Practice Calculation: 5 qubits (A, B, C, D, E)

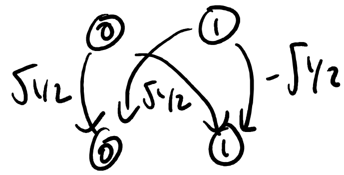
Say your current q. state is:

$$\left\{\begin{array}{lll} .36 \text{ amplitude on } & 00000 \\ .48 \quad " & " \quad 00100 \\ -.8 \quad " & " \quad 10000 \end{array}\right\} \quad (.36)^2 + (.48)^2 + (-.8)^2 = 1 \checkmark$$

$$\rightarrow \; .36|00000\rangle + .48|00100\rangle - .8|10000\rangle$$

What if we do H on 3rd qubit C?

$$\begin{array}{l} 00000 \rightarrow \\ \\ \\ 00100 \rightarrow \\ \\ \\ 10000 \rightarrow \\ \\ \end{array} \begin{bmatrix} .36 \\ 0 \\ 0 \\ .48 \\ \vdots \\ \vdots \\ -.8 \\ 0 \\ \vdots \end{bmatrix}$$



$$\equiv \begin{array}{cc} & \begin{array}{cc} 0 & \text{in} \quad 1 \end{array} \\ \begin{array}{c} 0 \\ \text{out} \\ 1 \end{array} & \begin{bmatrix} \sqrt{1/2} & \sqrt{1/2} \\ \sqrt{1/2} & -\sqrt{1/2} \end{bmatrix} \end{array}$$

Result:

With ampl. .36, ABCDE is 00000
→ Doing H on C, go to 00000 w/ ampl $\sqrt{1/2}$
go to 00100 w/ ampl $\sqrt{1/2}$

With ampl. .48, ABCDE is 00100
→ Doing H on C, go to 00000 w/ ampl $\sqrt{1/2}$
go to 00100 w/ ampl $-\sqrt{1/2}$

So far: $(.36\sqrt{1/2} + .48\sqrt{1/2})$ ampl. on 00000
$(.36\sqrt{1/2} - .48\sqrt{1/2})$ ampl. on 00100

$.84\sqrt{1/2}$
$-.12\sqrt{1/2}$

With ampl -.8, ABCDE is 10000
→ Doing H on C, go to 10000 w/ ampl $\sqrt{1/2}$
go to 10100 w/ ampl $\sqrt{1/2}$

With ampl. 0, ABCDE is 10100
→ Doing H on C, go to 10000 w/ ampl $\sqrt{1/2}$
go to 10100 w/ ampl $-\sqrt{1/2}$

Final result:

{ $.84\sqrt{1/2}$ ampl on 00000, $-.12\sqrt{1/2}$ ampl on 00100 }

$-.8\sqrt{1/2}$ " " 10000, $-.9\sqrt{1/2}$ " " 10100

<u>Summary</u>: If you're doing H on 3rd out of 5 qubits:
Pair up all the 5-bit strings
like $\left\{\begin{array}{l} ab0de \to \text{ampl. } p \\ ab1de \to \text{ampl } q \end{array}\right\}$ Do Had. instruct on $\begin{bmatrix} p \\ q \end{bmatrix}$
gives new ampls on
ab0de
ab1de

Add up all results.

Lecture 6: Enter Minus Signs

To get <u>some</u> minus signs: Apply H to 1.

Make-m: // assume A=0
- Add 1 to A
- H on A

Result: $\text{Ampl}[0] = \sqrt{1/2}$ $\text{Ampl}[1] = -\sqrt{1/2}$: $\begin{array}{c} 0 \\ 1 \end{array}\begin{bmatrix} +\sqrt{1/2} \\ -\sqrt{1/2} \end{bmatrix} =: \text{"}\vec{m}\text{"}$

(real name: "|->")

<u>Fun fact</u>:
If we do "logical negation" (bool NOT) ("Add 1 to A")
the resulting q. state $\begin{array}{c} 0 \\ 1 \end{array}\begin{bmatrix} -\sqrt{1/2} \\ +\sqrt{1/2} \end{bmatrix} = -\vec{m}$
"amplitude negation"

Q: What if we now did $(\text{Make-m})^{-1}$?
- H on A
- Add 1 to A

$(\text{Make-m})^{-1}$ $\vec{m}$

$[\ \vdots\ ]$ $[\ ] =: \begin{array}{c} 0 \\ 1 \end{array}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Longrightarrow$ $(\text{Make-m})^{-1}(-\vec{m})$

$[\ \vdots\ ]\left(-[\ \vdots\ ]\right) = -\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}\begin{array}{c} 0 \\ 1 \end{array}$

Final state: "Ampl -1 on $A=0$"
(Ampl 0 on $A=1$)"

Q: What if did the same but $\exists$ vars $X, Y, Z\ldots$
vals $1\ 1\ 0\ldots$

Coming in: $XYZ,A = 110,0$: Ampl$[1100] = +1$
Make-m on A: Ampl$[1100] = +\sqrt{1/2}$, Ampl$[1101] = -\sqrt{1/2}$
Add 1 To A : Ampl$[1101] = +\sqrt{1/2}$ Ampl$[1100] = -\sqrt{1/2}$
$(\text{Make-m})^{-1}$ on A: Ampl$[1100] = -1$

Idea: X, Y, Z have certain 0/1 values
$A = 0$. temp1, temp2, ... $= 0$ ["ancillas"]

Make-m on A
Add $F(X,Y,Z)$ to A
$(\text{Make-m})^{-1}$ on A

Result:
temp1=0
temp2=0
...
If $F(X,Y,Z)=0$, Ampl$[X,Y,Z, A=0] = +1$
If $F(X,Y,Z)=1$, Ampl$[X,Y,Z, A=0] = -1$

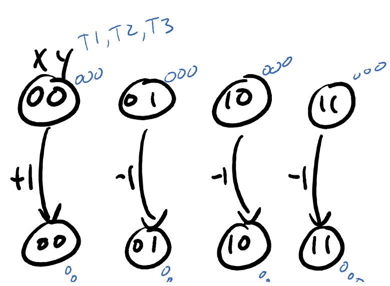
No bits changed vals, but ampl. sign changed if $F(X,Y,Z)=1$.

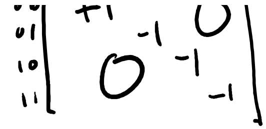
"If $F(X,Y,Z)$ Then Minus":
// assume some # of temp vars (incl. "A") init'd to 0
Make-m on A
Add $F(X,Y,Z)$ to A
$(\text{Make-m})^{-1}$ on A
// post condition: all temp vars 0 again

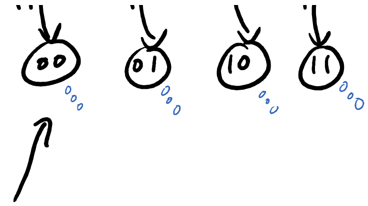
"Sign-computing F" or "$F^{\pm}$" (only appropriate for $F: \{0,1\}^n \to \{0,1\}$)

New operation:
"If (X OR Y) Then Minus":

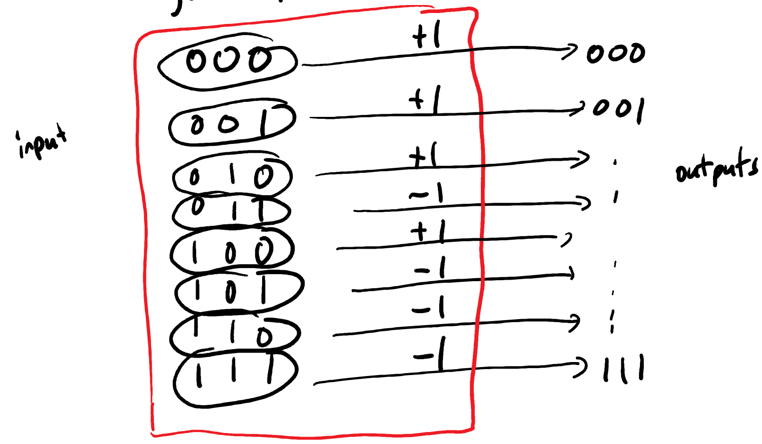
$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 00 & +1 & & & 0 \\ 01 & & -1 & & \\ 10 & 0 & & -1 & \\ 11 & & & & -1 \end{array}$$







"If $\mathrm{Maj}_3(X,Y,Z)$ Then Minus":

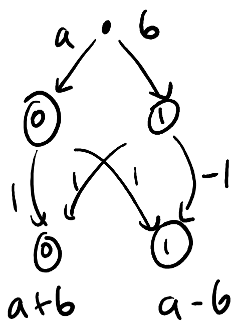


HAD. Say you have two #'s like 8 and 6.

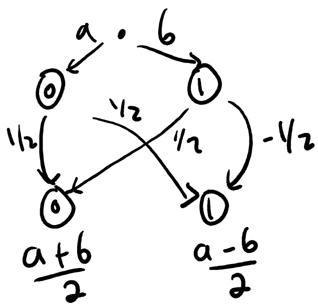
Average & Displacement $(8,6) = (7,1)$ because $7 \pm 1 = 8,6$

Add & Difference $(7,1) = (8,6)$ " $7 \pm 1 = 8,6$

$\mathrm{Add\&Diff}(\mathrm{Avg\&Disp}(\cdot,\cdot))$ = "do nothing"



← Add & Diff → $\begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix} = \sqrt{2}\begin{bmatrix} H \end{bmatrix}$



← Avg& Disp → $\begin{bmatrix} +\frac{1}{2} & +\frac{1}{2} \\ +\frac{1}{2} & -\frac{1}{2} \end{bmatrix} = \sqrt{\frac{1}{2}}\begin{bmatrix} H \end{bmatrix}$

Hadamard → $\begin{pmatrix} +\sqrt{\frac{1}{2}} & +\sqrt{\frac{1}{2}} \\ +\sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} \end{pmatrix} = H$ "scaling split the diff. between 1 & $\frac{1}{2}$"

Avg & Disp is "basically" H except extra scalar factor of $\sqrt{\frac{1}{2}}$

Add & Diff " " " " " " " " $\sqrt{2}$

TRICK: If you do an even # of H instructions,
OK to pretend half are Avg & Disp
& " " Add & Diff

TRICK: OK to change H instructions to A&D willy-nilly
as long as you tell the reader you're working
"unnormalized quantum states".

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example:

Say you have one qubit: "ampl. .8 on 0, ampl. −.6 on 1"

$$\begin{array}{c} 0 \\ 1 \end{array}\begin{bmatrix} .8 \\ -.6 \end{bmatrix} \qquad \left(.8|0\rangle - .6|1\rangle \right) \qquad (.8^2 + (-.6)^2 = .64 + .36 = 1 \checkmark)$$

Apply Add & Diff: $\left(= \sqrt{2} \cdot \begin{bmatrix} \text{H} \end{bmatrix} \right)$

$$\begin{array}{c} 0 \\ 1 \end{array}\begin{bmatrix} .2 \\ 1.4 \end{bmatrix} \Bigg\} \text{ "unnormalized state"} \quad (.2)^2 + (1.4)^2 = .04 + 1.96 = 2 \checkmark$$

OR Apply Avg & Disp: $\left(= \sqrt{\frac{1}{2}} \cdot \begin{bmatrix} \text{H} \end{bmatrix} \right)$

$$\begin{array}{c} 0 \\ 1 \end{array}\begin{bmatrix} .1 \\ .7 \end{bmatrix} \Bigg\} \text{ "unnormalized"} \quad (.1)^2 + (.7)^2 = .01 + .49 = .5 = \frac{1}{2} \checkmark$$

And followed with Add & Diff

$$\begin{bmatrix} .8 \\ -.6 \end{bmatrix} \Bigg\} \text{ normalized } \checkmark$$