Model Checking

Lecture 12: CPEN 400P

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(Slides based on Arie Gurfinkel's slides at the University of Waterloo in ECE 750T)

Outline

What is model checking ?

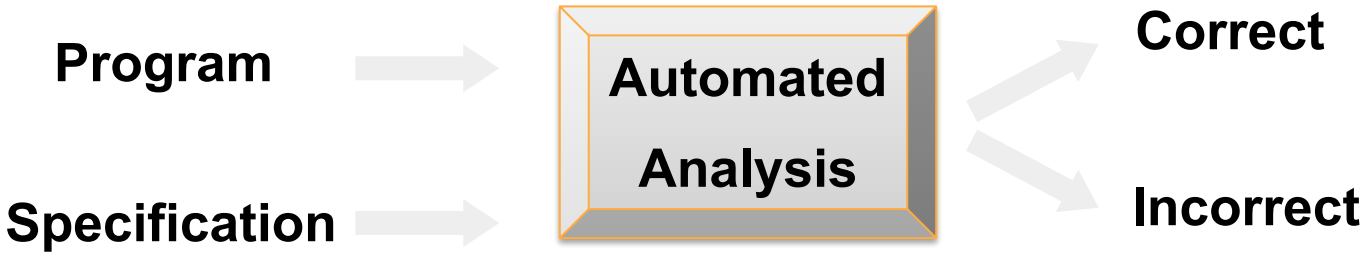
Kripke Structures

CTL (Computation Tree Logic)

Model Checking of Programs

Counter Example Guided Abstraction Refinement (CEGAR)

Static Program Analysis



Reasoning statically about behavior of a program without executing it

- compile-time analysis
- exhaustive, considers all possible executions under all possible environments and inputs

The *algorithmic* discovery of *properties* of program by *inspection* of the *source text*

Manna and Pnueli, “Algorithmic Verification”

Also known as static analysis, program verification, formal methods, etc.

Undecidability

The halting problem

- does a program P terminates on input I
- proved undecidable by Alan Turing in 1936
- https://en.wikipedia.org/wiki/Halting_problem

Rice's Theorem

- for any non-trivial property of partial functions, no general and effective method can decide whether an algorithm computes a partial function with that property
- in practice, this means that there is no machine that can always decide whether the language of a given Turing machine has a particular nontrivial property
- https://en.wikipedia.org/wiki/Rice%27s_theorem

Living with Undecidability

"Algorithms" that occasionally diverge

Limit programs that can be analyzed

- finite-state, loop-free

Partial (unsound) verification

- analyze only some executions up-to a fixed number of steps

Incomplete verification / Abstraction

- analyze a superset of program executions

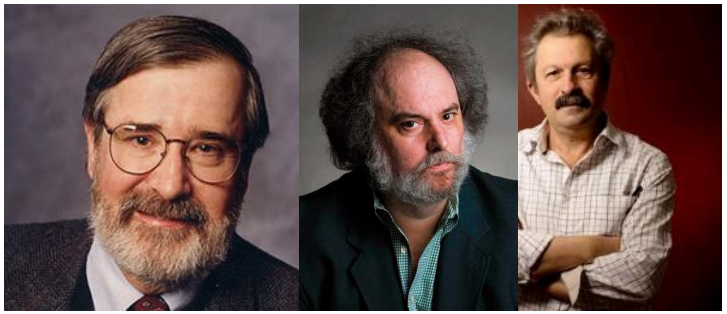
Programmer Assistance

- annotations, pre-, post-conditions, inductive invariants

(Temporal Logic) Model Checking

Automatic verification technique for finite state concurrent systems.

- Developed independently by Clarke and Emerson and by Queille and Sifakis in early 1980's.
- ACM Turing Award 2007



Specifications are written in propositional temporal logic. (Pnueli 77)

- Computation Tree Logic (CTL), Linear Temporal Logic (LTL), …

Verification procedure is an intelligent exhaustive search of the state space of the design

- Statespace explosion

Model Checking since 1981

1981 Clarke / Emerson: CTL Model Checking
Sifakis / Quielle

10^5

1982 EMC: Explicit Model Checker
Clarke, Emerson, Sistla

1990 Symbolic Model Checking
Burch, Clarke, Dill, McMillan

10^{10}

1992 SMV: Symbolic Model Veri
McMillan

1990s: Formal Hardware Verification in Industry: Intel, IBM, Motorola, etc.

1998 Bounded Model Checking using SAT
Biere, Clarke, Zhu

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2000 Counterexample-guided Abstraction Refinement
Clarke, Grumberg, Jha, Lu, Veith

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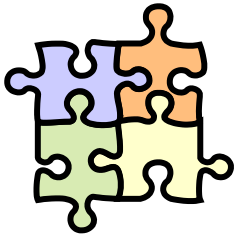

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SLAM, MAGIC, BLAST, …

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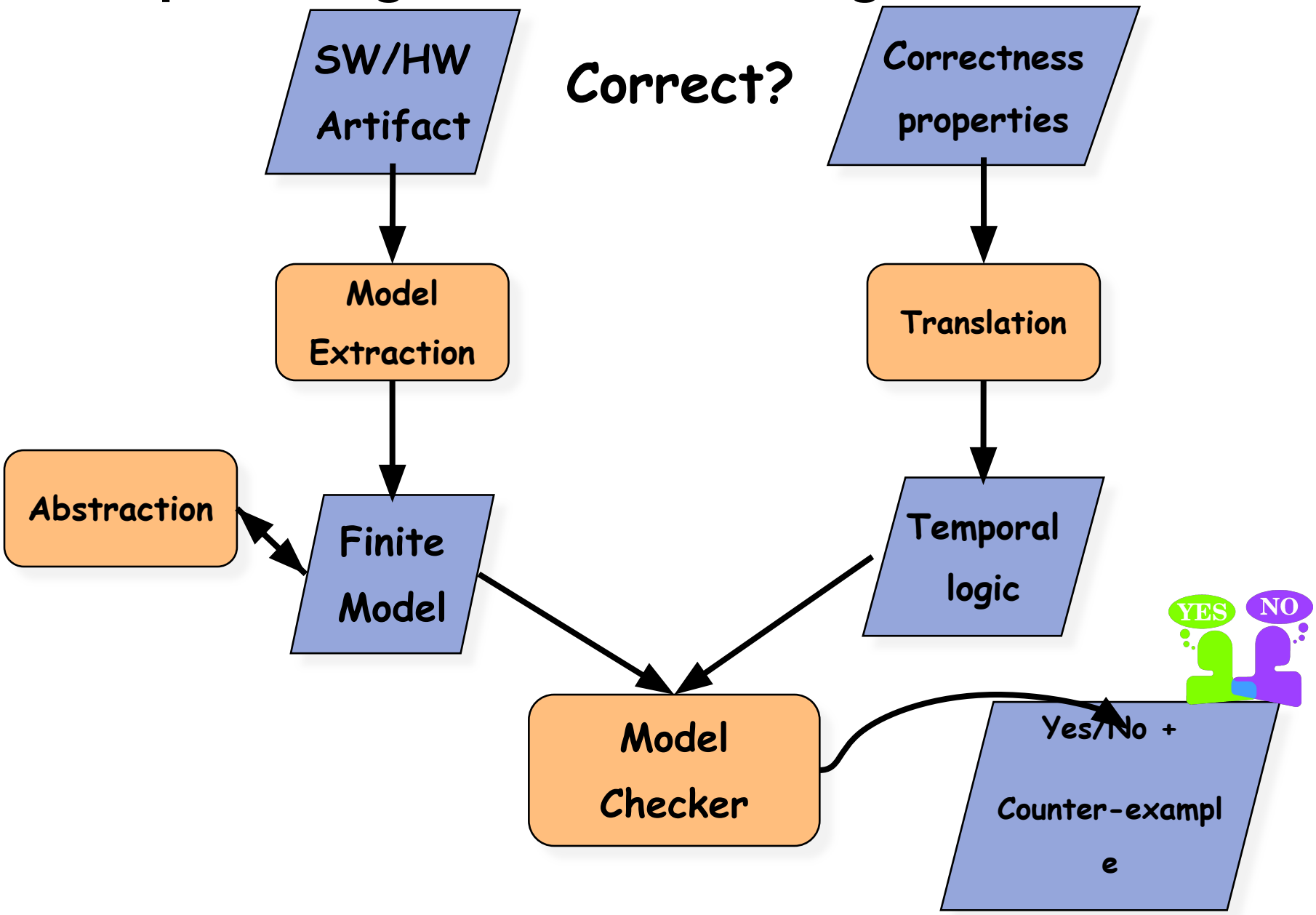
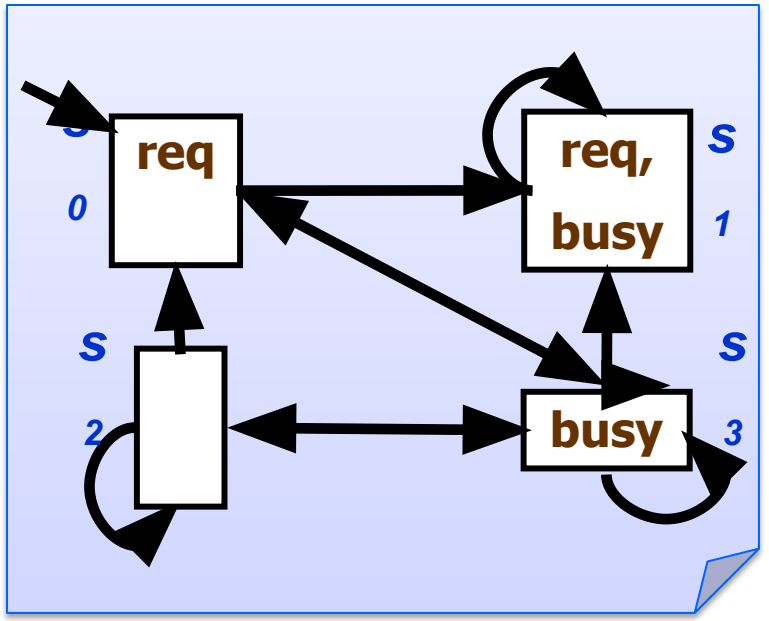
Temporal Logic Model Checking



Models: Kripke Structures

Conventional state machines

- $K = (V, S, s_0, I, R)$
- V is a (finite) set of atomic propositions
- S is a (finite) set of states
- $s_0 \in S$ is a start state
- $I: S \rightarrow 2^V$ is a labelling function that maps each state to the set of propositional variables that hold in it
  - That is, $I(S)$ is a set of interpretations specifying which propositions are true in each state
- $R \subseteq S \times S$ is a transition relation



Propositional Variables

Fixed set of atomic propositions, e.g, *{p, q, r}*

Atomic descriptions of a system

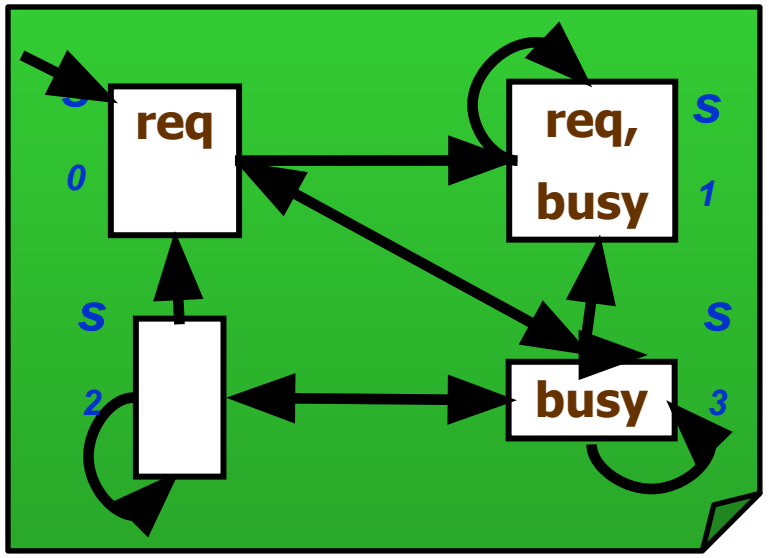
- “Printer is busy”
- “There are currently no requested jobs for the printer”
- “Conveyer belt is stopped”

Do not involve time!

Representing Models Symbolically

A system state represents an interpretation (truth assignment) for a set of propositional variables V

- Formulas represent sets of states that satisfy it
  - False = ∅, True = S
  - req – set of states in which req is
  - true – {s0, s1}
  - busy – set of states in which busy is
  - true – {s1, s3}
  - req ∨ busy = {s0, s1 , s3}



- State transitions are described by relations over two sets of variables: V (source state) and V' (destination state)
  - Transition (s2, s3) is ¬req ∧ ¬ busy ∧ ¬req' ∧ busy'
  - Relation R is described by disjunction of formulas for individual transitions

Modal Logic

Extends *propositional logic* with modalities to qualify propositions

- *"it is raining"* – *rain*
- "it will rain tomorrow" – □*rain*
  - it is raining in all possible futures
- "it might rain tomorrow" - ◇*rain*
  - it is raining in some possible futures

Modal logic formulas are interpreted over a collection of *possible worlds* connected by an *accessibility relation*

Temporal logic is a modal logic that adds temporal modalities: next, always, eventually, and until

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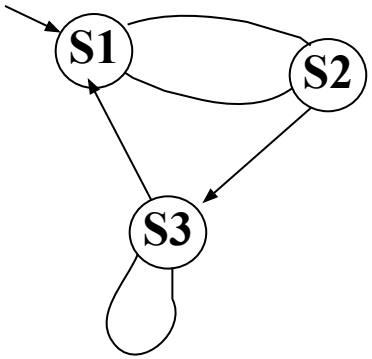
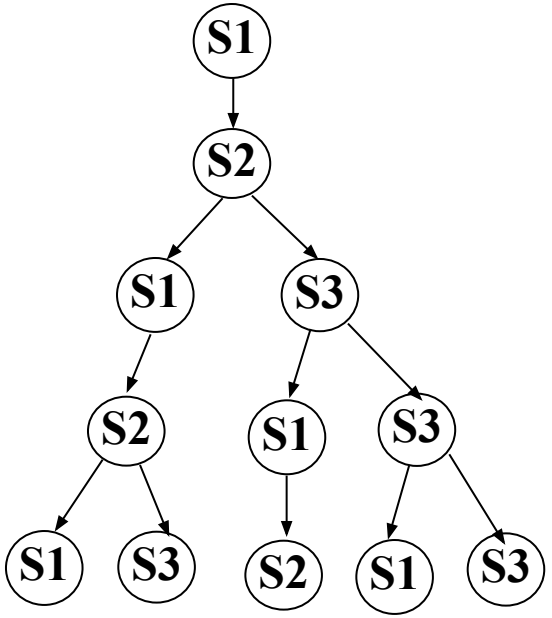
Computation Tree Logic (CTL)

CTL: Branching-time propositional temporal logic
Model - a tree of computation paths



Kripke Structure



Tree of computation

CTL: Computation Tree Logic

Propositional temporal logic with explicit quantification over possible futures

Syntax:

True and *False* are CTL formulas;
propositional variables are CTL formulas;

If ϕ and ψ are CTL formulae, then so are: $\neg \phi$, $\phi \wedge \psi$, $\phi \vee \psi$

EX ϕ : ϕ holds in some next state

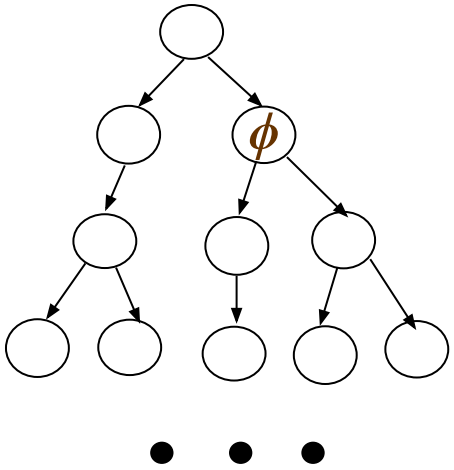
EF ϕ : along some path, ϕ holds in a future state

E[ϕ U ψ] : along some path, ϕ holds until ψ holds

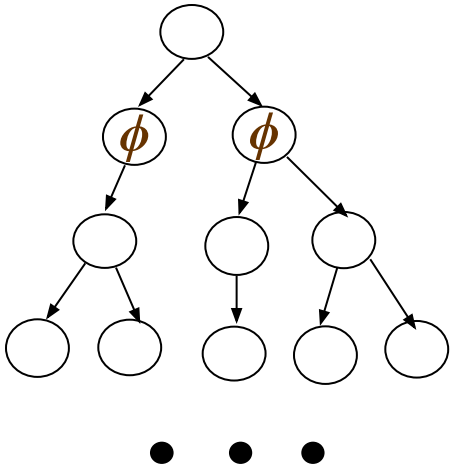
EG ϕ : along some path, ϕ holds in every state

- Universal quantification: AX ϕ , AF ϕ , A[ϕ U ψ], AG ϕ

Examples: EX and AX

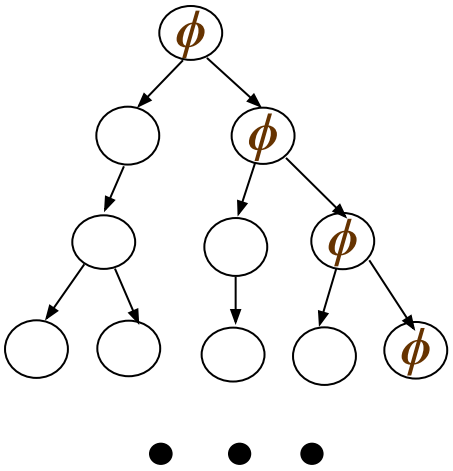


EX ϕ (exists next)

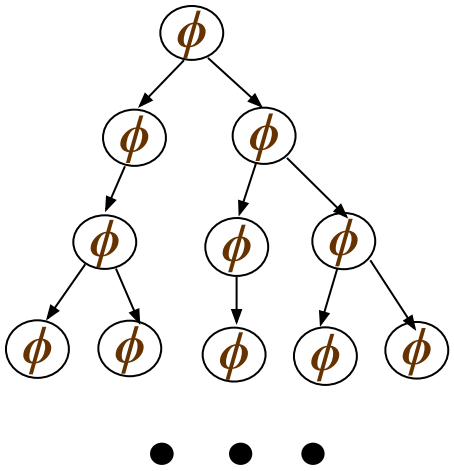


AX ϕ (all next)

Examples: EG and AG

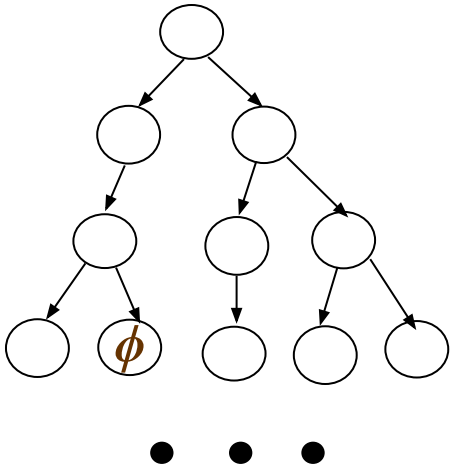


EG ϕ (exists global)

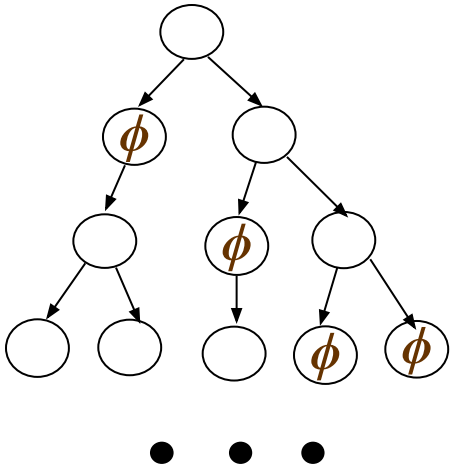


AG ϕ (all global)

Examples: EF and AF

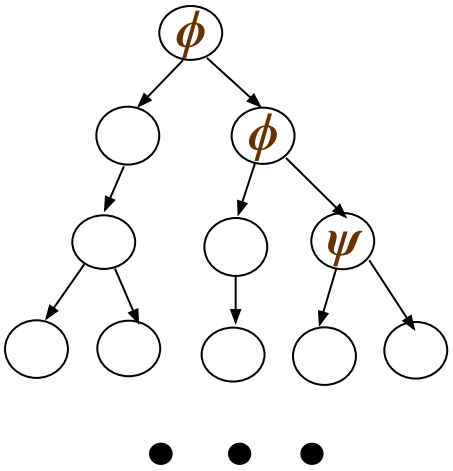


EF ϕ (exists future)

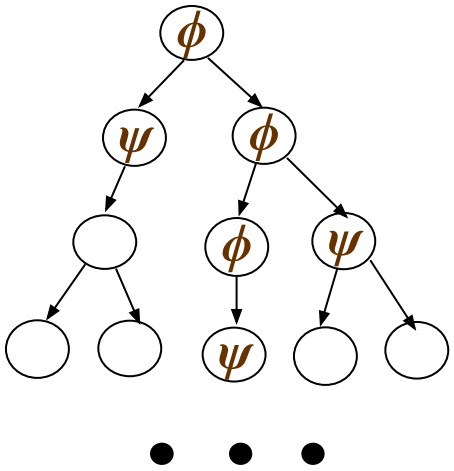


AF ϕ (all future)

Examples: EU and AU



E[ϕ U ψ] (exists until)



A[ϕ U ψ] (all until)

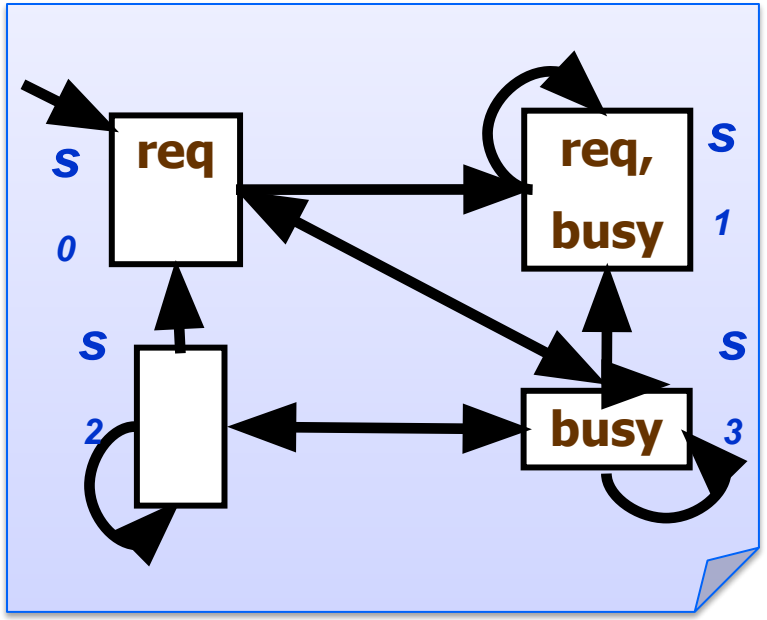
CTL Examples

Properties that hold:

- (AX busy)(s_0)
- (EG busy)(s_3)
- A (req U busy) (s_0)
- E (¬req U busy) (s_1)
- AG (req ⇒ AF busy) (s_0)

Properties that fail:

- (AX (req ∨ busy))(s_3)



Semantics of CTL

$K,s \models \phi$ – means that formula ϕ is true in state s. K is often omitted since we always talk about the same Kripke structure

- E.g., $s \models p \wedge \neg q$

$\pi = \pi^0 \pi^1 \ldots$ is a path

π^0 is the current state (root)

π^{i+1} is a successor state of π^i. Then,

$AX\ \phi = \forall \pi \cdot \pi^1 \models \phi$ $\quad$ $EX\ \phi = \exists \pi \cdot \pi^1 \models \phi$

$AG\ \phi = \forall \pi \cdot \forall i \cdot \pi^i \models \phi$ $\quad$ $EG\ \phi = \exists \pi \cdot \forall i \cdot \pi^i \models \phi$

$AF\ \phi = \forall \pi \cdot \exists i \cdot \pi^i \models \phi$ $\quad$ $EF\ \phi = \exists \pi \cdot \exists i \cdot \pi^i \models \phi$

$A[\phi\ U\ \psi] = \forall \pi \cdot \exists i \cdot \pi^i \models \psi \wedge \forall j \cdot 0 \leq j < i \Rightarrow \pi^j \models \phi$

$E[\phi\ U\ \psi] = \exists \pi \cdot \exists i \cdot \pi^i \models \psi \wedge \forall j \cdot 0 \leq j < i \Rightarrow \pi^j \models \phi$

Safety and Liveness

Safety: Something “bad” will never happen

- AG ¬bad
- e.g., mutual exclusion: no two processes are in their critical section at once
- Safety = if false then there is a finite counterexample
- Safety = reachability

Liveness: Something “good” will always happen

- AG AF good
- e.g., every request is eventually serviced
- Liveness = if false then there is an infinite counterexample
- Liveness = termination

Every universal temporal logic formula can be decomposed into a conjunction of safety and liveness

Class Activity: Write the CTL formula for these

An elevator can remain idle on the third floor with its doors closed

- EF (state=idle ∧ floor=3 ∧ doors=closed)

When a request occurs, it will eventually be acknowledged

-

A process is enabled infinitely often on every computation path

-

A process will eventually be permanently deadlocked

-

Action *s* precedes *p* after *q*

-

- Note: hard to do correctly.

Class Activity: Solution

An elevator can remain idle on the third floor with its doors closed

- EF (state=idle $\wedge$ floor=3 $\wedge$ doors=closed)

When a request occurs, it will eventually be acknowledged

- AG (request $\Rightarrow$ AF acknowledge)

A process is enabled infinitely often on every computation path

- AG AF enabled

A process will eventually be permanently deadlocked

- AF AG deadlock

Action *s* precedes *p* after *q*

- A[$\neg q$ U ($q \wedge$ A[$\neg p$ U s])]
- Note: hard to do correctly.

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Model Checking of Programs

Counter Example Guided Abstraction Refinement (CEGAR)

Pros and Cons of Model Checking

Largely automatic and fast

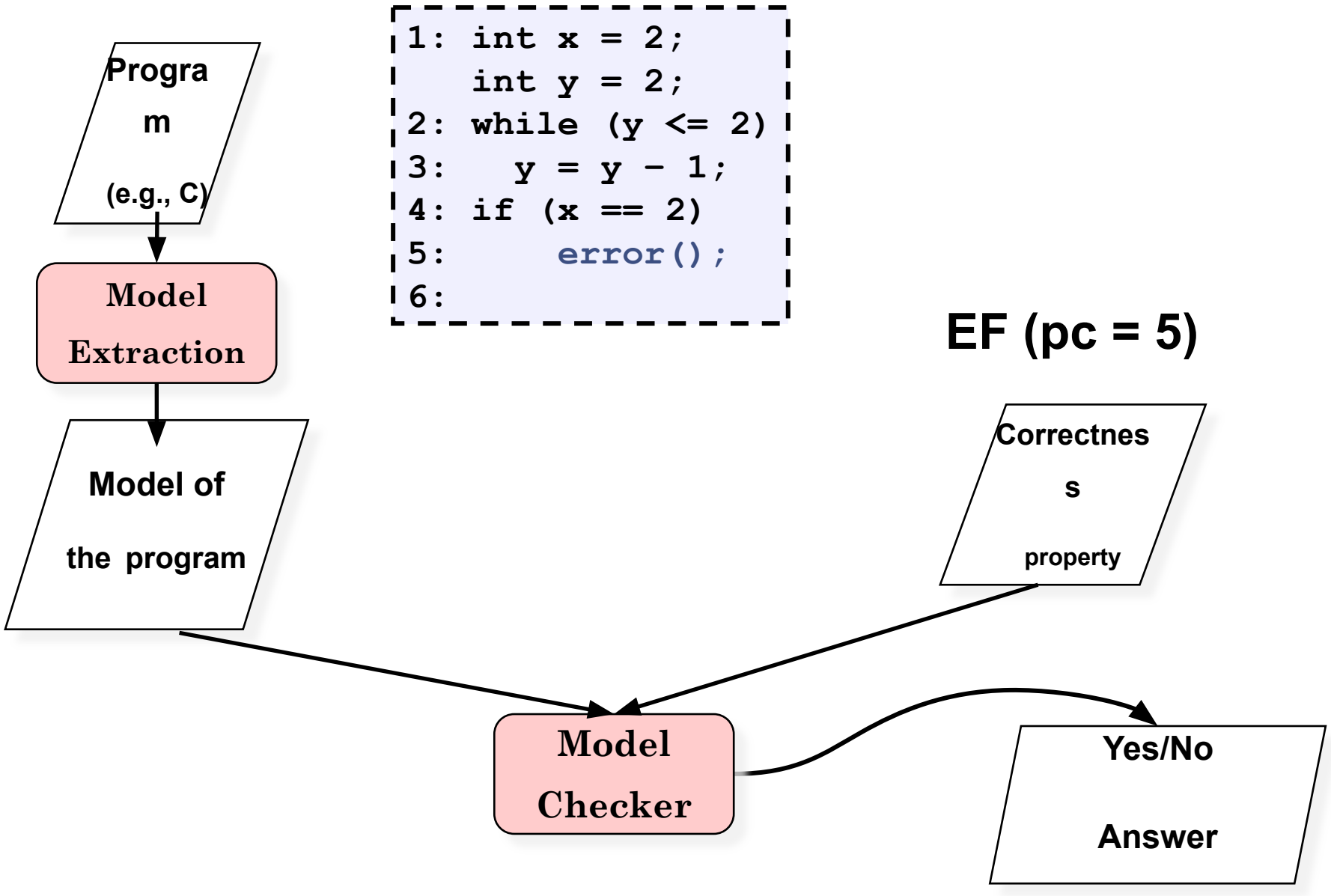
Better suited for debugging

- … rather than assurance

Testing vs model-checking

- Usually, find more problems by
  exploring all behaviours of a downscaled system
  than by
  testing some behaviours of the full system

Software Model Checking



Assumptions: In Our Programming Language…

All variables are global

Functions are in-lined

`int` is integer

- i.e., no overflow

Special statements:

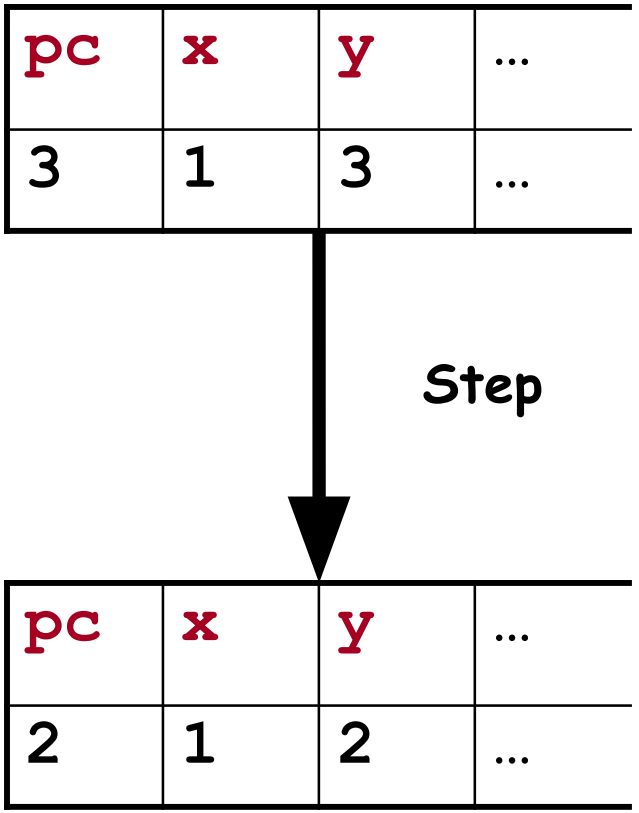
`skip`	**do nothing**
`assume(e)`	**if `e` then `skip` else abort**
`x,y=e1,e2`	**`x, y` are assigned `e1,e2` in parallel**
`x=nondet()`	**`x` gets an arbitrary value**
`goto L1,L2`	**non-deterministically go to `L1` or `L2`**

From Programs to Kripke Structures

Program

```
1: int x = 2;
   int y = 2;
2: while (y <= 2)
3:   y = y - 1;
4: if (x == 2)
5:     error();
6:
```

State



Property: EF (pc = 5)

Programs as Control Flow Graphs

Program

```
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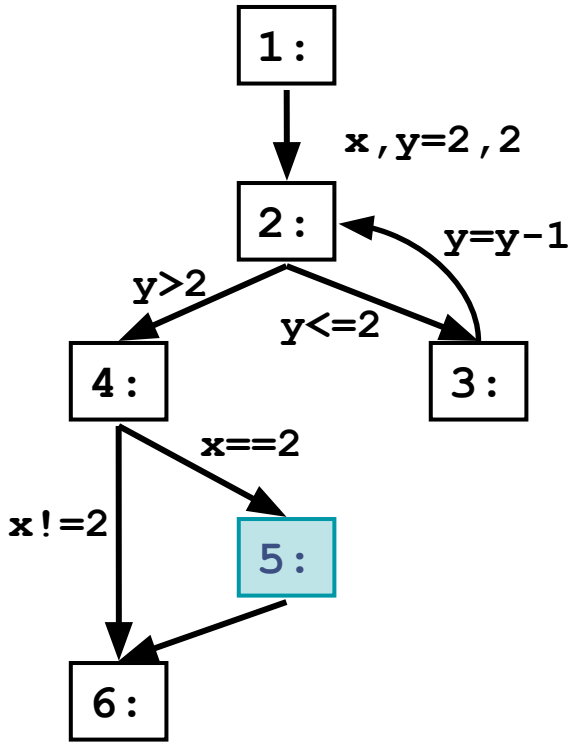
Semantics S

Labeled CFG



Modeling in Software Model Checking

Software Model Checker works directly on the source code of a program

- but it is a whole-program-analysis technique
- requires the user to provide the model of the environment with which the program interacts
  - e.g., physical sensors, operating system, external libraries, specifications

Programing languages already provide convenient primitives to describe behavior

- programming languages are extended to modeling and specification languages by adding three new features
  - non-determinism: like random values, but without a probability distribution
  - assumptions: constraints on “random” values
  - assertions: an indication of a failure

From Programming to Modeling

Extend C programming language with 3 modeling features

Assertions

- assert(e) – aborts an execution when e is false, no-op otherwise

```
void assert (bool b) { if (!b)  error(); }
```

Non-determinism

- nondet_int() – returns a non-deterministic integer value

```
int nondet_int () { int x; return x; }
```

Assumptions

- assume(e) – “ignores” execution when e is false, no-op otherwise

```
void assume (bool e) { while (!e) ;  }
```

Non-determinism vs. Randomness

A *deterministic* function always returns the same result on the same input

- e.g., F(5) = 10

A *non-deterministic* function may return different values on the same input

- e.g., G(5) in [0, 10] “G(5) returns a non-deterministic value between 0 and 10”

A *random* function may choose a different value with a probability distribution

- e.g., H(5) = (3 with prob. 0.3, 4 with prob. 0.2, and 5 with prob. 0.5)

Non-deterministic choice cannot be implemented !

- used to model the worst possible adversary/environment

Modeling with Non-determinism

```
int x, y;

void main (void)
{
  x = nondet_int ();

  assume (x > 10);
  assume (x <= 100);
  y = x + 1;

  assert (y > x);
  assert (y < 200);

}
```

What happens in this program ? Is there an execution of the program for which either assert is violated ?

Using nondet for modeling

Library spec:

- "foo is given via grab_foo(), and is busy until returned via return_foo()"

Model Checking stub:

```
int nondet_int ();
int is_foo_taken = 0;
int grab_foo () {
  if (!is_foo_taken)
    is_foo_taken = nondet_int ();
  return is_foo_taken; }
```

```
void return_foo ()
{ is_foo_taken = 0; }
```

Dangers of unrestricted assumptions

Assumptions can lead to vacuous correctness claims!!!

Is this program correct?

```
if (x > 0) {
  assume (x < 0);
  assert (0); }
```

Assume must either be checked with assert or used as an idiom:

```
x = nondet_int ();
y = nondet_int ();
assume (x < y);
```

Software Model Checking Workflow

1. Identify module to be analyzed
   - e.g., function, component, device driver, library, etc.

2. Instrument with property assertions
   - e.g., buffer overflow, proper API usage, proper state change, etc.
   - might require significant changes in the program to insert monitors

3. Model environment of the module under analysis
   - provide stubs for functions that are called but are not analyzed

4. Write verification harness that exercises module under analysis
   - similar to unit-test, but can use symbolic values
   - tests many executions at a time

5. Run Model Checker

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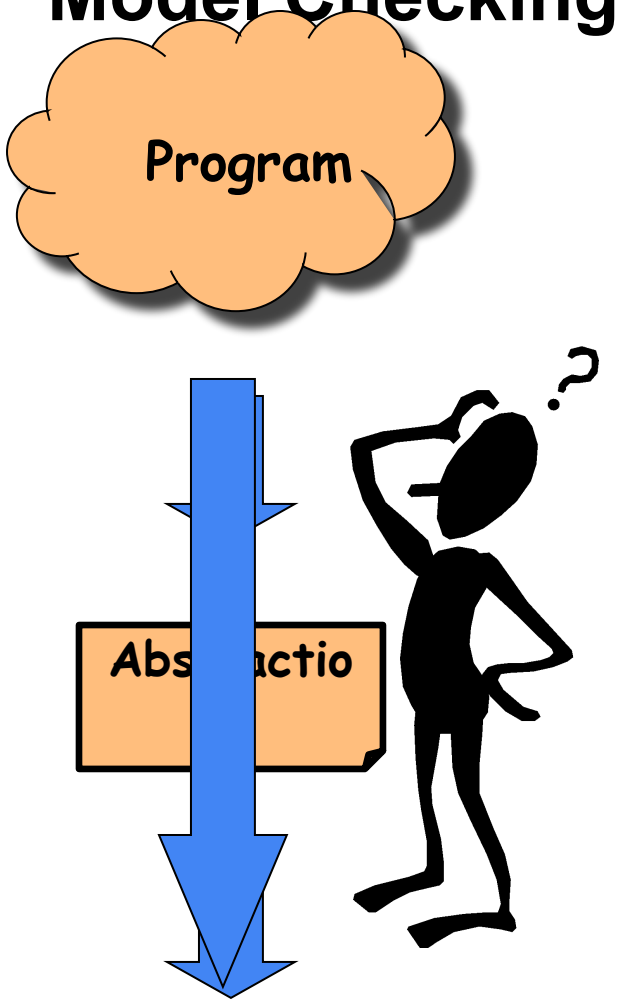
Model Checking Software by Abstraction

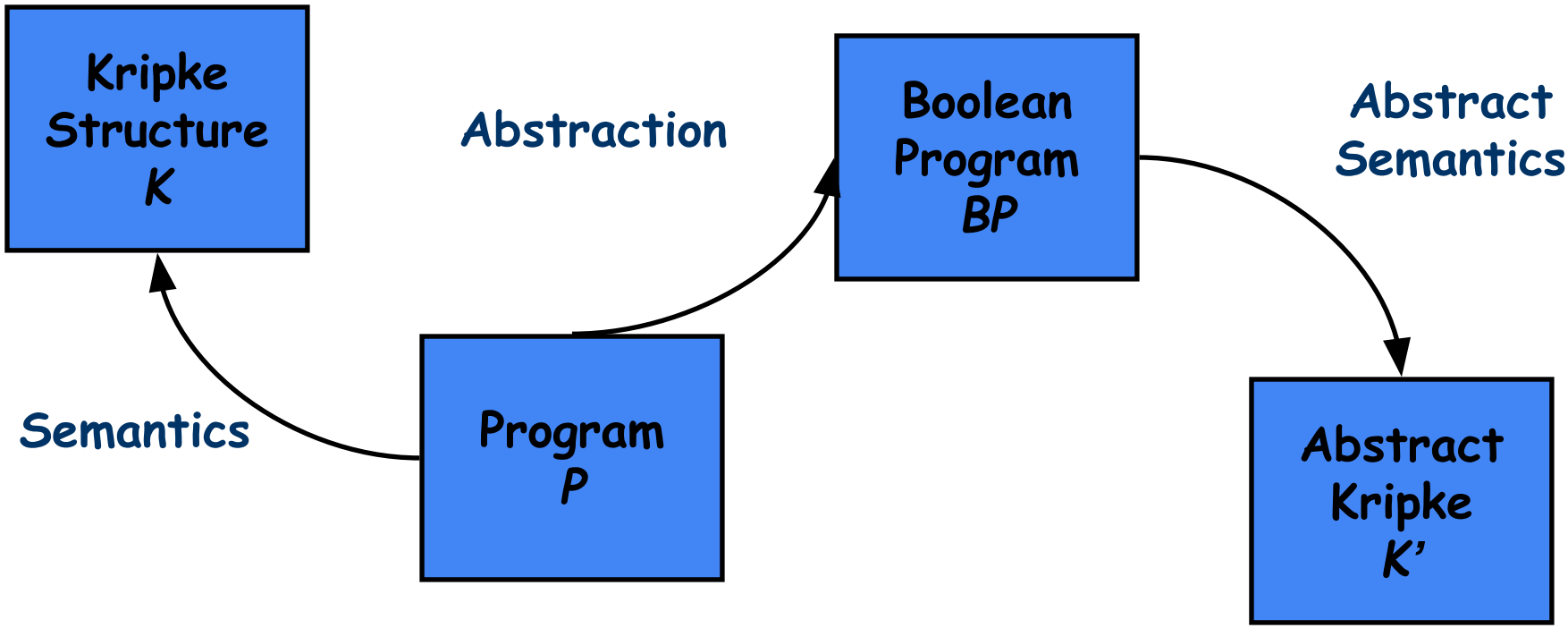


Programs are not finite state machines

- integer variables
- recursion
- unbounded data structures
- dynamic memory allocation
- dynamic thread creation
- pointers

- **Build a finite abstraction**
  - **… small enough to analyze**
  - **… rich enough to give conclusive results**

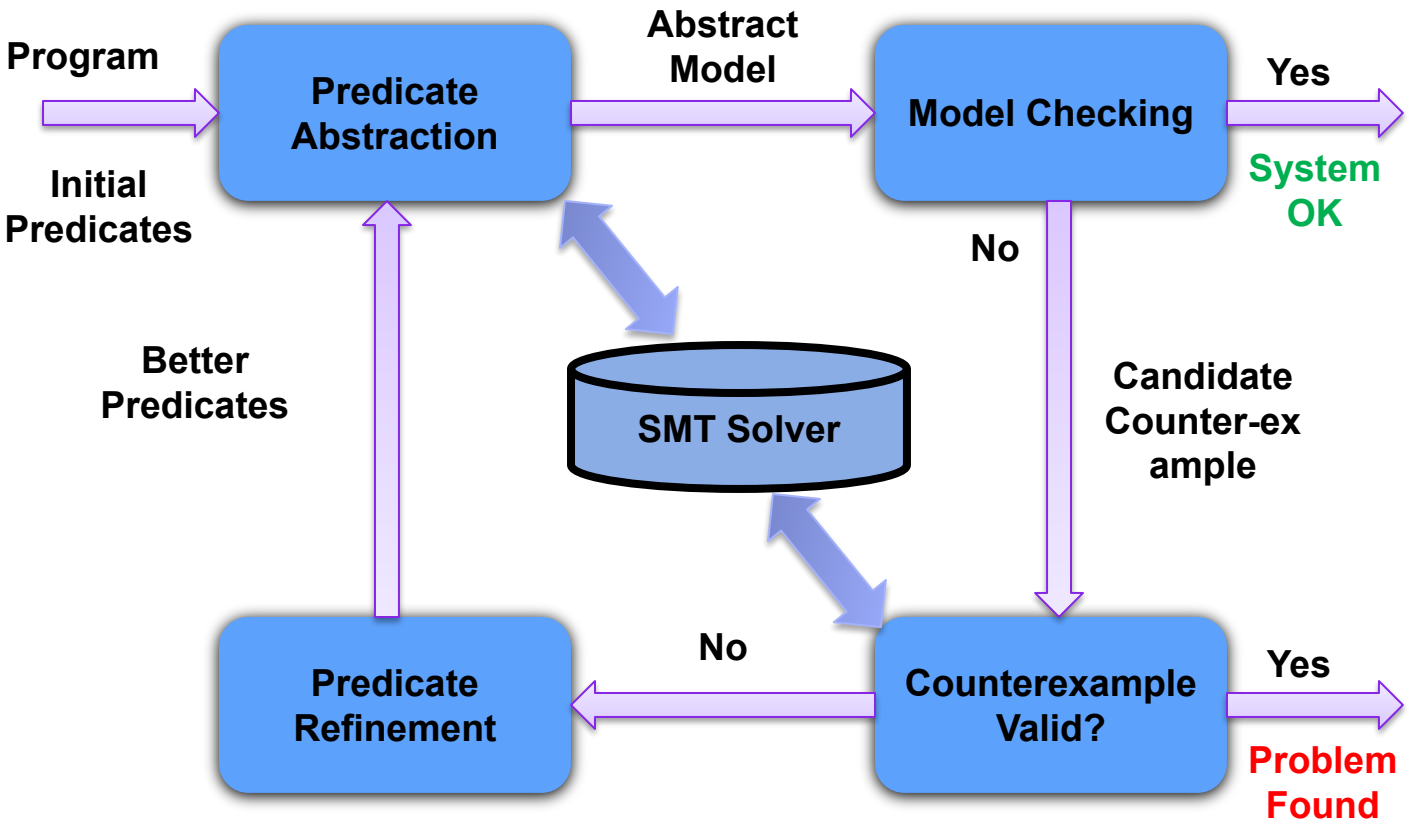
Software Model Checking and Abstraction



Soundness of Abstraction:

BP abstracts *P* implies that *K'* approximates *K*

CounterExample Guided Abstraction Refinement (CEGAR)



Localization Reduction, Kurshan, Bell Labs

Counterexample-guided Abstraction Refinement for Symbolic Model Checking, Clarke et al., CMU

Software Model Checking, SLAM Project, Microsoft, Ball & Rajamani

The Running Example

Program	Property	Expected Answer
1: int x = 2; int y = 2; 2: while (y <= 2) 3: y = y - 1; 4: if (x == 2) 5: error(); 6:	EF (pc = 5)	False

An Example Abstraction

Program

```
1: int x = 2;
   int y = 2;
2: while (y <= 2)
3:   y = y - 1;
4: if (x == 2)
5:     error();
6:
```

Abstraction

(with y<=2)

```
bool b is (y <= 2)
1: b = T;

2: while (b)
3:   b = ch(b,f);
4: if (*)
5:     error();
6:
```

Boolean (Predicate) Programs (BP)

Variables correspond to predicates

Usual control flow statements

while, if-then-else, goto

Expressions

usual Boolean expressions, plus

```
*
ch(a,b)
```

unknown

```
if a then
   true
else
   if b then
     false
   else *
```

Parallel Assignment

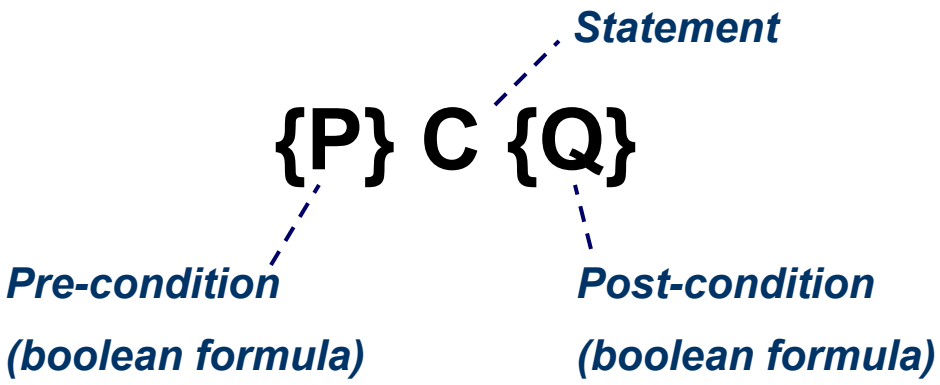
p_1 = ch(a_1,b_1), p_2 = ch(a_2,b_2), ...

b_1 = ch(b_1,¬b_1), b_2 = ch($b_1 \vee b_2$, f), b_3=ch(f,f)

Detour: Pre- and Post-Conditions

A *Hoare triple* {P} C {Q} is a logical statement that holds when

For any state *s* that satisfies P, if executing statement C on *s* terminates with a state *s'*, then *s'* satisfies Q.



Boolean Program Abstraction

Update `p = ch(a, b)` is an approximation of a concrete statement `S` iff {a}S{p} and {b}S{¬p} are valid

- i.e., `y = y - 1` is approximated by
  - `(x == 2) = ch(x ==2, x!=2)`, and
  - `(y <= 2) = ch(y<=2,false)`

Parallel assignment approximates a concrete statement `S` iff all of its updates approximate `S`

- i.e., `y = y - 1` is approximated by

```
(x == 2) = ch(x ==2, x!=2),
(y <= 2) = ch(y<=2,false)
```

A Boolean program approximates a concrete program iff all of its statements approximate corresponding concrete statements

The result of abstraction

Program

```
1: int x = 2;
   int y = 2;
2: while (y <= 2)
3:   y = y - 1;
4: if (x == 2)
5:     error();
6:
```

Abstraction

(with y<=2)

```
bool b is (y <= 2)
1: b = T;

2: while (b)
3:   b = ch(b,f);
4: if (*)
5:     error();
6:
```

But what is the semantics of Boolean programs?

BP Semantics: Overview

Over-Approximation

- treat “unknown” as non-deterministic
- good for establishing correctness of universal properties

Under-Approximation

- treat “unknown” as abort
- good for establishing failure of universal properties

Exact Approximation

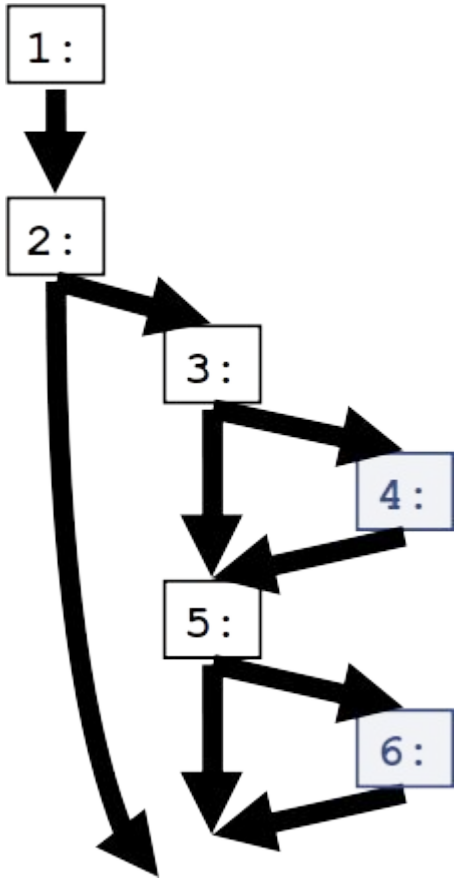
- Treat “unknown” as a special unknown value
- good for verification and refutation
- good for universal, existential, and mixed properties

BP Semantics: Over-Approximation

Abstraction

```
1: ;
2: if (nondet) {
3:   if (*)
4:     error();
5:   if (nondet)
6:     error();
7: }
```

Over-Approximation

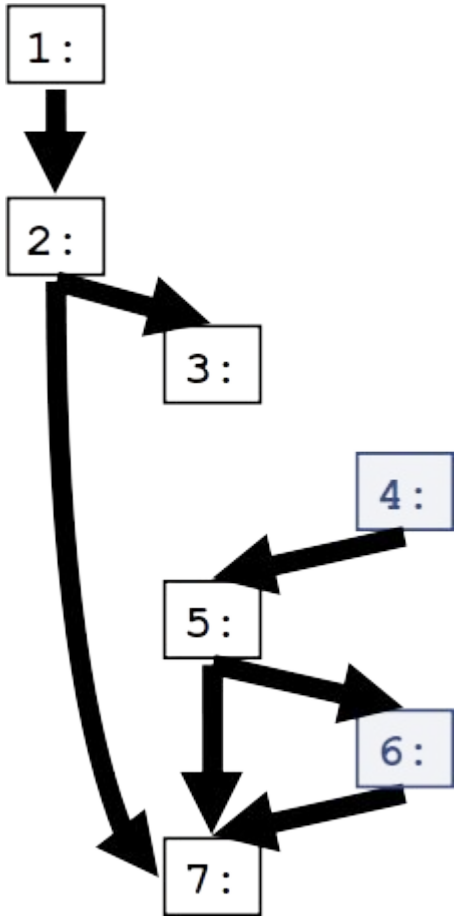


BP Semantics: Under-Approximation

Abstraction

```
1: ;
2: if (nondet) {
3:   if (*)
4:     ERROR;
5:   if (nondet)
6:     ERROR;
7: }
```

Under-Approximation

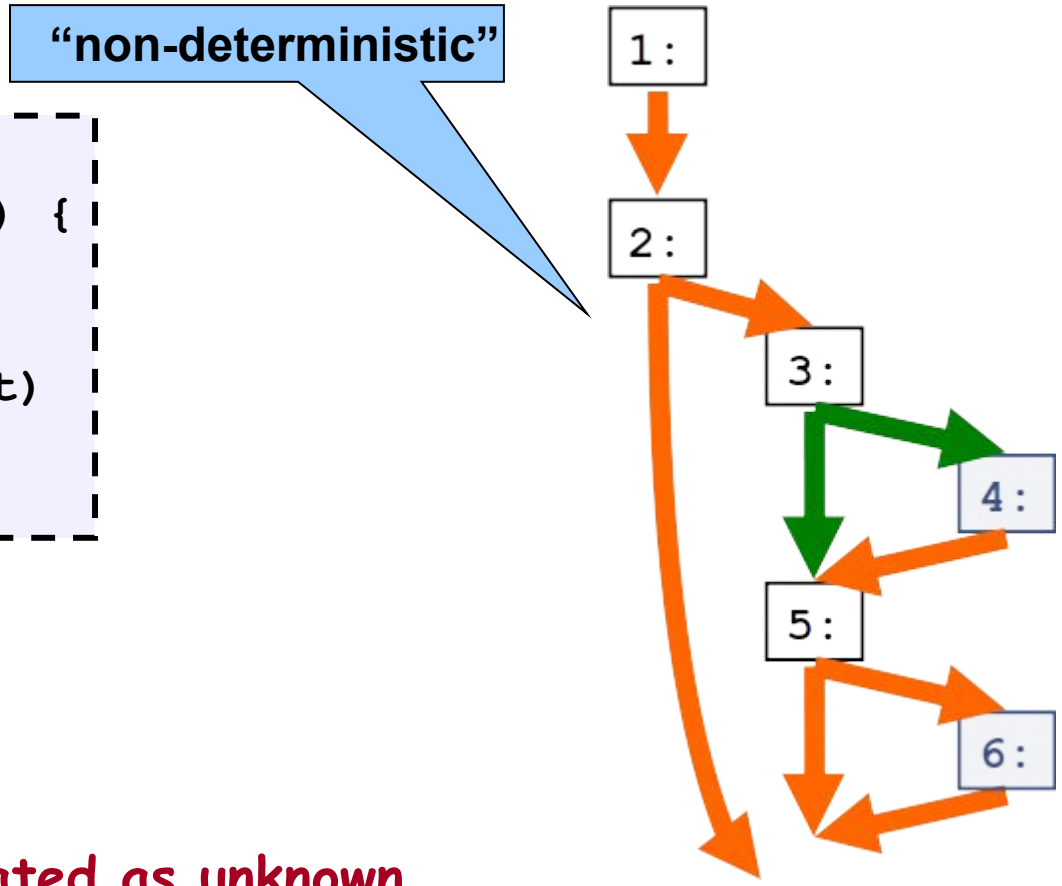


Unknown is treated as abort

BP Semantics: Exact Approximation

Abstraction

```
1:  ;
2:  if (nondet) {
3:    if (*)
4:      ERROR;
5:    if (nondet)
6:      ERROR;
7:  }
```



Unknown is treated as unknown

Summary: The Three Semantics

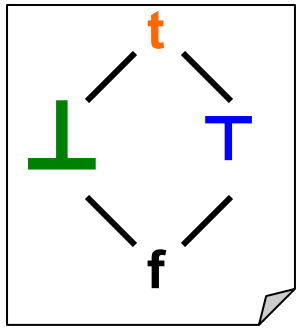
Concrete

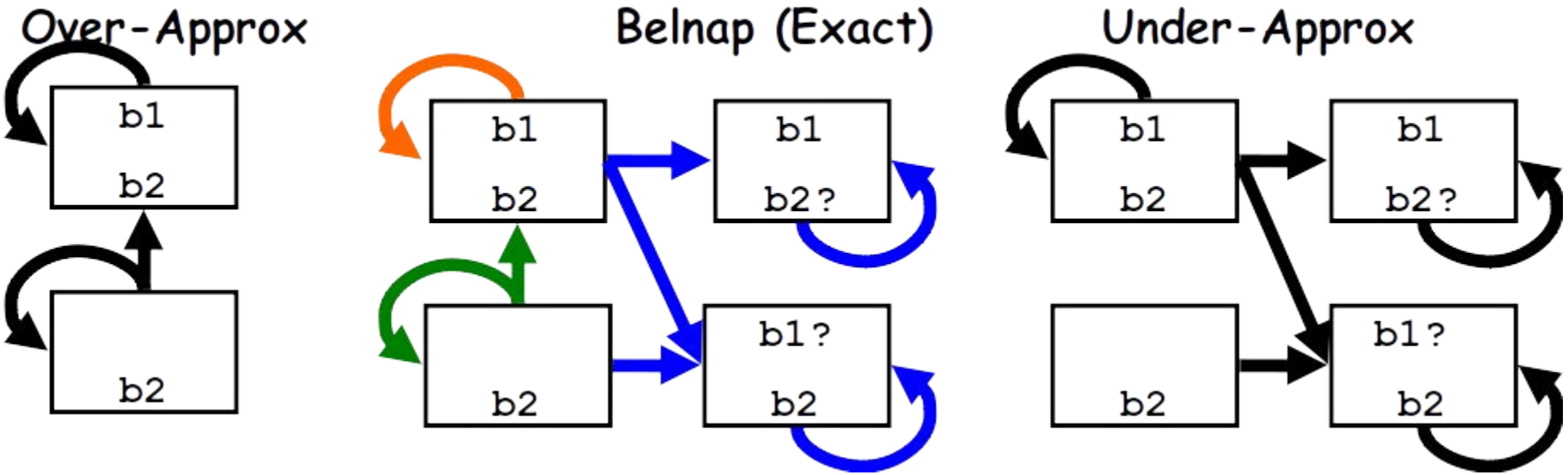
```
y = y - 1;
```

Abstract

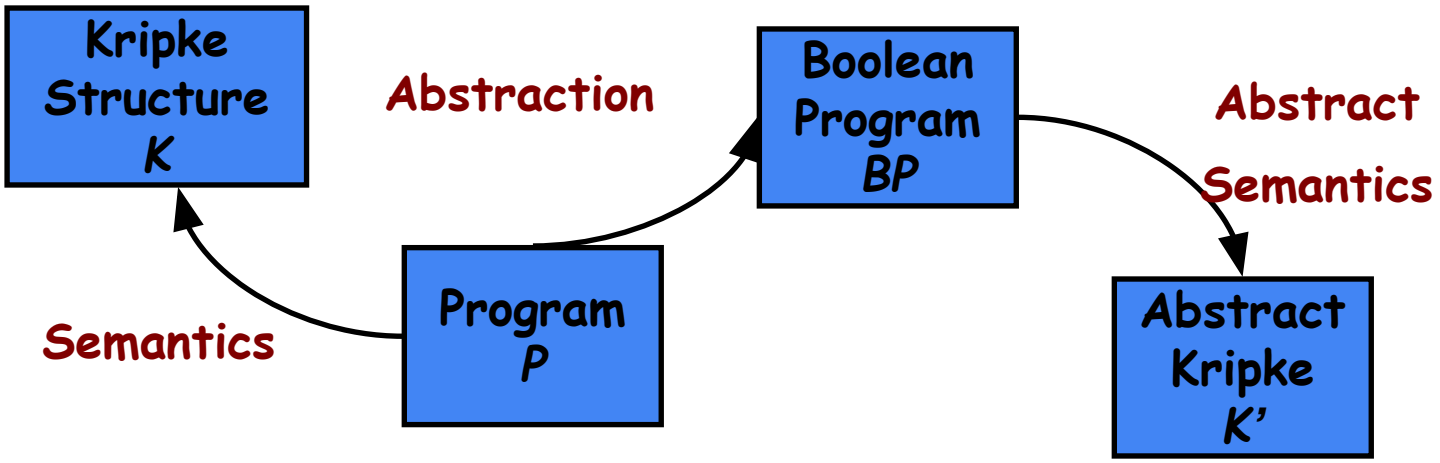
b1 is (y <= 2)
b2 is (x == 2)

```
b1 = ch(b1,f);
b2 = ch(b2,¬b2)
```





Summary: Program Abstraction



Abstract a program P by a Boolean program BP

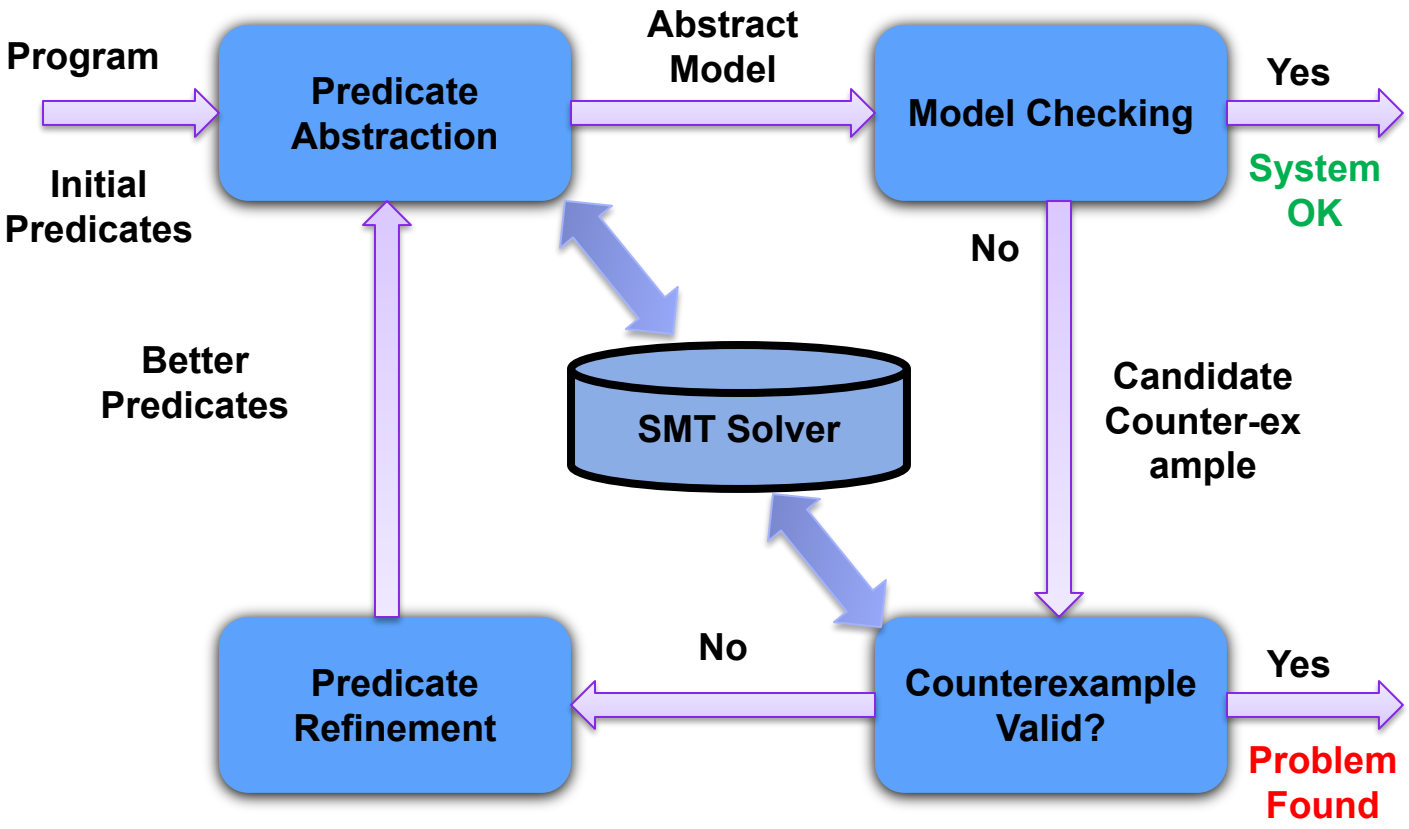
Pick an abstract semantics for this BP:

- Over-approximating
- Under-approximating
- Belnap (Exact)

Yield relationship between K and K':

- Over-approximation
- Under-approximation
- Belnap abstraction

CounterExample Guided Abstraction Refinement (CEGAR)



Localization Reduction, Kurshan, Bell Labs

Counterexample-guided Abstraction Refinement for Symbolic Model Checking, Clarke et al., CMU

Software Model Checking, SLAM Project, Microsoft, Ball & Rajamani

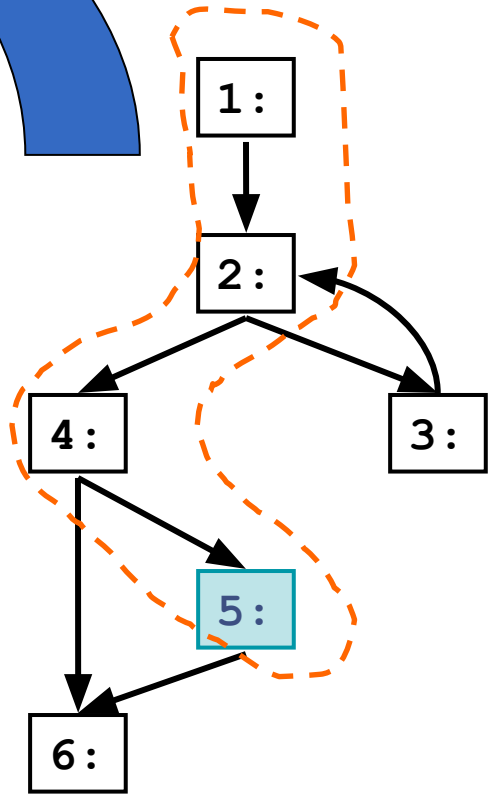
Example: Is ERROR Unreachable?

Need This!

```
1: int x = 2;
   int y = 2;
2: while (y <= 2)
3:       y = y - 1;
4: if (x == 2)
5:       error();
6:
```

```
1: ;

2: while (*)
3:       ;
4: if (*)
5:       error();
6:
```



CEGAR steps

Abstract ⟹ Translate ⟹ Check ⟹ Validate ⟹ Repeat

Example: Is ERROR Unreachable?

Program

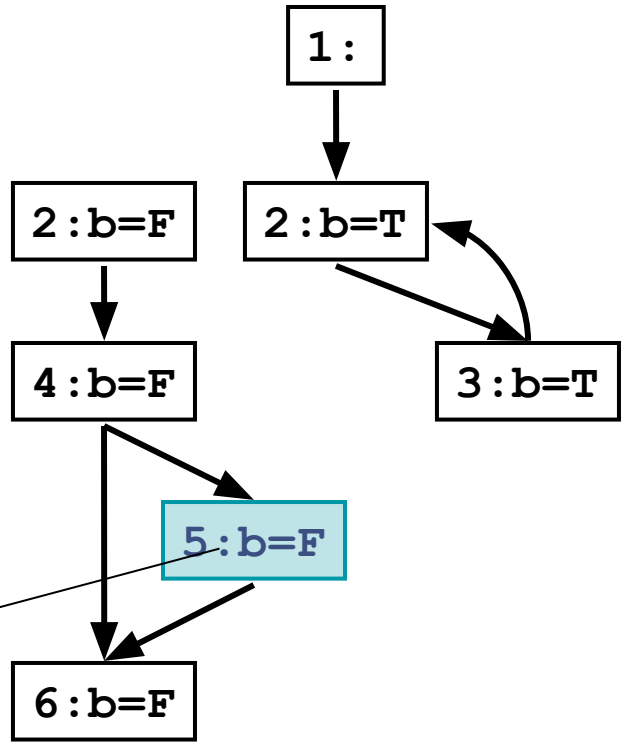
```
1: int x = 2;
   int y = 2;
2: while (y <= 2)
3:   y = y - 1;
4: if (x == 2)
5:     error();
6:
```

Abstraction (with y<=2)

```
bool b is (y <= 2)
1: b = T;

2: while (b)
3:   b = ch(b,f);
4: if (*)
5:     error();
6:
```

Over-Approximation



CEGAR steps

Abstract ⟹ Translate ⟹ Check ⟹ NO ERROR

Summary: Predicate Abstraction and CEGAR

Predicate abstraction with CEGAR is an effective technique for analyzing behavioral properties of software systems

Combines static analysis and traditional model-checking

Abstraction is essential for scalability

- Boolean programs are used as an intermediate step
- Different abstract semantics lead to different abs.
  - over-, under-, Belnap

Automatic abstraction refinement finds the “right” abstraction incrementally

Outline

What is model checking ?

Kripke Structures

CTL (Computation Tree Logic)

Model Checking of Programs

Counter Example Guided Abstraction Refinement (CEGAR)