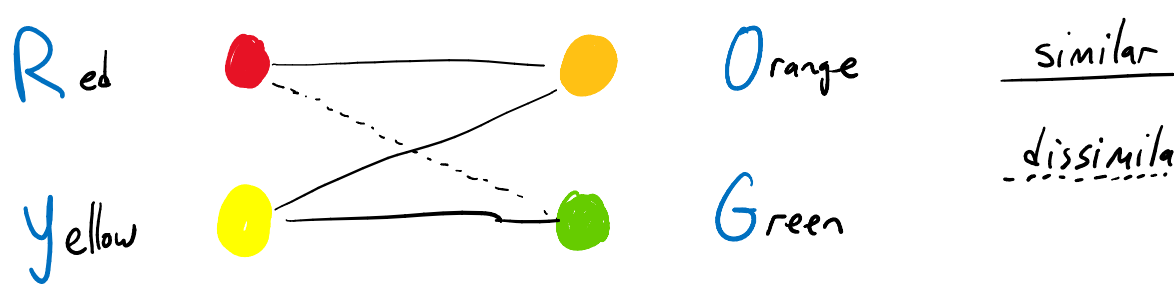
Lecture 15 — The RY:OG game

[[Real name: "CHSH game"]]

[[Named after Clauser – who just won Nobel Prize for this, Horne, Shimony, Holt. Based on an earlier version by Bell. Clauser got the prize by managing to be not dead yet, though :/]]

[[Here's a fact about colors:]]



A co-op game for Alice & Bob.

[[They can prep before the game however they want. But once game starts, they are separated & not allowed to communicate. To ensure this with no cheating, you can make Alice go to Mars (w/ a referee), Bob go to Jupiter (w/ a referee), and play the game over the course of, say, 5 minutes. Since Mars & Jupiter are 30 light-minutes apart, it's impossible for them to communicate: Law of Physics that info. can't travel faster than speed of light. Here's the RY:OG game: At noon, Pittsburgh time…⟧

Alice's ref flips a coin with sides R/Y

Alice has 5 minutes to respond w/ bit a

Bob's ref flips a coin with sides O/G

Bob has 5 mins to respond w/ bit b

Refs convene. Al & Bob win iff a, b have same "similarity" as coin outcomes

⟦That is, they always want $a = b$ unless the flips were R for Alice & G for Bob, in which case they want $a \neq b$.⟧

⟦Try playing this game! Obviously can win always if allowed to communicate in those 5 mins.⟧

[[It would seem Alice & Bob's best chance of winning is 75%. Let's prove this....]]

- Can win w/ prob. 75%:

  Just ignore coins. Alice outputs $a = 1$ [[say]]

  Bob outputs $b = 1$

  always. Win $\Leftrightarrow$ don't get Red & Green

  Prob. 25%.

- Can't win w/ prob. > 75%?

[[What is the definition of a "strategy"? Code!]]

```
~~ create some global vars ~~

A_strat(RYcoin):                B_strat(OGcoin):

    return a                        return b
```

Case 1: All code deterministic.

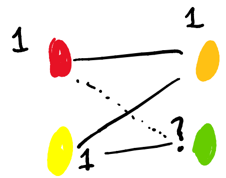
Then $a = f(\text{RYcoin})$ for some deterministic
$b = g(\text{OGcoin})$

$f : \{\bullet, \bullet\} \to \{0,1\}$

$g : \{\bullet, \bullet\} \to \{0,1\}$.

[[f, g can depend on global vars too, but they're det'ically set]]

[So in deterministic case, a strategy boils down to a labeling of this graph with 0's & 1's, and easy to see cannot get all 4 edges as desired...]

eg: 

Must be "wrong" on ≥ 1 edge

prob. 1/4 of being chosen by coin flips

$\Rightarrow \Pr[\text{loss}] \geq 25\%$

Case 2: Strategy uses probabilistic code.

[Now all the can include RNG calls.]

eg:
```
A_strat(RYcoin):
    if RYcoin == Red:
        D1 := RandInt(10)
        D2 := RandInt(20)
        a := D1+D2 mod 2
    else:
        D3 := RandInt ___
        D4 ___
        a := DetFunc(D3, D4, ...)
```

But: Alice can move all RNG to begining...
& return some DetFunc(D_1, D_2, D_3, D_4, RYcoin).

In fact, can do all RNG before going to Mars [/seeing RYcoin]
[same with Bob!]

[So we equivalently get to...]

~ Global, shared RNG ~

~ $D_1, D_2, \ldots$ ~

Mars

Jupiter

A_strat (RYcoin):

return $f(D_1, D_2, \ldots, RYcoin)$

B_strat (OGcoin):

return $g(D_1, D_2, \ldots, OGcoin)$

[for f, g deterministic]

Now for each outcome/realization of random $D_1, D_2, \ldots$

e.g. $8, 17, \ldots,$ A_strat, B_strat are just some deterministic labeling $RYcoin \mapsto \{0,1\}$, $OGcoin \mapsto \{0,1\}$, hence yield win prob. $\le 75\%$!

[So effectively, they're just randomly agreeing on a deterministic strategy.... which we know wins with prob. $\le 75\%$.

This fact, that even w/ prob. code they can win with prob $\le 75\%$, is called "Bell's Inequality"]

Key obs: with prob. code, you may as well REALIZE all RNG outcomes before playing

[and then it's like strats are deterministic].

[So... what if they use qubits...? We'll see they can win with prob. $\approx 85\%$!!]

Recall: $|MEP\rangle$ is like a shared steering wheel.

Alice's right hand U

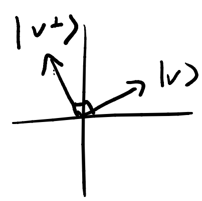
Bob's left hand $U^\dagger$

These do the same thing

[For U a real unitary, I forgot to mention.]

$$\sqrt{\tfrac{1}{2}}\left(|\overset{AB}{00}\rangle + |\overset{AB}{11}\rangle\right)$$

$$= \sqrt{\tfrac{1}{2}}\left(|v\rangle \otimes |v\rangle + |v^\perp\rangle \otimes |v^\perp\rangle\right) \quad \text{for any basis}$$



$\mathbb{R}^2$

Say Bob does Rot_β to B

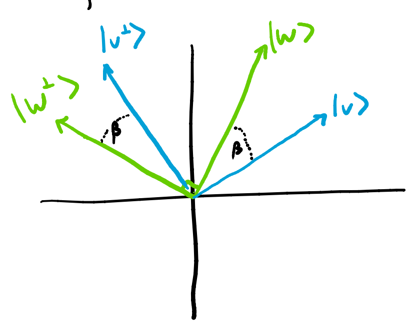
[equivalent to Alice doing $Rot_{-\beta}$ to A.]

Get to "$|MEP$ rot'd β tow Bob$\rangle$" $= \sqrt{\tfrac{1}{2}}\left(|0\rangle \otimes Rot_\beta|0\rangle + |1\rangle \otimes Rot_\beta|1\rangle\right)$

$$= \sqrt{\tfrac{1}{2}}\left(|v\rangle \otimes |w\rangle + |v^\perp\rangle \otimes |w^\perp\rangle\right)$$

for any bases $v, v^\perp$ $w, w^\perp$ having angle β from v to w:

eg:



We're at $\frac{1}{\sqrt{2}}\left(|0\rangle \otimes Rot_\beta|0\rangle + |1\rangle \otimes Rot_\beta|1\rangle\right)$.

Suppose now Alice measures A, then Bob measures B.

⟦as we've seen before, equivalent if they measure in opposite order, temporally. Also, as we saw last time...⟧

• $\Pr[\text{Alice reads out "0"}] = 1/2$

$\longrightarrow$ state collapses to $|0\rangle \otimes Rot_\beta|0\rangle$

$\left(Rot_\beta|0\rangle = (\cos\beta)|0\rangle + (\sin\beta)|1\rangle\right)$

$\longrightarrow \Pr[\text{Bob reads "0"}] = (\cos\beta)^2$

$\Pr[\text{Bob reads "1"}] = (\sin\beta)^2$

• $\Pr[\text{Alice reads out "1"}] = 1/2$

$\longrightarrow$ state collapses to $|1\rangle \otimes Rot_\beta|1\rangle$

$\longrightarrow \Pr[\text{Bob reads "1"}] = (\cos\beta)^2$

so $\Pr[\text{Bob reads "0"}] = (\sin\beta)^2$

Check: Overall prob Bob reads out "0" is

$$(1/2)(\cos\beta)^2 + (1/2)(\sin\beta)^2 = \left(\tfrac{1}{2}\right)\left((\cos\beta)^2 + (\sin\beta)^2\right)$$
$$= (1/2)(1) = 1/2. \checkmark$$

So. A & B both see 50-50 random bits.

[But they're not independent, they're] correlated.

$\Pr(A=B) = (1/2)(\cos\beta)^2 + \left(\tfrac{1}{2}\right)(\cos\beta)^2 = (\cos\beta)^2.$ ⟦Prob of getting "v" when measuring in $w, w^\perp$ basis - or vice versa.⟧

Wild: Alice & Bob can affect the amount of correlation from afar, by "turning the steering wheel".

E.g.: $|MEP\rangle$: measuring now would have $\Pr[A=B] = (\cos 0°)^2 = 1$

Bob does $Rot_{30°}$

$\to$ "$|MEP$ rot'd 30° to Bob$\rangle$": measuring both now would have $\Pr[A=B] = (\cos 30°)^2 = 3/4$

Alice does $Rot_{30°}$

$\to$ $|MEP\rangle$ again $\quad (\Rightarrow \Pr[A=B] = 1)$

Alice does $Rot_{45°}$

$\to$ "$|MEP$ rot'd 45° to Al.$\rangle$": measuring both now would give $\Pr[A=B] = (\cos(-45°)^2)$
$= (\cos 45°)^2 = 1/2$.

Bob does $Rot_{-45°}$

$\to$ "$|MEP$ rot'd 90° tow. Al$\rangle$" : measuring both now would
$= \frac{1}{\sqrt{2}}(|1\rangle \otimes |0\rangle - |0\rangle \otimes |1\rangle)$ give $\Pr[A=B] = (\cos(-90°))^2$
$= 0$

etc.

Q: Can Alice, from far away, affect what Bob sees?

A: No! No matter what, Bob just sees 50-50 random bit.

[Alice cannot convey info, though she can affect correlation.]

Vs. classical probabilistic bits

Alice & Bob close eyes, run this code:

```
A := RandBit()
B := A
```

Alice [[eyes still closed]] takes A to Mars
Bob " " " " B " Jupiter.

Now if Alice, Bob "measure" (open eyes), each bit is 50-50, with correlation, $\Pr[A=B]=1$.

Instead... [[before opening eyes, they could do operations on bits]].

Alice: NOT on A. Now $\Pr[A=B]=0$.
Bob : NOT on B. Now $\Pr[A=B]=1$ again.

Alice: If RandInt(1...4) = 1 Then NOT on A.
Now $\Pr[A=B]=75\%$

DIFFERENCE w/ QUANTUM: Now there is nothing Alice & Bob can do (remotely/separately) to make $\Pr[A=B]$ outside the range $25\% \ldots 75\%$.

[[Einstein, Podolsky, Rosen "EPR" didn't like this "ability" of qubits. They believed QM would eventually be replaced by new laws that let you "realize" qubit values before Alice & Bob get far apart, so Nature doesn't have to "remember" that distant qubits A, B are entangled.

Bell ('64) imagined the ~~RY:OG game~~ a more complicated thing which Clauser-Horne-Shimony-Holt (CHSH) simplified to RY:OG game, to prove that actually Nature cannot be like EPR hoped.]]

Claim: By pre-sharing $|\mathrm{MEP}\rangle$, Alice & Bob can win RY:OG game w. prob. $\approx 85\%$.

[[Tsirelson, '80s: you can't beat this 85%]]

How: Alice : sees : does $\mathrm{Rot}_{0°}$ on A

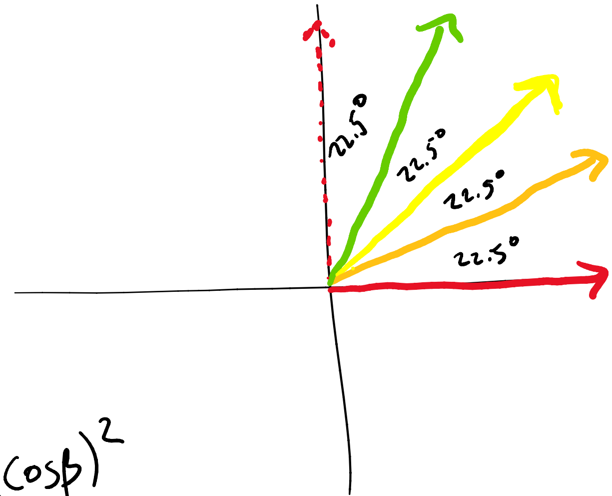
sees : does $\mathrm{Rot}_{45°}$ on A

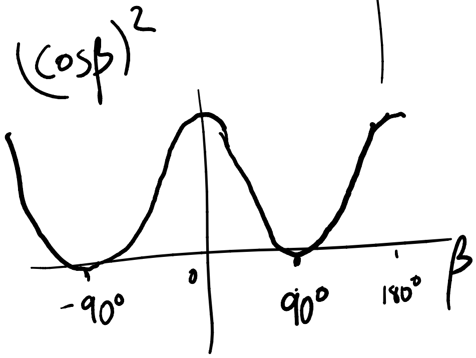
Then measures, sets $a =$ readout.

Bob : sees : does $\mathrm{Rot}_{22.5°}$ on B

sees : does $\mathrm{Rot}_{67.5°}$ on B

Then measures, sets $b =$ readout.





Observe: in 3 cases where Alice & Bob want $a = b$, they create "$|\text{MEP rot'd } \pm 22.5°\rangle$"

$\Rightarrow \Pr[\text{A readout} = \text{B readout}]$

$= (\cos \pm 22.5°)^2 \approx 85°$

$> 75°$

[[precisely: $\frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{2}}$]]

And, in one case (Red/Green) where they want $a \neq b$, they have "$|\text{MEP rot'd by } 67.5° \text{ (tow. Bob)}\rangle$",

so $\Pr[A = B] = \cos(67.5°)^2 = \sin(22.5°)^2$

$\Rightarrow \Pr[A \neq B] \qquad = \cos(22.5°)^2 \approx 85°$ again.

[[So in all 4 cases they win w. prob. $\approx 85°$.

Not possible if the world has only prob. bits, or otherwise reifiable, "local hidden variables".]]

[History: CHSH game described in '69.

Experiments done, mostly showing win rate $> 75\%$, near 85%, by '72, including by Freedman & Clauser. But not super-careful/clear. In particular, "Alice" & "Bob" acted more slowly than the time for light to travel between them.

Aspect-Dalibard-Roger '82 finally did it with A&B 12m ($= 40$ light-nanosecs) apart, with actions in < 20 nanosecs.

just got Nobel Prize

Win prob. was $\approx 84\%$!

Technically still some loopholes; e.g. did not flip RY/OG coins, but instead looped coin results in "quasiperiodic" way. Aspect did some more better ones, as did many others. Culmination: super-careful version in Delft, 2015, Hanson et al.... diamonds, 1.3km apart.

Zeilberg's Nobel was for more entanglement theory, plus "quantum teleportation" experiments.

see next quiz!!]