Spring 2022: Quantum Field Theory (PH6418/ EP4618)

Notes for lecture 10*

March 31, 2022

1 The Yukawa meson field

Just as in the case of Maxwell's electrodynamics, one has propagating wave solutions in the absence of sources (charges and currents), we found in the absence of any sources (homogeneous equations) the real scalar field theory also admits wave solutions. Another important question in Maxwell theory is what is the electric field produced say at a point $\boldsymbol{x}$ by a source e.g. a point charge of strength q located at the point, $\boldsymbol{y}$. The answer takes the form of the famous Coulomb law,

$$\mathbf{E}(\boldsymbol{x}) = \frac{q}{4\pi r^2}\hat{\boldsymbol{r}},$$

where $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y}$. One might ask the same question for the real scalar theory, namely what is the expression for the field produced by a point source of unit strength, located at position, $\boldsymbol{y}$, namely, a source density function

$$\rho(\boldsymbol{x}) = \delta^3(\boldsymbol{x} - \boldsymbol{y}). \tag{1}$$

How do we include this source term in the real scalar theory? We will do this following Maxwell theory by inserting a potential term $\rho(x)\,\varphi(x)$ in the action (or lagrangian),

$$I\left[\varphi\right] = \int d^4x\ \left(\frac{1}{2}\partial_\mu\varphi\ \partial^\mu\varphi - \frac{m}{2}\,\varphi^2 - \rho\,\varphi\right),$$

with the specified source density (1). The equation of motion in this case is,

$$\left(\Box + m^2\right)\varphi(x) = -\rho(x).$$

Since the source (1) is time-independent (static), one can expect the field it creates to be also time independent (static), i.e. $\varphi(x) = \varphi(\boldsymbol{x})$, just a function of the spatial coordinates. In such a time-independent field, time-derivatives vanish : $\partial_t\varphi = 0$, and the equation reduces to a purely spatial equation,

$$\left(\boldsymbol{\nabla}^2 - m^2\right)\varphi(\boldsymbol{x}) = \delta^3(\boldsymbol{x} - \boldsymbol{y}). \tag{2}$$

This equation is reminiscent of the Coulomb Green's function equation in electrodynamics and we shall solve this equation using the same method we did for the Coulomb Green's function i.e. by Fourier transforming to momentum space,

$$\varphi(\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\ G(\boldsymbol{k})\ e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}. \tag{3}$$

*Typos and errors should be emailed to the Instructor: Shubho Roy (email: sroy@phy.iith.ac.in)

Here $G(\boldsymbol{k})$ are the Fourier components of the field φ (we use the letter G because it is a Green's function i.e. for a delta function source). Plugging this in the LHS of equation (2) and on the RHS plugging the integral/Fourier representation of the Dirac delta function

$$\delta^3(\boldsymbol{x}-\boldsymbol{y}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})},$$

one gets,

$$\left(\boldsymbol{\nabla}^2 - m^2\right) \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, G(\boldsymbol{k})\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})} = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})},$$

or,

$$\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, G(\boldsymbol{k})\, \left(\boldsymbol{\nabla}^2 - m^2\right) e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})} = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}.$$

Next recall, $\boldsymbol{\nabla} e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})} = -i\boldsymbol{k} e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}$ and one has $\boldsymbol{\nabla}^2 \to (-i\boldsymbol{k})\cdot(-i\boldsymbol{k}) = -\boldsymbol{k}^2$ and the above equation becomes,

$$\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, G(\boldsymbol{k})\, \left(-\boldsymbol{k}^2 - m^2\right) e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})} = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}.$$

As before the $e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}$'s form a basis and the only way the above equation can hold iff for all $\mathbf{k}$, the basis coefficients are equal on both sides, i.e.

$$G(\boldsymbol{k})\, \left(-\boldsymbol{k}^2 - m^2\right) = 1,$$

which immediately give the Fourier components of the field to be,

$$G(\boldsymbol{k}) = -\frac{1}{\boldsymbol{k}^2 + m^2}. \tag{4}$$

Plugging this back in the mode expansion (3), we get the expression for the field

$$\varphi(\boldsymbol{x}) = -\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, \frac{1}{\boldsymbol{k}^2 + m^2}\, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{y})}.$$

Next we have to perform the momentum integrals. It will be convenient if we switch from Cartesian coordinates to spherical polar coordinates in momentum space, i.e. $(k^1, k^2, k^3) \to (q, \theta, \phi)$ where $q = \sqrt{\boldsymbol{k}\cdot\boldsymbol{k}}$. Further it will be even more convenient to choose, without any loss of generality the z-axis in the momentum space to be along the vector, $\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y}$. In these new coordinates the expression for the field is,

$$\varphi(\boldsymbol{x}) = -\int \frac{q^2 dq\, \sin\theta d\theta\, d\phi}{(2\pi)^3}\, \frac{1}{q^2 + m^2}\, e^{-iqr\cos\theta}.$$

Here $r = |\boldsymbol{r}|$. The ϕ integral can be performed easily as nothing in the integrand depends on it. So we replace $\int d\phi = 2\pi$, and get,

$$\varphi(\mathbf{x}) = -\frac{1}{4\pi^2}\int_0^\infty dq \frac{q^2}{q^2 + m^2} \int_0^\pi d\theta\, \sin\theta\, e^{-iqr\cos\theta}.$$

Next the θ-integral is performed, namely,

$$\int_0^\pi d\theta \ \sin\theta \ e^{-iqr\cos\theta} = \int_{-1}^1 d(\cos\theta) \ e^{-iqr(\cos\theta)} = \frac{e^{iqr} - e^{-iqr}}{iqr}.$$

After this one has,

$$\begin{aligned}\varphi(\mathbf{x}) &= -\frac{1}{4\pi^2}\int_0^\infty dq \frac{q^2}{q^2+m^2} \ \frac{e^{iqr} - e^{-iqr}}{iqr} \\ &= \frac{1}{4\pi^2 r}\int_0^\infty dq \frac{iq}{q^2+m^2} \ \left(e^{iqr} - e^{-iqr}\right) \\ &= \frac{1}{4\pi^2 r}\left(\int_0^\infty dq \frac{iq}{q^2+m^2} \ e^{iqr} - \int_0^\infty dq \frac{iq}{q^2+m^2} \ e^{-iqr}\right).\end{aligned}$$

Now on the second integral we perform a change of variables, $q \to -q$. Under this change of variables the second integral becomes,

$$-\int_0^\infty dq \frac{iq}{q^2+m^2} \ e^{-iqr} = \int_{-\infty}^0 dq \frac{iq}{q^2+m^2} e^{iqr}.$$

which is the same integral as the first term except the range is now $(-\infty, 0)$. Summing these two contributions, the field expression becomes a single integral ranging from $(-\infty, \infty)$,

$$\varphi(\boldsymbol{x}) = \frac{1}{4\pi^2 r}\int_{-\infty}^\infty dq \frac{iq}{q^2+m^2} \ e^{iqr}.$$

Since $\frac{d}{dr}(e^{iqr}) = iq \, e^{iqr}$, we can rewrite the above expression as,

$$\varphi(\boldsymbol{x}) = \frac{1}{4\pi^2 r}\frac{d}{dr}\left(\int_{-\infty}^\infty dq \frac{e^{iqr}}{q^2+m^2}\right). \tag{5}$$

To get to the final form of the field one has to perform the q-integral, namely,

$$J(r) = \int_{-\infty}^\infty dq \frac{e^{iqr}}{q^2+m^2}.$$

We note that the integrand has two poles on the imaginary q-axis, , while the integration contour is along the real q-axis from $-\infty$ to ∞. To evaluate the integral, J using the residue theorem one has to close the contour, and in the process encircle either of the two poles, i.e. one can choose to close the contour from above (in the upper half-complex q-plane) anticlockwise, or close the contour from below (in the lower half complex q-plane) clockwise as shown in figure (1). The choice of the contour is determined by the physical boundary conditions, one must have

$$\varphi(\boldsymbol{x}), J(r) \to 0, \ \text{as } r \to \infty. \tag{6}$$

This physically means the field weakens and becomes zero as one moves away from the location of the source to infinitely far distance. This boundary condition selects the anticlockwise contour in the upper half complex q-plane which encircles the pole, $q = +im$. Then using the residue theorem one has

$$J = \oint dq \frac{e^{iqr}}{q^2+m^2} = \frac{2\pi i \ e^{i(im)r}}{2(im)} = \frac{\pi}{m} e^{-mr}.$$

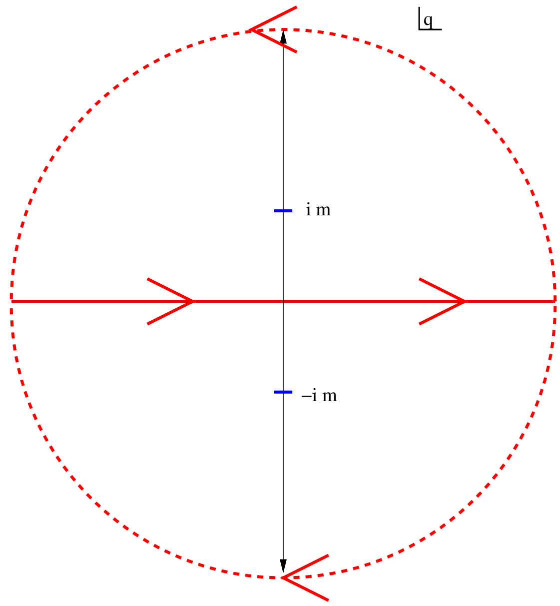

Figure 1: Two possible contours for the Yukawa meson field (Green's function) in the complex q-plane

Thus indeed we find $J(r) \to 0$ as $r \to \infty$ [1]. Plugging this result back in (5), we get the final expression of the scalar field produced by a point source of strength g located at $\boldsymbol{y}$,

$$\varphi(\boldsymbol{x}) = -\frac{e^{-mr}}{4\pi r}, \; r = |\boldsymbol{x} - \boldsymbol{y}| \,. \tag{7}$$

Although the factor $\frac{1}{4\pi r}$ is perhaps familiar from the Coulomb Green's function for electrodynamics for a unit point charge (source), there are a couple massive differences.

- First is the factor e^{-mr}, an exponential suppression of the field as one goes away from the source. This effectively means the effect of the sources dissipates to negligible amount $r > \frac{1}{m}$. Thus unlike electromagnetic force, the Klein-Gordon scalar field represents a **short-range interaction (force)**.

- Second, the overall negative sign implies that the force is an attractive one. The best way to see this is to ask what is the potential energy when we introduce a test source $\rho(\boldsymbol{x}) = \varepsilon\, \delta^3(\boldsymbol{x} - \boldsymbol{x}')$ of very weal strength ε in the field (7). The interaction energy is,

  $$U = \int d^3\boldsymbol{x}\; \rho(\boldsymbol{x})\varphi(\boldsymbol{x}) = -\varepsilon\, \frac{e^{-m|\boldsymbol{x}'-\boldsymbol{y}|}}{|\boldsymbol{x}' - \boldsymbol{y}|} < 0.$$

  Since the potential energy is negative, this implies an **attractive interaction**.

Such a real scalar field was proposed to be the force-field responsible for holding two protons or two neutrons together in the nucleus of an atom by H. Yukawa in 1935. That real scalar field went

[1]Had we selected the other contour, closing it from down in the lower half of complex q-plane, one would get $J(r) \propto e^{mr}$ thus leading to $J(r) \to \infty$ as $r \to \infty$. This would have been unphysical as the field (effect of the source) grows larger as we move farther from the source.

by the name of Yukawa's meson (field). From the range of the nuclear force, i.e. $\sim 10^{-15}m$, he was able to predict the mass parameter, $m \sim 10^{2-3}$ times the mass of the electron. On quantizing the Yukawa meson field thus one should expect the quanta of the meson field, namely the meson particles to have appear in nature with the same mass. Indeed in 1947, such meson particles were observed by Cecil Powell in cosmic rays (following up on observations of muon tracks on photographic plates by D.M. Bose and Bibha Chowdhuri seven years earlier).

However this real scalar is electrically neutral and cannot describe the interaction (force) between the neutron and the proton. For that one needs a theory of (electrically) charged scalars and the correct theory for that would necessarily involve a *complex* scalar field. We study the complex scalar field in the next section.

2 Complex Scalar field theory

A quick generalization of the free real scalar field theory is obtained by complexifying the field. We will denote the complex field by upper case Greek symbol, $\Phi(x)$. The main contrast with the real scalar field theory is that the complex scalar field theory will admit an **internal** symmetry called the global $U(1)$ symmetry which we will see via Noether's theorem to lead to a conserved charge. Further, we will identify this charge as the electric charge when we couple the complex scalar to a Maxwell gauge field, A_μ. As usual, the very first step in the study of any physical system, here in this case the complex scalar field theory, is writing down the action functional. Since the action must be real, the action is constrained to be the following,

$$I\left[\Phi(x), \Phi^\dagger(x)\right] = \int d^4x \, \left[(\partial_\mu \Phi)^\dagger \, \partial^\mu \Phi - V\left(\Phi^\dagger \Phi\right)\right]. \tag{8}$$

Here $\Phi^\dagger$ is the complex conjugate of Φ.

2.1 Free complex scalar field theory

For simplicity we take, $V\left(\Phi^\dagger\Phi\right) = m^2 \Phi^\dagger \Phi$, which as expected will give rise to a free theory i.e. one with equations of motion linear in Φ or $\Phi^\dagger$. The classical equation of motion for the complex field theory are,

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \Phi\right)}\right) - \frac{\partial \mathcal{L}}{\partial \Phi} = 0 = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \Phi^\dagger\right)}\right) - \frac{\partial \mathcal{L}}{\partial \Phi^\dagger}.$$

Plugging the expression for $\mathcal{L}$ from (8) is same as that of the real scalar field, i.e. the Klein-Gordon equation,

$$\left(\partial^2 + m^2\right)\Phi = \left(\partial^2 + m^2\right)\Phi^\dagger = 0. \tag{9}$$

Upon rewriting this complex scalar field into it's real and imaginary components,

$$\Phi = \frac{\phi_1 + i\,\phi_2}{\sqrt{2}}, \Phi^\dagger = \frac{\phi_1 - i\,\phi_2}{\sqrt{2}}, \tag{10}$$

we find out that the complex scalar field theory is a theory of two **non-interacting** real scalar field theories. This can be seen two ways, first by adding and subtracting respective sides of the two complex equations of motion, (9), one obtains two real Klein-Gordon equations,

$$\left(\partial^2 + m^2\right)\varphi_1 = \left(\partial^2 + m^2\right)\varphi_2 = 0$$

Second, on plugging field in terms of real and imaginary components (10) into the action, splits the action into two decoupled real free scalar field theory actions,

$$\begin{aligned} I\left[\Phi(x), \Phi^\dagger(x)\right] &= \int d^4x \, \left[(\partial_\mu \Phi)^\dagger \, \partial^\mu \Phi - m^2 \Phi^\dagger \Phi\right], \\ &= \int d^4x \, \left(\frac{1}{2}\partial_\mu \phi_1 \, \partial^\mu \phi_1 - \frac{1}{2} m^2 \phi_1^2\right) + \int d^4x \, \left(\frac{1}{2}\partial_\mu \phi_2 \, \partial^\mu \phi_2 - \frac{1}{2} m^2 \phi_2^2\right) \\ &= I\left[\varphi_1(x)\right] + I\left[\varphi_2(x)\right]. \end{aligned}$$

Homework: Using the solutions for the free real scalar field equations in terms of plane waves, write down the solution for a free complex scalar field.

3 Symmetries of the real scalar field theories

The theory of the real scalar field, $\varphi(x)$ can be described by the action,

$$I\left[\varphi(x)\right] = \int d^4x \, \mathcal{L},$$

where the Lagrangian $\mathcal{L}$ is a function of the scalar, $\varphi(x)$ and it's spacetime derivatives $\partial_\mu \varphi(x)$,

$$\mathcal{L} = \mathcal{L}\left(\varphi(x), \partial_\mu \varphi(x)\right) = \mathcal{L} = \frac{1}{2}\partial_\mu \varphi \, \partial^\mu \varphi - V\left(\varphi(x)\right)$$

The integration range is over all space and time. In particular, for the **free scalar,** the Lagrangian can be taken to be,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi \, \partial^\mu \varphi - \frac{1}{2} m^2 \varphi^2. \tag{11}$$

This form is dictated by Lorentz invariance i.e. the Lagrangian density **must be Lorentz scalar**. m is a Lorentz invariant quantity with the dimensions of mass (or energy). (Upon quantizing the system, the parameter, m will turn out to be the mass of the scalar field quanta/particles). The symmetries/invariances of the real scalar field action are:

- Lorentz invariance

$$x \to x' = \Lambda x,$$
$$\varphi(x) \to \varphi'(x') = \varphi(x).$$

- Translation invariance

$$x \to x' = x + a,$$
$$\varphi(x) \to \varphi'(x') = \varphi(x)$$

- Discrete internal symmetry,

$$\varphi \to \varphi' = -\varphi.$$

(Only if the lagrangian contains **even powers** of φ).

Checks:

- Lorentz invariance is rather obvious because most terms in the action is Lorentz invariant, d^4x, m^2, φ^2. Even the kinetic term, $\partial_\mu\varphi\,\partial^\mu\varphi$, is because the Lorentz index μ is contracted, viz:

$$\partial_\mu\varphi(x) \to \partial'_\mu\varphi'(x') = \Lambda_\mu{}^\nu\,\partial_\nu\varphi(x),$$

$$\begin{aligned}
\partial_\mu\varphi(x)\,\partial^\mu\varphi(x) \to \partial'_\mu\varphi'(x')\,\partial'^\mu\varphi'(x') &= \Lambda_\mu{}^\nu\,\partial_\nu\varphi(x)\,\Lambda^\mu{}_\alpha\partial^\alpha\varphi(x) \\
&= (\Lambda_\mu{}^\nu)\,(\Lambda^\mu{}_\alpha)\,\partial_\nu\varphi(x)\,\partial^\alpha\varphi(x) \\
&= \delta^\nu_\alpha\,\partial_\nu\varphi(x)\,\partial^\alpha\varphi(x) \\
&= \partial_\nu\varphi(x)\,\partial^\nu\varphi(x).
\end{aligned}$$

Thus the action

$$\begin{aligned}
I\left[\varphi'(x')\right] &= \int d^4x'\,\left[\frac{1}{2}\partial'_\mu\varphi'(x')\partial'^\mu\varphi'(x') - \frac{1}{2}m^2\varphi^2(x)\right] \\
&= \int d^4x\,\left[\frac{1}{2}\partial_\mu\varphi(x)\partial^\mu\varphi(x) - \frac{1}{2}m^2\varphi^2(x)\right] \\
&= I\left[\varphi(x)\right],
\end{aligned}$$

remains invariant.

- Translation invariance is also obvious because the action integral being defined over all space and time i.e ranges of integration being $(-\infty,\infty)$, is independent of the origin of coordinates and there is no *explicit* dependence on the coordinates, x. Recall that under translations, namely,

$$x \to x' = x + a,$$

the field φ transforms as,

$$\varphi'(x') = \varphi(x),$$

What about the kinetic piece containing terms such as $\partial_\mu\varphi$. Such a term seems to care about the spacetime coordinate through the derivative, $\partial_\mu = \frac{\partial}{\partial x^\mu}$. Actually even this derivative is independent of the shift in origin because under a shift of origin of coordinates,

$$x \to x' = x + a,$$

Conversely,

$$x = x' - a$$

the derivative transforms as

$$
\begin{aligned}
\frac{\partial}{\partial x^\mu} \to \frac{\partial}{\partial x'^\mu} &= \frac{\partial x^\nu}{\partial x'^\mu}\frac{\partial}{\partial x^\nu}, \\
&= \frac{\partial\left(x'^\nu - a^\nu\right)}{\partial x'^\mu}\frac{\partial}{\partial x^\nu} \\
&= \frac{\partial x'^\nu}{\partial x'^\mu}\frac{\partial}{\partial x^\nu} \\
&= \partial^\nu_\mu \frac{\partial}{\partial x^\nu} \\
&= \frac{\partial}{\partial x^\mu}.
\end{aligned}
$$

So the derivative remains unchanged. (Here since a is a constant its derivative vanishes, and we get $\frac{\partial(x'^\nu - a^\nu)}{\partial x'^\mu} = \frac{\partial x'^\nu}{\partial x'^\mu}$).

- Discrete internal symmetry such as $\varphi \to \varphi' = -\varphi$ is also obvious when the Lagrangian contains even powers of φ. However as they are discrete (non-continuous) and do not obey Noether's theorem and they will not give rise to conserved charges.

4 Symmetries of the complex scalar field theories

In addition to Lorentz and translation symmetries the complex scalar field theories have an extra couple of symmetries

4.1 Global $U(1)$ Symmetry of the complex field theory

One can easily check that the complex scalar field theory action Eq. (8) is invariant under multiplication by a **constant** complex phase factor $e^{i\,\alpha}$,

$$
\begin{aligned}
\Phi \to \Phi' &= e^{-i\,\alpha}\Phi, \\
\Phi^\dagger \to \Phi'^\dagger &= e^{i\,\alpha}\Phi^\dagger,
\end{aligned} \tag{12}
$$

where $\alpha \in \mathbb{R}$. Since a complex phase is unitary i.e. the complex conjugation is also the inverse,

$$
\left(e^{-i\,\alpha}\right)^\dagger = \left(e^{-i\,\alpha}\right)^{-1},
$$

such phases are also called $U(1)$ factors (U stands for Unitary matrix and since a number is a 1×1 matrix, $U(1)$ is unitary matrix of size 1×1). Since this symmetry transformation does not touch spacetime but only changes the fields (configuration space variables), such a symmetry is called an **internal symmetry**. Also note that since α is a constant i.e. not a function of spacetime, it is a **global** symmetry (**global = same everywhere = independent of spacetime location**).

Check: Under the U(1) symmetry Eq. (12), the potential term is obviously invariant,

$$
\begin{aligned}
\Phi'^\dagger\Phi' &= \left(e^{i\,\alpha}\Phi^\dagger\right)\left(e^{-i\,\alpha}\Phi\right) \\
&= \Phi^\dagger\Phi
\end{aligned}
$$

and this is true whether α is a constant or a function of spacetime i.e. $\alpha(x)$. Now let's look at the kinetic term,

$$\begin{aligned} \left(\partial_\mu \Phi^\dagger\right)\left(\partial^\mu \Phi\right) \rightarrow \left(\partial_\mu \Phi'^\dagger\right)\left(\partial^\mu \Phi'\right) &= \partial_\mu\left(e^{i\alpha}\Phi^\dagger\right)\partial^\mu\left(e^{-i\alpha}\Phi\right), \\ &= e^{i\alpha}\left(\partial_\mu \Phi^\dagger\right)\, e^{-i\alpha}\left(\partial^\mu \Phi\right) \\ &= \left(\partial_\mu \Phi^\dagger\right)\left(\partial^\mu \Phi\right). \end{aligned}$$

So this kinetic term in the action is also invariant because α is a constant and the derivative does not act on it. If α was a function of spacetime, $\alpha = \alpha(x)$, the derivative would have acted on it and the term would not be invariant. Incidentally, a spacetime dependent phase $\alpha(x)$ is called a **local** $U(1)$ **transformation. Noether's theorem implies this global continuous symmetry will give rise to a conserved quantity (Noether charge) which we will shortly identify as the electric charge of the mesons.**

4.2 The charge conjugation symmetry of the complex scalar field theory

Note that in addition to the continuous global $U(1)$ symmetry (12), there is another discrete internal symmetry of the complex field theory, namely, interchanging the field Φ with its complex conjugate, $\Phi^\dagger$,

$$\Phi \leftrightarrow \Phi^\dagger.$$

Under this symmetry of course the charge (polarity) of the scalar field also changes,

$$Q \rightarrow -Q.$$

This is why this discrete symmetry is dubbed as ***charge conjugation symmetry***. In the quantum theory, this discrete internal symmetry will transform particles to antiparticles and vice-versa.