Lecture 10 — The Nature of Qubits

Physical objects have properties that can be in different "basic states".

[What's a "basic state"? Best to just give some examples...]

- Coin  (on a table): "Heads" (up) / "Tails" (up)
- Switch  : "On" / "Off"
- Die  (on table): "1", "2", ..., "6"

If 2 basic states, can model a logical bit. [an abstract concept]
0/1

[assign, e.g., "Heads" = 0, "Tails" = 1]

[In practical computers/circuits, often....]

- Wire's voltage: "5V" / "0V"

[Not the crispest example because voltage actually has a continuum of values... but engineers have some methods for coercing voltages to be either ≈ 5V or ≈ 0V.]

Q: Why focus on bits (2 basic states)?

A: Just proved to be convenient.

[[You don't have to. You could try to build computers based around logical trits (0/1/2), get objects w/ 3 basic states (3-sided dice, voltages ≈ 0V, ≈ 5V, ≈ 10V, ...) Just seems like 80 years of experience suggests bits are most pleasant.]]

2 states: bit (0/1)

3 states: trit (0/1/2)

4 states: "dit" with $d=4$ (0/1/2/3) ≡ 00, 01, 10, 11 → 2 bits

[[d-sided die implements a general "d"it]]

dit with $d=2^k$: ≡ k bits

Law of Physics:

If <u>any</u> object can be in 2 basic states ("0", "1"), it can also be in a <u>superposition</u> state: { "a ampl. on 0, b ampl. on 1" where $a^2+b^2=1$. (a,b can even be complex #'s, with $|a|^2+|b|^2=1$).

This logical concept called: <u>qubit</u>.

If d basic states ⟶ "qudit"

amplitude a_0 on "0", a_1 on "1", ..., a_{d-1} on "d−1"

such that $|a_0|^2+|a_1|^2+\dots+|a_{d-1}|^2=1$.

[Like python, us C.S. people like 0-based indexing...]

Q: <u>Any</u> object?? Yes!

[So... why don't we ever see a die in a superposition of 6 faces, or Schrödinger's Cat in a superposition of ~~Alive~~ & ~~Dead~~ Asleep & Awake?

Well, turns out it's hard to get macroscopic objects into superpositions other than "1 amplitude on ___".

Reason has to do w/ shielding them from the "environment."

More on this later....]

[Well, but for microscopic particles, we <u>can</u> have these states. As we have seen, it's often convenient to stack the amplitudes into column vectors...]

A qubit: $\begin{matrix} 0 \\ 1 \end{matrix}\begin{bmatrix} a \\ b \end{bmatrix}$

A die's quantum state: $\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$

Two qubit eg: $\vec{V} = \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix}\begin{bmatrix} \sqrt{.1} \\ \sqrt{.2} \\ \sqrt{.3} \\ \sqrt{.4} \end{bmatrix}$

unit vectors → with $|a|^2 + \dots + |f|^2 = 1$

Dirac's "Bra-Ket" Notation:

[This notation is awesome notation for LINEAR ALGEBRA. Has nothing inherently to do with quantum, except that only quantum physicists seem wise enough to use it.]

[Has 3 cool features I'll tell 2 today, save the 3rd (bras) for later.]

Feature 1:

[[Some people signify vectors with boldface, like $\mathbf{v}$. This is disgusting.]]

(broke)

[[Much better is when people write $\vec{v}$ to mean "a vector named v". Or even $\overrightarrow{foo}$ for "a vector named foo".]]

(woke)

[[But best is Dirac's way....]]

(bespoke)

$|v\rangle$: a <u>column</u> <u>vector</u> named v

or $|foo\rangle$: " " " " foo.

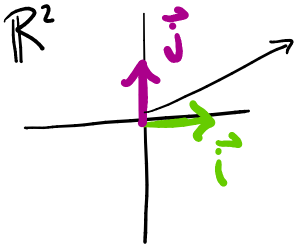
"$|\cdot\rangle$", pronounced "ket", signifies the data type <u>column</u> <u>vector</u>.

[[Later we'll see $\langle\cdot|$, "bra", signifying row vector]]

Feature 2:

[[In high school, you learn about...]] vectors in 2-d:

$\mathbb{R}^2$



$$\vec{i} = \begin{bmatrix}1\\0\end{bmatrix} \qquad \vec{j} = \begin{bmatrix}0\\1\end{bmatrix}$$

$$\begin{bmatrix}2.8\\1.3\end{bmatrix} = 2.8\,\vec{i} + 1.3\,\vec{j}$$

(broke)

[[Later, you move to...]] Vectors in 3-d:

$$\vec{i} = \begin{bmatrix}1\\0\\0\end{bmatrix} \quad \vec{j} = \begin{bmatrix}0\\1\\0\end{bmatrix} \quad \vec{k} = \begin{bmatrix}0\\0\\1\end{bmatrix}, \qquad \begin{bmatrix}.2\\-3\\1\end{bmatrix} = .2\vec{i} - 3\vec{j} + \vec{k}.$$

(woke)

[[College:]] Vectors in d dims:

$$\vec{e}_0 = \begin{bmatrix}1\\0\\0\\\vdots\\0\end{bmatrix}, \quad \vec{e}_1 = \begin{bmatrix}0\\1\\0\\\vdots\\0\end{bmatrix}, \quad \vec{e}_2 = \begin{bmatrix}0\\0\\1\\\vdots\\0\end{bmatrix}, \quad \ldots, \quad \vec{e}_{d-1} = \begin{bmatrix}0\\0\\\vdots\\0\\1\end{bmatrix}$$

[[Dirac: why bother to write "e"?!]] (bespoke)

$$|0\rangle = \begin{bmatrix}1\\0\\0\\\vdots\\0\end{bmatrix}, \quad |1\rangle = \begin{bmatrix}0\\1\\0\\\vdots\\0\end{bmatrix}, \quad \ldots, \quad |d-1\rangle = \begin{bmatrix}0\\0\\0\\\vdots\\0\\1\end{bmatrix}$$

When $d = 2^n$, use base-2 notation.

E.g., $d = 4$:

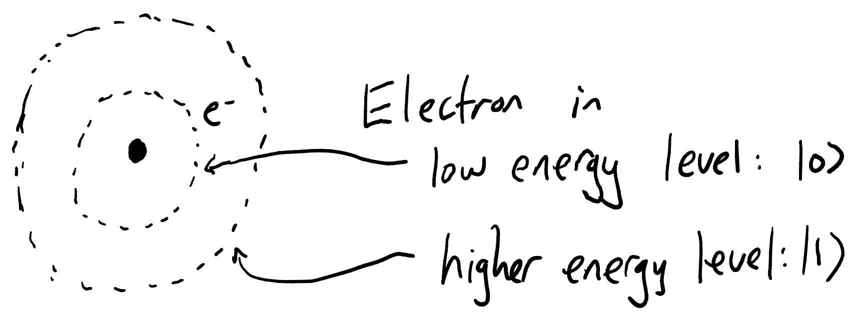
$$|00\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \quad |01\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \quad |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \quad |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

eg: $\left\{ \begin{array}{l} \text{".8 ampl. on 00,} \\ \text{−.6 ampl. on 11"} \end{array} \right\} = \begin{bmatrix}.8\\0\\0\\-.6\end{bmatrix} \begin{matrix}00\\01\\10\\11\end{matrix}$

$= .8|00\rangle - .6|11\rangle$

[[End of linear algebra interlude for now. Let's go back to talking about objects, properties, basic states, & superpositions. We'll give some examples with which you can actually make a qubit.]]

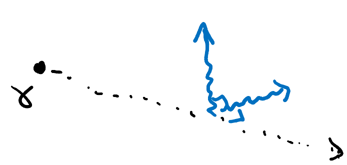
- Hydrogen atom:



[[Actually, there are theoretically ∞'ly many levels, 0, 1, 2, ... Kinda like voltage in a wire, you have to engineer things so only first 2 levels are used.]]

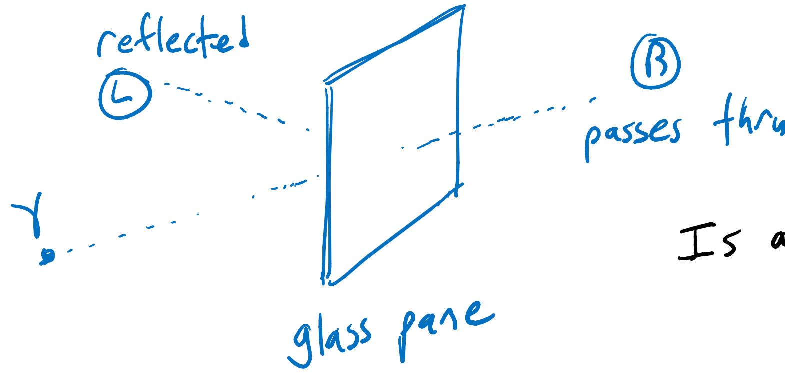
- Photon [[light particle]] Property: "polarization".

[[My favorite example, because — as we'll see — you can actually experience this one in day-to-day life. Roughly speaking, if a photon is travelling



it can be "wiggling" ("polarized") horizontally: |0⟩ or vertically: |1⟩

- **Photon** Property: **Position** [Same object, different property]



Is at position L or position R.

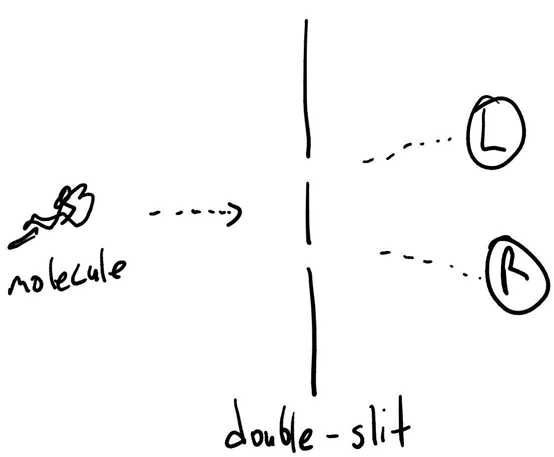
[Indeed, when a photon strikes an ordinary piece of glass, it will get into a superposition (!) of being at 0 or 1. It's why you can see faint reflections in windows....]

Remark: By tracking one photon's polarization & position, can get 2 qubits: amplitudes

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \begin{matrix} H\ L \\ H\ R \\ V\ L \\ V\ R \end{matrix}$$

(polarization ↗, position ↑)

- Molecule with 1000s of atoms [they can actually do this!]



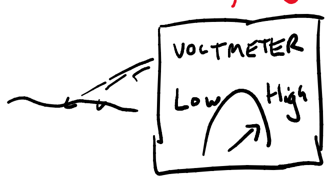
Property: position

- <u>Electron</u> Property: "spin" ["magnetic moment"]

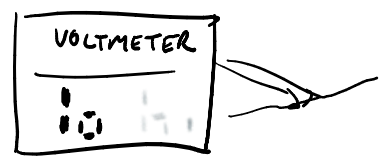
Basic states: "down" & "up".

$|0\rangle$ $|1\rangle$

[But... what <u>is</u> it? Well, some people burble about it being some kind of "internal angular momentum", but at the end of the day, it's just... some intrinsic property. Like "electrical charge"! What <u>is</u> charge? Who knows. But... it manifests itself in some macroscopic phenomena. If you hook a voltmeter up to a wire, you get...]

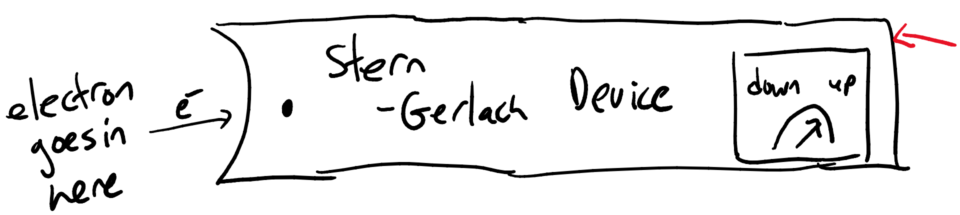


[old-fashioned needle]



[new-fashioned LCD 7-segment display]

[Same for electron "spin". Can build a big old apparatus, as they did 100 years ago...]



[needle or LCD "readout", shows "down" or "up" when you put in electron]

[has a magnetic screen & a screen & stuff]

Physically implements the "Print" instruction for qubit stored via "spin"

Real name: "MEASURE"

Incoming electron has spin in state $\begin{bmatrix} a \\ b \end{bmatrix}^{\text{down}}_{\text{up}} = a|\text{down}\rangle + b|\text{up}\rangle$

$= a|0\rangle + b|1\rangle$

$\Rightarrow \Pr[\text{readout is "down"}] = |a|^2$

$\Pr[\text{readout is "up"}] = |b|^2$

[[NOTE: Readout can literally only say "down" or "up".]]

[[There are also ways to change ("Add 1 To", "Had") the spin state of an electron (magnetic fields?) and prepare electrons in state $|\text{down}\rangle$ or state $|\text{up}\rangle$.

[[They built these measuring devices, got two kinds of readouts, found they conformed to the above rules, and just named the property "spin".]]

[[What about for the "double slit" experiments, where the property was "Position"?
The "basic states" are actually determined by the nature of your...]] Measuring Device:

[[Again, this implements the Print/Measure instruction:]]

$$a|\text{Left}\rangle + b|\text{Right}\rangle \Rightarrow \Pr[\text{readout} = \text{"Left"}] = |a|^2.$$

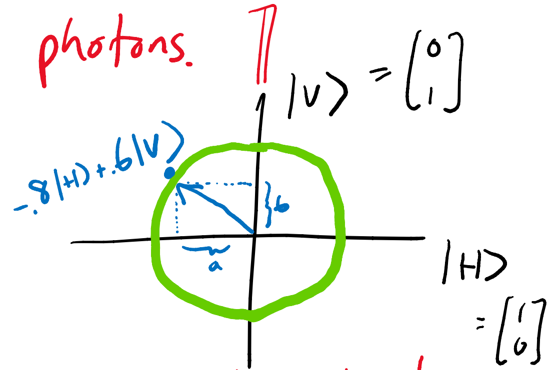
[[Another kind of measuring device for photons:
retinas. Unclear if human retinas can be sensitive to 1 photon (ten photons: for sure).
But, e.g., frog retinas definitely are.

These convert qubit superpositions to classical bits of info (electrochemical impulses on nerves).]]

⟦Finally, let's talk about my favorite quantum property: polarization for photons.⟧

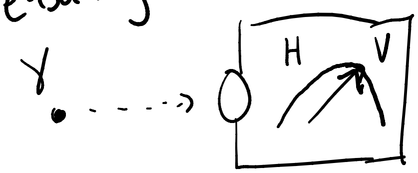
$a|H\rangle + b|V\rangle$:

horz wiggling — vert wiggling



⟦Here, superposition doesn't seem that weird at all. Just diagonal wiggling?⟧

One can build polarization measuring device:



⟦In fact, you can buy a device that does something very similar to this, for $0.80...⟧

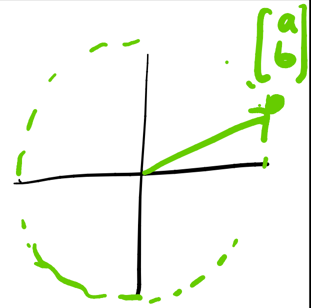
→ "Horizontal Polarizing Filter" ↩ its behavior, in "code"

↑ "polaroid"

⟦show the plastic square⟧

```
HPF(A)
 . measure A           // like "Print", destroys A
 . if Output = "V"     // vertical
       GenerateHeat()
       return
 . else                // Output = "H", horizontal
       Make A
       return A        // pass thru A, in deterministic state |H⟩
```

Incoming photon $\begin{bmatrix} a \\ b \end{bmatrix} = a|H\rangle + b|V\rangle \in \mathbb{R}^2$



- with prob a^2, photon passes thru, now in state $|H\rangle$
- with prob b^2, heat generated.

[[Doubles as a measure ("Print") instruction: output is whether photon comes thru (heat is too little to detect)

& as a device for (trying to) initialize a random photon to $|H\rangle$ ("Make").]]

Can also build VPF [[Vertically polarizing filter]]:

- passes thru in state $|V\rangle$ w. prob b^2
- heat w. prob a^2

[[At 3-d movies, they give you glasses where left eye has HPF, right eye has VPF.

They project one image using only photons in state $|H\rangle$: left eye sees them, all blocked for right eye.

Other image projected with only $|V\rangle$ photons — seen only by right eye. Voila, 3-d.]]

A little "circuit":

any photon here definitely $|H\rangle$

0% probability a photon gets here.

[[show it blocks all light]]

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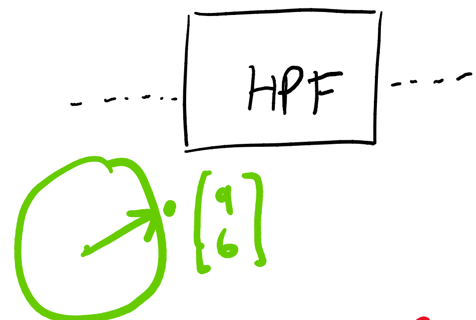
random photon $a|H\rangle + b|V\rangle$

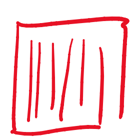
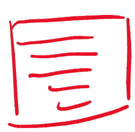


[[Prob. gets thru is... $\mathbb{E}[a^2]$ for $\begin{bmatrix} a \\ b \end{bmatrix}$ random on circle.

By symmetry, $\mathbb{E}[a^2] = \mathbb{E}[b^2]$, but $\mathbb{E}[a^2+b^2]=1$, so both are $1/2$.]]

Conclusion: 50% of photons get thru.
Also true if you stack 2, 3, 4, ... HPFs

[[Show 50% intensity with 1 or 2 HPFs]]

[[Know how to build a VPF  from an HPF ? Just physically rotate it 90°!

But then... wait... what if you rotate an HPF 42°? What does that do? Unitary $\begin{pmatrix} \cos 42° & -\sin 42° \\ \sin 42° & \cos 42° \end{pmatrix}$ involved ...]]