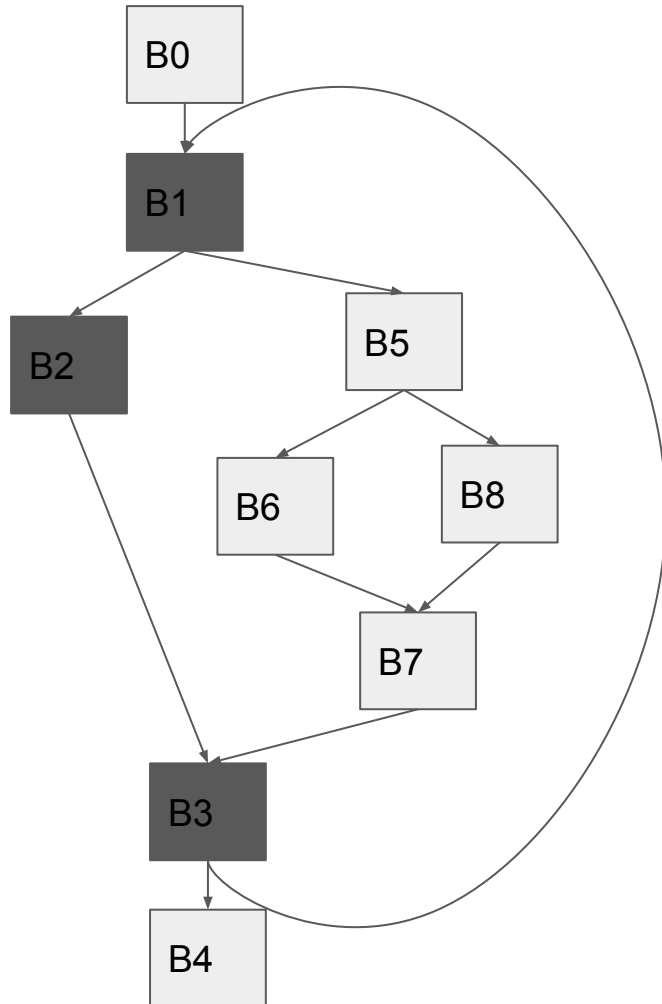
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ }
B6	5	{ }
B7	5	{ }
B8	5	{ }



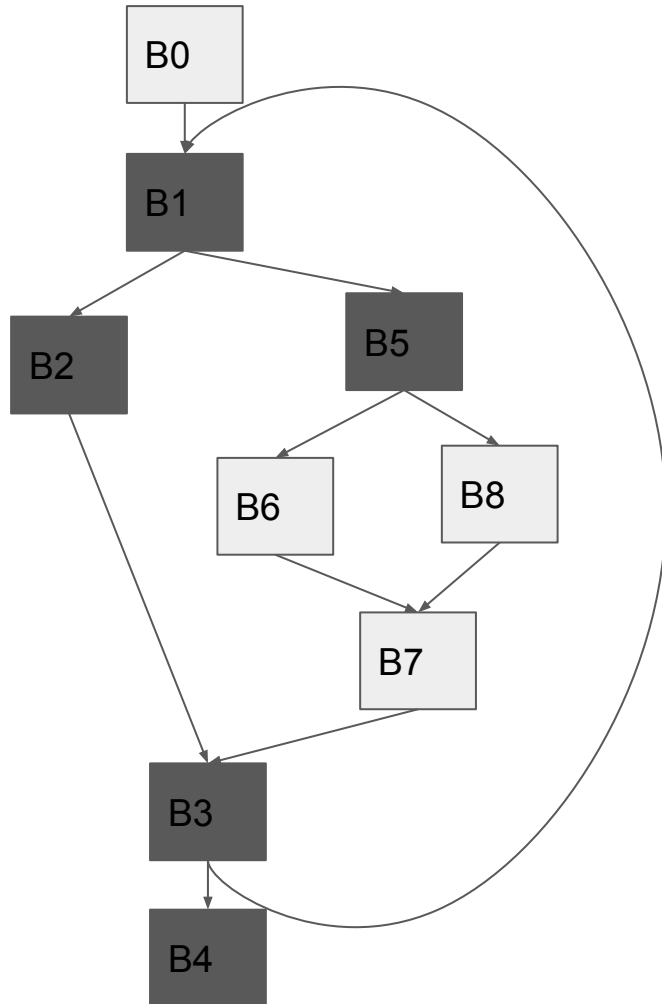
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ }
B6	5	{ }
B7	5	{ }
B8	5	{ }



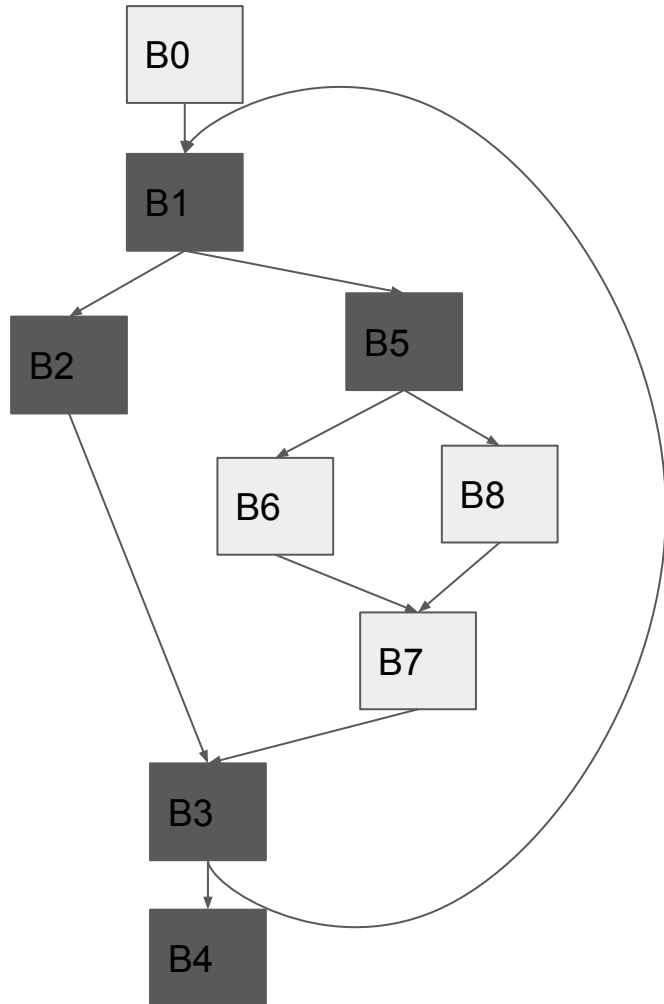
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ }
B6	5	{ }
B7	5	{ 3 }
B8	5	{ }



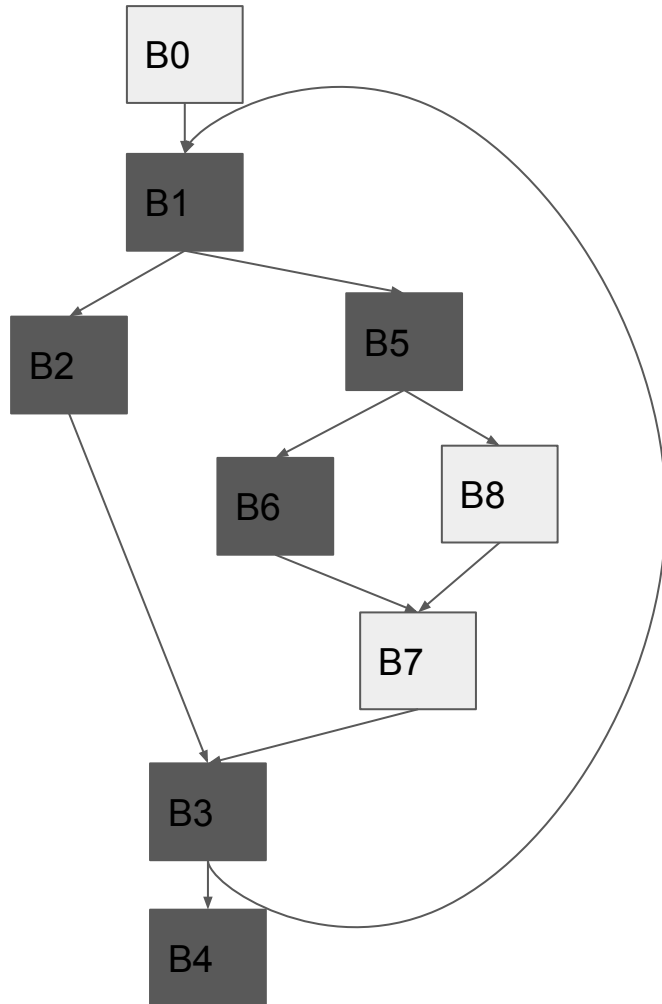
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ 3 }
B6	5	{ }
B7	5	{ 3 }
B8	5	{ }



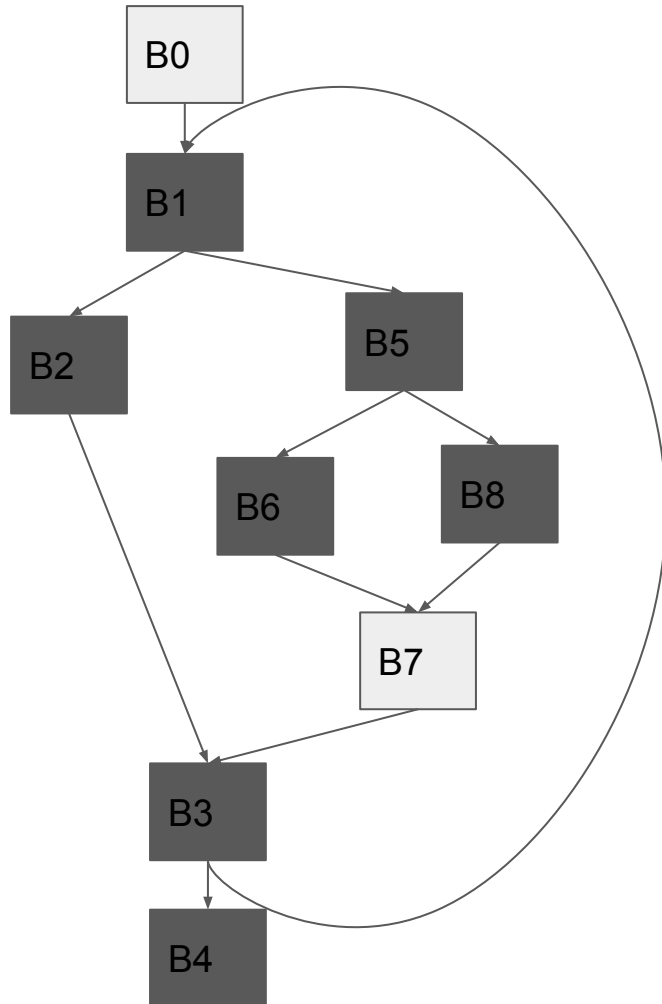
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ 3 }
B6	5	{ }
B7	5	{ 3 }
B8	5	{ }



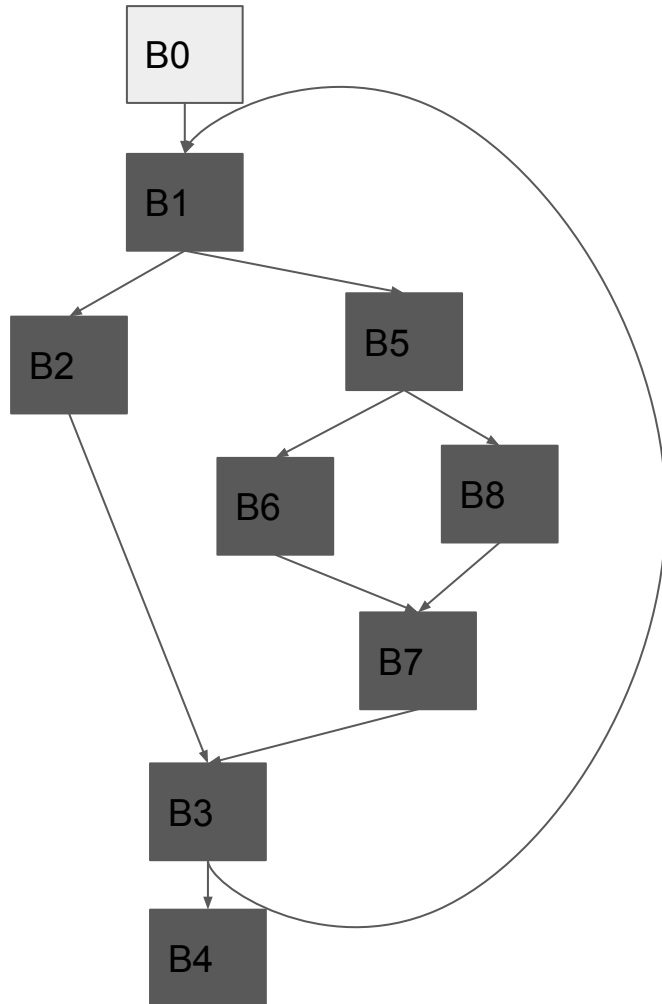
Basic Block	IDOM	DF
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B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ 3 }
B6	5	{ }
B7	5	{ 3 }
B8	5	{ }



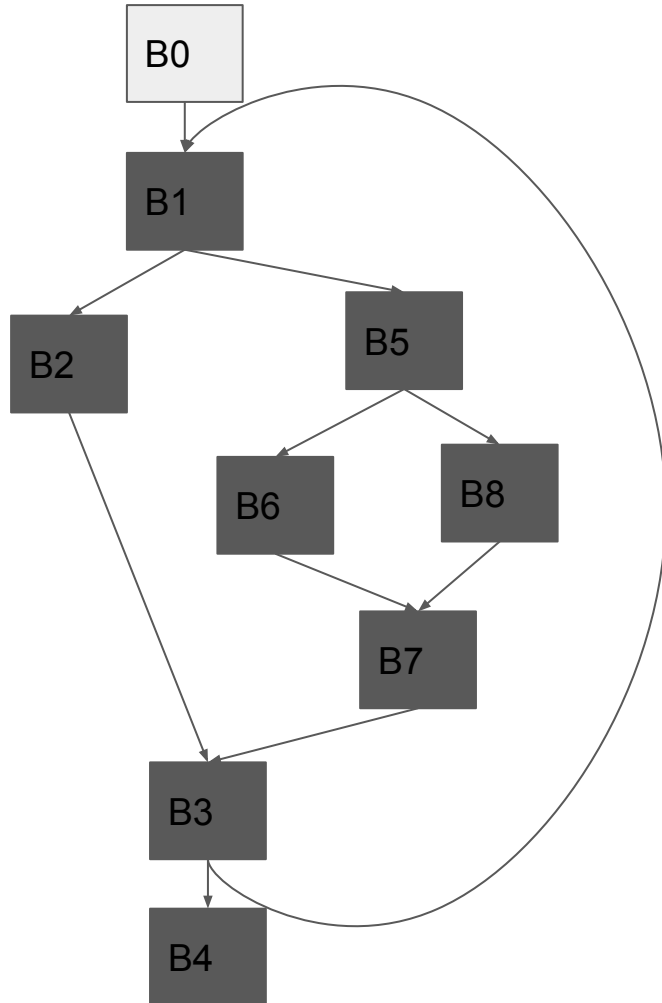
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ 3 }
B6	5	{ }
B7	5	{ 3 }
B8	5	{ }



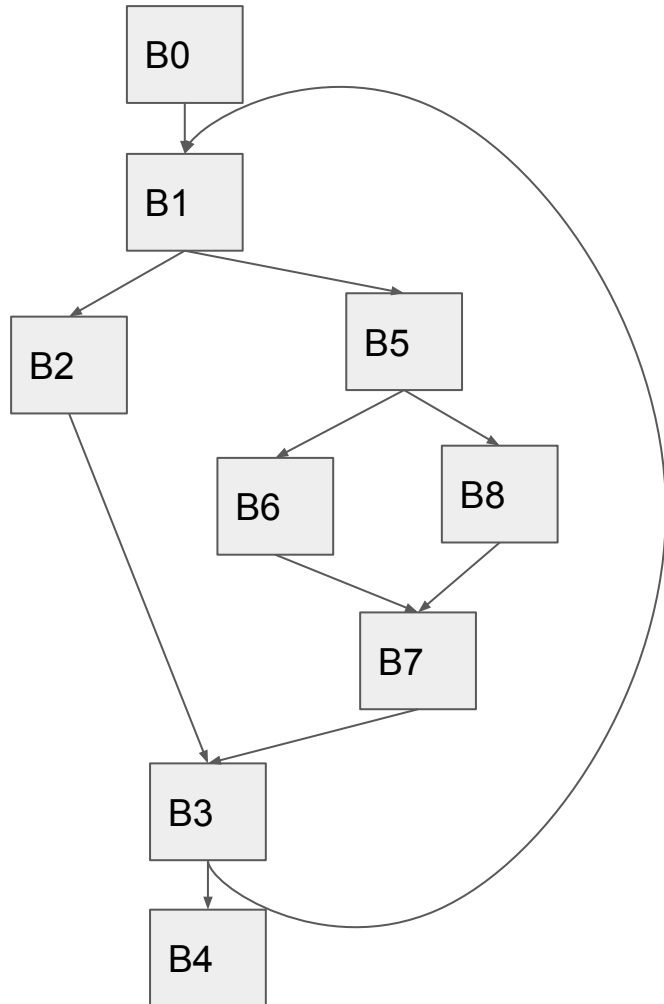
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ 3 }
B6	5	{ }
B7	5	{ 3 }
B8	5	{ }



Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ 3 }
B6	5	{ 7 }
B7	5	{ 3 }
B8	5	{ }



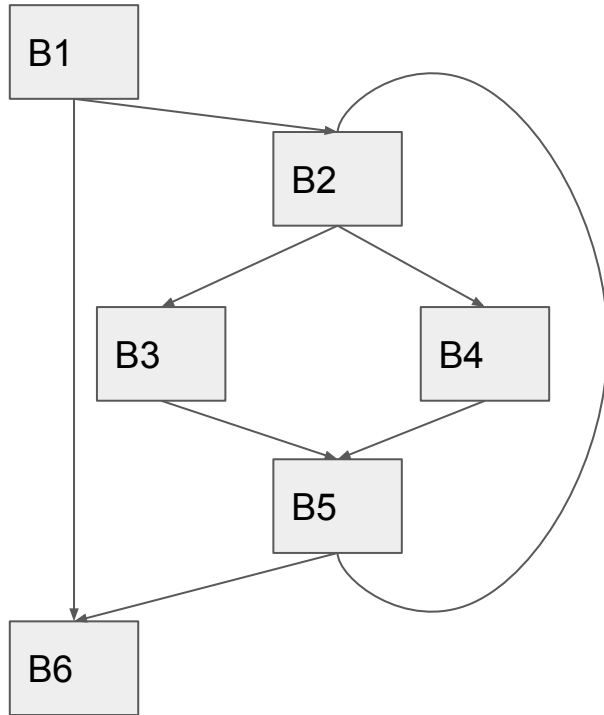
Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ 3 }
B6	5	{ 7 }
B7	5	{ 3 }
B8	5	{ 7 }



Basic Block	IDOM	DF
B0	-	{ }
B1	0	{ 1 }
B2	1	{ 3 }
B3	1	{ 1 }
B4	3	{ }
B5	1	{ 3 }
B6	5	{ 7 }
B7	5	{ 3 }
B8	5	{ 7 }

Class Activity

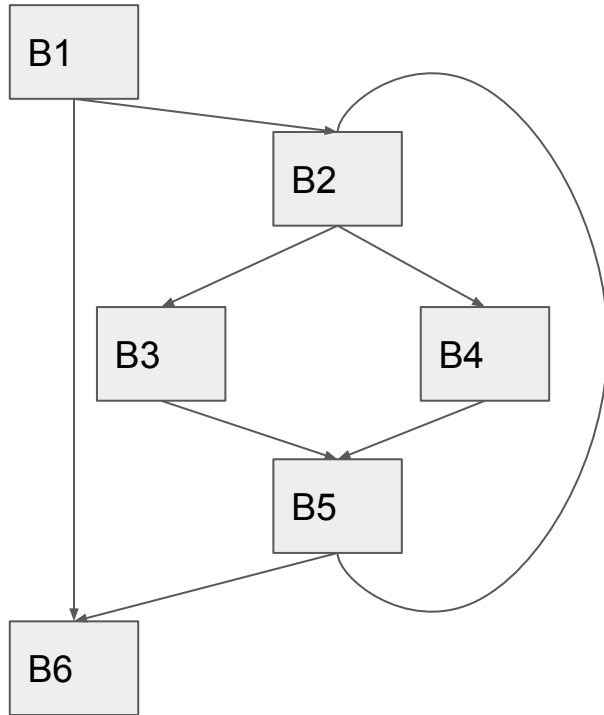
Calculate the DF for each node in the CFG below using the DF algorithm.



Block	DOM	IDOM	DF
B1			
B2			
B3			
B4			
B5			
B6			

Class Activity: Solution

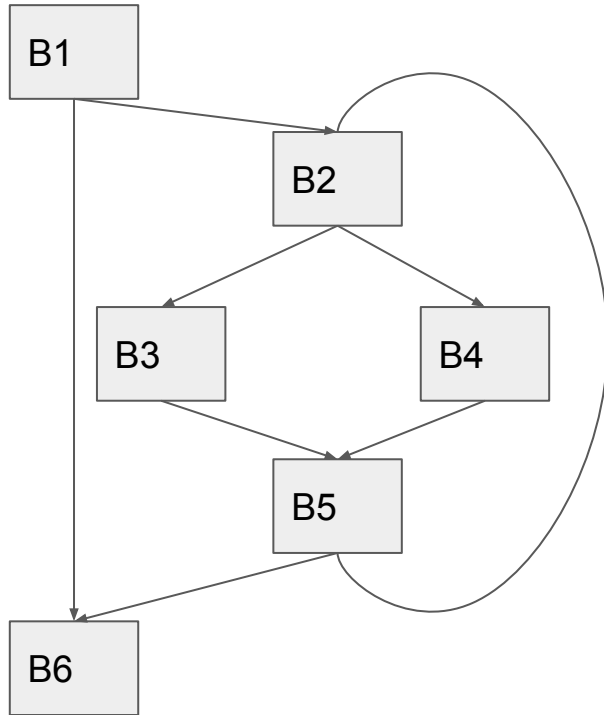
Calculate the DF for each node in the CFG below using the DF algorithm.



Block	DOM	IDOM	DF
B1	{ }		
B2	{ 1 , 2 }		
B3	{ 1, 2, 3 }		
B4	{ 1, 2, 4 }		
B5	{ 1, 2, 5 }		
B6	{ 1, 6 }		

Class Activity: Solution

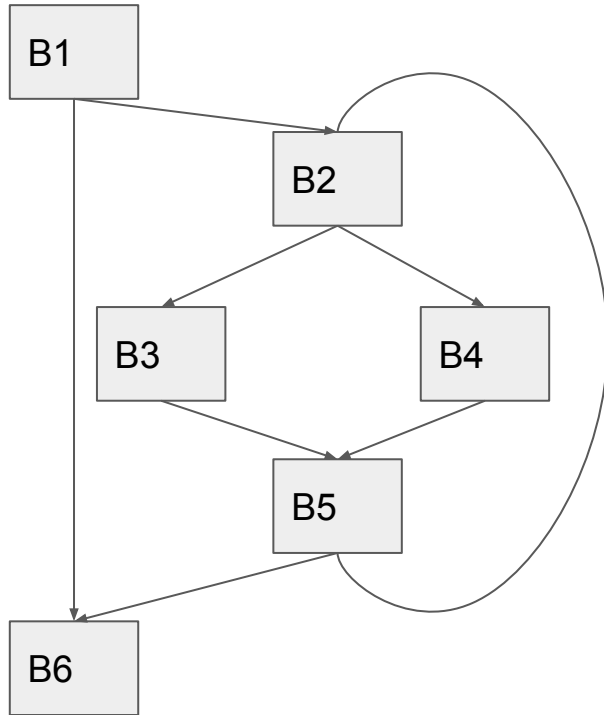
Calculate the DF for each node in the CFG below using the DF algorithm.



Block	DOM	IDOM	DF
B1	{ }	-	{
B2	{ 1 , 2 }	1	
B3	{ 1, 2, 3 }	2	
B4	{ 1, 2, 4 }	2	
B5	{ 1, 2, 5 }	2	
B6	{ 1, 6 }	1	

Class Activity: Solution

Calculate the DF for each node in the CFG below using the DF algorithm.



Block	DOM	IDOM	DF
B1	{ }	-	{ }
B2	{ 1 , 2 }	1	{ 2 , 6 }
B3	{ 1 , 2 , 3 }	2	{ 5 }
B4	{ 1 , 2 , 4 }	2	{ 5 }
B5	{ 1 , 2 , 5 }	2	{ 2 , 6 }
B6	{ 1 , 6 }	1	{ }

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Renaming Variables

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Now that we have DFs, how do we insert the Phi-Nodes ?

Insert phi nodes in all blocks that are in the $DF(b)$, where b is the block in which the variable 'x' is defined

- $DF(b)$ represents the first join-point in the CFG in which 'x' does NOT dominate any downstream uses of 'x'.
- Phi-node ensures that the invariant of SSA is preserved, every def dominates all its uses
- Can be pruned if the variable 'x' is not Live across multiple basic blocks (any variable that is live across multiple blocks is added to *Globals*)
- Variables that are live across basic blocks can be computed as $UEVar(b)$

Pseudo-code for algorithm

For each name x in *Globals*:

$WorkList \leftarrow Blocks(x)$ // Set of Basic Blocks in which ' x ' is defined

 For each block b in the $WorkList$:

 For each block d in $DF(b)$:

 If d has no ϕ for x in d then:

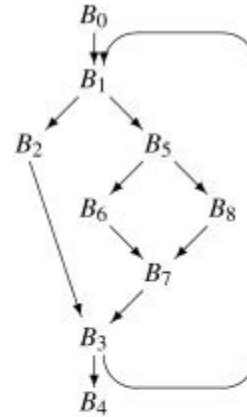
 Insert a ϕ function for x in d

$WorkList \leftarrow WorkList \cup \{ d \}$ // Why do we need this ?

Running Example (Same as Class Activity in DFA lecture)

B_0 : $i \leftarrow 1$
 $\rightarrow B_1$
 B_1 : $a \leftarrow \dots$
 $c \leftarrow \dots$
 $(a < c) \rightarrow B_2, B_5$
 B_2 : $b \leftarrow \dots$
 $c \leftarrow \dots$
 $d \leftarrow \dots$
 $\rightarrow B_3$
 B_3 : $y \leftarrow a + b$
 $z \leftarrow c + d$
 $i \leftarrow i + 1$
 $(i \leq 100) \rightarrow B_1, B_4$
 B_4 : return
 B_5 : $a \leftarrow \dots$
 $d \leftarrow \dots$
 $(a \leq d) \rightarrow B_6, B_8$
 B_6 : $d \leftarrow \dots$
 $\rightarrow B_7$
 B_7 : $b \leftarrow \dots$
 $\rightarrow B_3$
 B_8 : $c \leftarrow \dots$
 $\rightarrow B_7$

(a) Code for the Basic Blocks



(b) Control-Flow Graph

	B_0	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8
UEVAR	\emptyset	\emptyset	\emptyset	$\{a, b, c, d, i\}$	\emptyset	\emptyset	\emptyset	\emptyset	\emptyset
VARKILL	$\{i\}$	$\{a, c\}$	$\{b, c, d\}$	$\{y, z, i\}$	\emptyset	$\{a, d\}$	$\{d\}$	$\{b\}$	$\{c\}$

(c) Initial Information

Block	IDOM	DF	Variables def
B0	-	$\{\}$	$\{i\}$
B1	0	$\{1\}$	$\{a, c\}$
B2	1	$\{3\}$	$\{b, c, d\}$
B3	1	$\{1\}$	$\{y, z, i\}$
B4	3	$\{\}$	$\{\}$
B5	1	$\{3\}$	$\{a, d\}$
B6	5	$\{7\}$	$\{d\}$
B7	5	$\{3\}$	$\{b\}$
B8	5	$\{7\}$	$\{c\}$

Algorithm for inserting Phi Nodes

Globals = { a, b, c, d, i }

Blocks

Var	Blocks	Phi Nodes
a	{ 1, 5 }	
b	{ 2, 7 }	
c	{ 1, 2, 8 }	
d	{ 2, 5, 6 }	
i	{ 0, 3 }	

WorkList: []

Block	IDOM	DF	Variables def
B0	-	{ }	{ i }
B1	0	{ 1 }	{ a, c }
B2	1	{ 3 }	{ b, c, d }
B3	1	{ 1 }	{ y, z, i }
B4	3	{ }	{ }
B5	1	{ 3 }	{ a, d }
B6	5	{ 7 }	{ d }
B7	5	{ 3 }	{ b }
B8	5	{ 7 }	{ c }

Algorithm for inserting Phi Nodes

Globals = { a, b, c, d, i }

Blocks

Var	Blocks	Phi Nodes
a	{ 1, 5 }	
b	{ 2, 7 }	
c	{ 1, 2, 8 }	
d	{ 2, 5, 6 }	
i	{ 0, 3 }	

WorkList: [1, 5]

Block	IDOM	DF	Variables def
B0	-	{ }	{ i }
B1	0	{ 1 }	{ a, c }
B2	1	{ 3 }	{ b, c, d }
B3	1	{ 1 }	{ y, z, i }
B4	3	{ }	{ }
B5	1	{ 3 }	{ a, d }
B6	5	{ 7 }	{ d }
B7	5	{ 3 }	{ b }
B8	5	{ 7 }	{ c }

Algorithm for inserting Phi Nodes

Globals = { a, b, c, d, i }

Blocks

Var	Blocks	Phi Nodes
a	{ 1, 5 }	{ 1 }
b	{ 2, 7 }	
c	{ 1, 2, 8 }	
d	{ 2, 5, 6 }	
i	{ 0, 3 }	

WorkList: [5, 1]

Block	IDOM	DF	Variables def
B0	-	{ }	{ i }
B1	0	{ 1 }	{ a, c }
B2	1	{ 3 }	{ b, c, d }
B3	1	{ 1 }	{ y, z, i }
B4	3	{ }	{ }
B5	1	{ 3 }	{ a, d }
B6	5	{ 7 }	{ d }
B7	5	{ 3 }	{ b }
B8	5	{ 7 }	{ c }

Algorithm for inserting Phi Nodes

Globals = { a, b, c, d, i }

Blocks

Var	Blocks	Phi Nodes
a	{ 1, 5 }	{ 1, 3 }
b	{ 2, 7 }	
c	{ 1, 2, 8 }	
d	{ 2, 5, 6 }	
i	{ 0, 3 }	

WorkList: [1, 3]

Block	IDOM	DF	Variables def
B0	-	{ }	{ i }
B1	0	{ 1 }	{ a, c }
B2	1	{ 3 }	{ b, c, d }
B3	1	{ 1 }	{ y, z, i }
B4	3	{ }	{ }
B5	1	{ 3 }	{ a, d }
B6	5	{ 7 }	{ d }
B7	5	{ 3 }	{ b }
B8	5	{ 7 }	{ c }

Algorithm for inserting Phi Nodes

Globals = { a, b, c, d, i }

Blocks

Var	Blocks	Phi Nodes
a	{ 1, 5 }	{ 1, 3 }
b	{ 2, 7 }	
c	{ 1, 2, 8 }	
d	{ 2, 5, 6 }	
i	{ 0, 3 }	

WorkList: []

Block	IDOM	DF	Variables def
B0	-	{ }	{ i }
B1	0	{ 1 }	{ a, c }
B2	1	{ 3 }	{ b, c, d }
B3	1	{ 1 }	{ y, z, i }
B4	3	{ }	{ }
B5	1	{ 3 }	{ a, d }
B6	5	{ 7 }	{ d }
B7	5	{ 3 }	{ b }
B8	5	{ 7 }	{ c }

Algorithm for inserting Phi Nodes

Globals = { a, b, c, d, i }

Blocks

Var	Blocks	Phi Nodes
a	{ 1, 5 }	{ 1, 3 }
b	{ 2, 7 }	
c	{ 1, 2, 8 }	
d	{ 2, 5, 6 }	
i	{ 0, 3 }	

WorkList: [2, 7]

Block	IDOM	DF	Variables def
B0	-	{ }	{ i }
B1	0	{ 1 }	{ a, c }
B2	1	{ 3 }	{ b, c, d }
B3	1	{ 1 }	{ y, z, i }
B4	3	{ }	{ }
B5	1	{ 3 }	{ a, d }
B6	5	{ 7 }	{ d }
B7	5	{ 3 }	{ b }
B8	5	{ 7 }	{ c }

Algorithm for inserting Phi Nodes

Globals = { a, b, c, d, i }

Blocks

Var	Blocks	Phi Nodes
a	{ 1, 5 }	{ 1, 3 }
b	{ 2, 7 }	{ 1, 3 }
c	{ 1, 2, 8 }	
d	{ 2, 5, 6 }	
i	{ 0, 3 }	

WorkList: [1, 2, 8]

Block	IDOM	DF	Variables def
B0	-	{ }	{ i }
B1	0	{ 1 }	{ a, c }
B2	1	{ 3 }	{ b, c, d }
B3	1	{ 1 }	{ y, z, i }
B4	3	{ }	{ }
B5	1	{ 3 }	{ a, d }
B6	5	{ 7 }	{ d }
B7	5	{ 3 }	{ b }
B8	5	{ 7 }	{ c }

Algorithm for inserting Phi Nodes

Globals = { a, b, c, d, i }

Blocks

Var	Blocks	Phi Nodes
a	{ 1, 5 }	{ 1, 3 }
b	{ 2, 7 }	{ 1, 3 }
c	{ 1, 2, 8 }	{ 1, 3, 7 }
d	{ 2, 5, 6 }	{ 1, 3, 7 }
i	{ 0, 3 }	

WorkList: [0, 3]

Block	IDOM	DF	Variables def
B0	-	{ }	{ i }
B1	0	{ 1 }	{ a, c }
B2	1	{ 3 }	{ b, c, d }
B3	1	{ 1 }	{ y, z, i }
B4	3	{ }	{ }
B5	1	{ 3 }	{ a, d }
B6	5	{ 7 }	{ d }
B7	5	{ 3 }	{ b }
B8	5	{ 7 }	{ c }

Algorithm for inserting Phi Nodes

Globals = { a, b, c, d, i }

Blocks

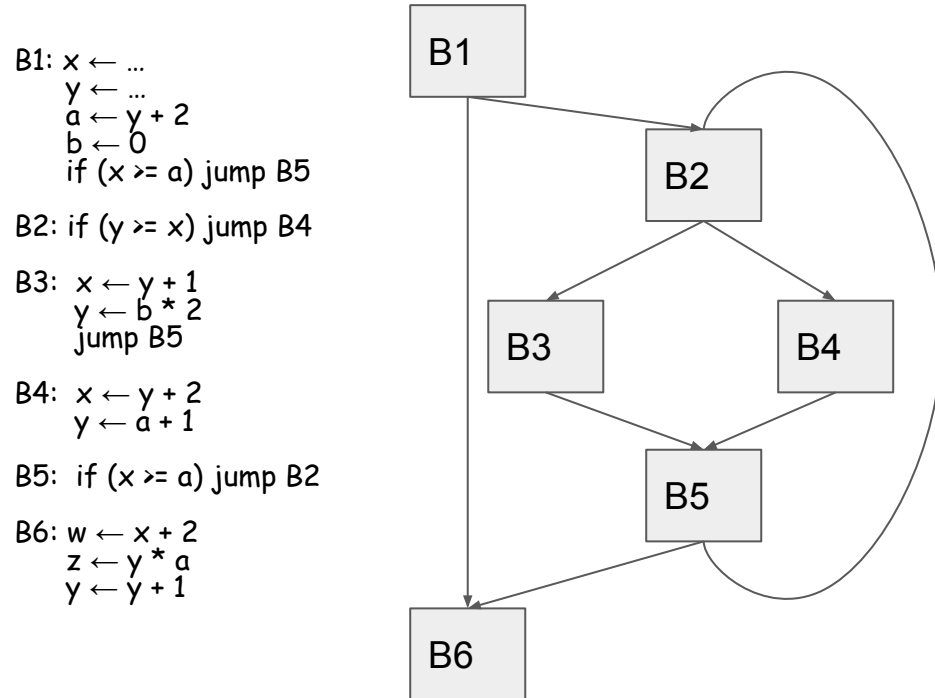
Var	Blocks	Phi Nodes
a	{ 1, 5 }	{ 1, 3 }
b	{ 2, 7 }	{ 1, 3 }
c	{ 1, 2, 8 }	{ 1, 3, 7 }
d	{ 2, 5, 6 }	{ 1, 3, 7 }
i	{ 0, 3 }	{ 1 }

WorkList: []

Block	IDOM	DF	Variables def
B0	-	{ }	{ i }
B1	0	{ 1 }	{ a, c }
B2	1	{ 3 }	{ b, c, d }
B3	1	{ 1 }	{ y, z, i }
B4	3	{ }	{ }
B5	1	{ 3 }	{ a, d }
B6	5	{ 7 }	{ d }
B7	5	{ 3 }	{ b }
B8	5	{ 7 }	{ c }

Class Activity

Insert the Phi-nodes for the example below by identifying the blocks for each variable (same example as in the class activity earlier in this lecture, and in lecture 2: SSA form)



Block	IDOM	DF
B1	-	{ }
B2	1	{ 2, 6 }
B3	2	{ 5 }
B4	2	{ 5 }
B5	2	{ 2, 6 }
B6	1	{ }

Class Activity: Solution

Insert the Phi-nodes for the example below by identifying the blocks for each variable (same example as in the class activity earlier in this lecture, and in lecture 2: SSA form)

Globals = { a, b, x, y, w }

Blocks

	Blocks	Phi-Nodes
x		
y		
a		
b		
w		

Block	IDOM	DF
B1	-	{ }
B2	1	{ 2, 6 }
B3	2	{ 5 }
B4	2	{ 5 }
B5	2	{ 2, 6 }
B6	1	{ }

Class Activity: Solution

Insert the Phi-nodes for the example below by identifying the blocks for each variable (same example as in the class activity earlier in this lecture, and in lecture 2: SSA form)

Globals = { a, b, x, y, w }

Blocks

	Blocks	Phi-Nodes
x	{ 1, 3, 4 }	
y	{ 1, 3, 4, 6 }	
a	{ 1 }	
b	{ 1 }	
w	{ 6 }	

Block	IDOM	DF
B1	-	{ }
B2	1	{ 2, 6 }
B3	2	{ 5 }
B4	2	{ 5 }
B5	2	{ 2, 6 }
B6	1	{ }

Class Activity: Solution

Insert the Phi-nodes for the example below by identifying the blocks for each variable (same example as in the class activity earlier in this lecture, and in lecture 2: SSA form)

Globals = { a, b, x, y, w }

Blocks

	Blocks	Phi-Nodes
x	{ 1, 3, 4 }	{ 2, 5, 6 }
y	{ 1, 3, 4, 6 }	
a	{ 1 }	
b	{ 1 }	
w	{ 6 }	

Block	IDOM	DF
B1	-	{ }
B2	1	{ 2, 6 }
B3	2	{ 5 }
B4	2	{ 5 }
B5	2	{ 2, 6 }
B6	1	{ }

Class Activity: Solution

Insert the Phi-nodes for the example below by identifying the blocks for each variable (same example as in the class activity earlier in this lecture, and in lecture 2: SSA form)

Globals = { a, b, x, y, w }

Blocks

	Blocks	Phi-Nodes
x	{ 1, 3, 4 }	{ 2, 5, 6 }
y	{ 1, 3, 4, 6 }	{ 2, 5, 6 }
a	{ 1 }	
b	{ 1 }	
w	{ 6 }	

Block	IDOM	DF
B1	-	{ }
B2	1	{ 2, 6 }
B3	2	{ 5 }
B4	2	{ 5 }
B5	2	{ 2, 6 }
B6	1	{ }

Class Activity: Solution

Insert the Phi-nodes for the example below by identifying the blocks for each variable (same example as in the class activity earlier in this lecture, and in lecture 2: SSA form)

Globals = { a, b, x, y, w }

Blocks

	Blocks	Phi-Nodes
x	{ 1, 3, 4 }	{ 2, 5, 6 }
y	{ 1, 3, 4, 6 }	{ 2, 5, 6 }
a	{ 1 }	{ }
b	{ 1 }	
w	{ 6 }	

Block	IDOM	DF
B1	-	{ }
B2	1	{ 2, 6 }
B3	2	{ 5 }
B4	2	{ 5 }
B5	2	{ 2, 6 }
B6	1	{ }

Class Activity: Solution

Insert the Phi-nodes for the example below by identifying the blocks for each variable (same example as in the class activity earlier in this lecture, and in lecture 2: SSA form)

Globals = { a, b, x, y, w }

Blocks

	Blocks	Phi-Nodes
x	{ 1, 3, 4 }	{ 2, 5, 6 }
y	{ 1, 3, 4, 6 }	{ 2, 5, 6 }
a	{ 1 }	{ }
b	{ 1 }	{ }
w	{ 6 }	

Block	IDOM	DF
B1	-	{ }
B2	1	{ 2, 6 }
B3	2	{ 5 }
B4	2	{ 5 }
B5	2	{ 2, 6 }
B6	1	{ }

Class Activity: Solution

Insert the Phi-nodes for the example below by identifying the blocks for each variable (same example as in the class activity earlier in this lecture, and in lecture 2: SSA form)

Globals = { a, b, x, y, w }

Blocks

	Blocks	Phi-Nodes
x	{ 1, 3, 4 }	{ 2, 5, 6 }
y	{ 1, 3, 4, 6 }	{ 2, 5, 6 }
a	{ 1 }	{ }
b	{ 1 }	{ }
w	{ 6 }	{ }

Block	IDOM	DF
B1	-	{ }
B2	1	{ 2, 6 }
B3	2	{ 5 }
B4	2	{ 5 }
B5	2	{ 2, 6 }
B6	1	{ }

Outline

Def-Use Chains

Why do we need SSA form ?

Dominator Trees and Dominance Frontiers

Inserting Phi-Nodes

Renaming Variables

Converting Out of SSA form

Intuition behind Renaming Algorithm

Renaming proceeds in a top-down fashion keeping track of variable lifetimes

- Use a stack to keep track of the latest name of a variable
- Push a new name onto the stack when seeing a new definition
- Use the version at the top of the stack in the uses
- Pop the name from the stack when exiting block and def goes out of scope

Renaming Variables: Algorithm Sketch

Once the Phi-Nodes have been inserted, we need to rename both Phi-nodes and regular variables (each def should have a unique index, and dominate its uses)

- Keep an array of stacks, one per variable - name at top of stack most recent
- Generate unique names for each Phi-function, and push them onto the stack
- Rewrite each operation in the block with version from top of stack
- Rewrite definition by inventing and pushing a new name
- Pop the names generated in block upon exit from the block

Perform the above operations in a recursive manner in a pre-order walk over the DOM tree (i.e., go from the root node to the children)

Renaming Algorithm: Pseudo-code

Generating new names...

for each global name i

$\text{counter}[i] \leftarrow 0$

$\text{stack}[i] \leftarrow \emptyset$

call $\text{Rename}(n_o)$

$\text{NewName}(n)$

$i \leftarrow \text{counter}[n]$

$\text{counter}[n] \leftarrow \text{counter}[n] + 1$

 push n_i onto $\text{stack}[n]$

 return n_i

$\text{Rename}(b)$

 for each ϕ -function in b , $x \leftarrow \phi(\dots)$

 rename x as $\text{NewName}(x)$

 for each operation “ $x \leftarrow y \text{ op } z$ ” in b

 rewrite y as $\text{top}(\text{stack}[y])$

 rewrite z as $\text{top}(\text{stack}[z])$

 rewrite x as $\text{NewName}(x)$

 for each successor of b in the CFG

 rewrite appropriate ϕ parameters

 for each successor s of b in dom. tree

$\text{Rename}(s)$

 for each operation “ $x \leftarrow y \text{ op } z$ ” in b

$\text{pop}(\text{stack}[x])$

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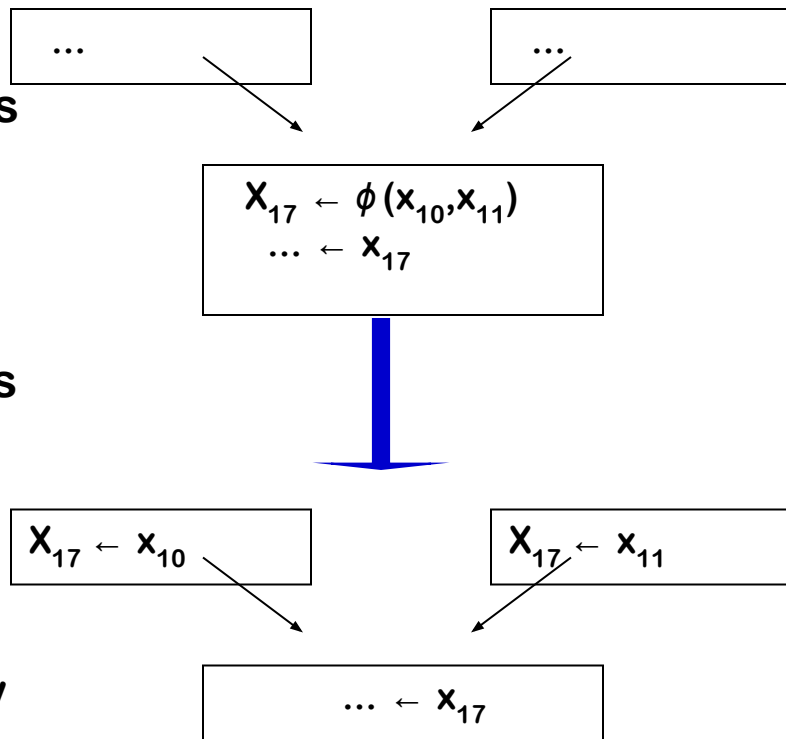
Renaming Variables

Converting Out of SSA form

SSA Deconstruction

At some point, we need executable code

- Few machines implement ϕ operations
- Need to fix up the flow of values
- Basic idea
 - > Insert copies in ϕ -function pred's
 - > Simple algorithm
 - Works in most cases
 - > Adds lots of copies
 - Most of them coalesce away



Translation Out of SSA Form

The Lost Copy Problem

Original code

```
graph TD
    Entry(( )) --> i1[i ← 1]
    i1 --> y1[y ← i]
    y1 --> i2[i ← i + 1]
    i2 --> d1[d ← y + ...]
```

Original code

In SSA form

```
graph TD
    Entry(( )) --> i0[i0 ← 1]
    i0 --> y0[y0 ← i0]
    y0 --> i1[i1 ← i0 + 1]
    i1 --> z0[z0 ← y0 + ...]
```

In SSA form

With copies folded

```
graph TD
    Entry(( )) --> i0[i0 ← 1]
    i0 --> i1[i1 ← Φ(i0, i2)]
    i1 --> z0[z0 ← i1 + ...]
```

With copies folded

Copies naïvely inserted

```
graph TD
    Entry(( )) --> i0[i0 ← 1]
    i0 --> i1[i1 ← i0]
    i1 --> i2[i2 ← i1 + 1]
    i2 --> z0[z0 ← i1 + ...]
```

Copies naïvely inserted

Copy folding has nothing to do with either *i* or SSA form.

The assignment to *z* now receives the wrong value.

To fix this problem, the compiler needs to create a temporary name to hold the penultimate value of *i*.

Translation Out of SSA Form

The Swap Problem

↓
x ← ...
y ← ...
↕
t ← x
x ← y
y ← t
↓

Original code

↓
x₀ ← ...
y₀ ← ...
↕
x₁ ← $\Phi(x_0, x_2)$
y₁ ← $\Phi(y_0, y_2)$
t₀ ← x₁
x₂ ← y₁
y₂ ← t₀
↓

In SSA Form

↓
x₀ ← ...
y₀ ← ...
↕
x₁ ← $\Phi(x_0, y_1)$
y₁ ← $\Phi(y_0, x_1)$
↓

Copies folded

↓
x₀ ← ...
y₀ ← ...
x₁ ← x₀
y₁ ← y₀
↕
x₁ ← y₁
y₁ ← x₁
↓

Copies naïvely
inserted

Code is incorrect

This problem arises when a Φ -function argument is defined by a Φ -function in the same block. Requires one or more copies & temporary names.

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