Language Technologies Institute

Carnegie Mellon University

Multimodal Machine Learning

Lecture 7.1: Multimodal Interaction

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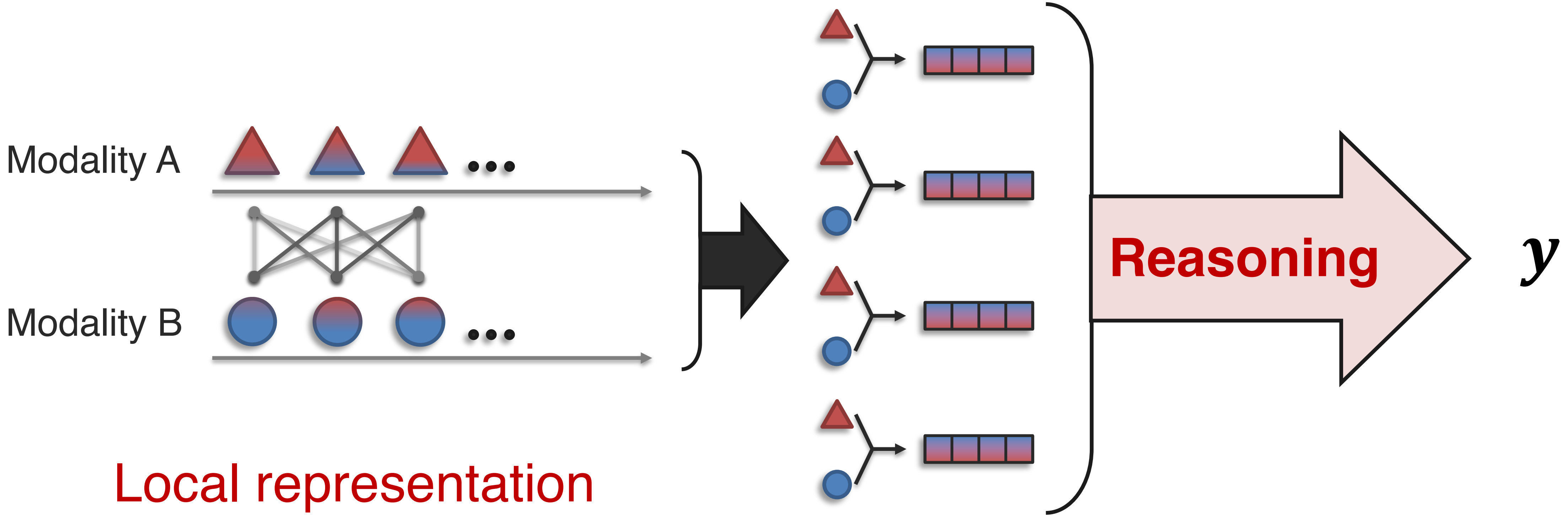
Reasoning

Definition: Combining knowledge, usually through multiple inferential steps, exploiting multimodal alignment and problem structure.



Local representation

+ Aligned representation

The Challenge of Compositionality

Definition: Combining knowledge, usually through multiple inferential steps, exploiting multimodal alignment and problem structure.

(a) some plants surrounding a lightbulb

(b) a lightbulb surrounding some plants

CLIP, ViLT, ViLBERT, etc.
All random chance

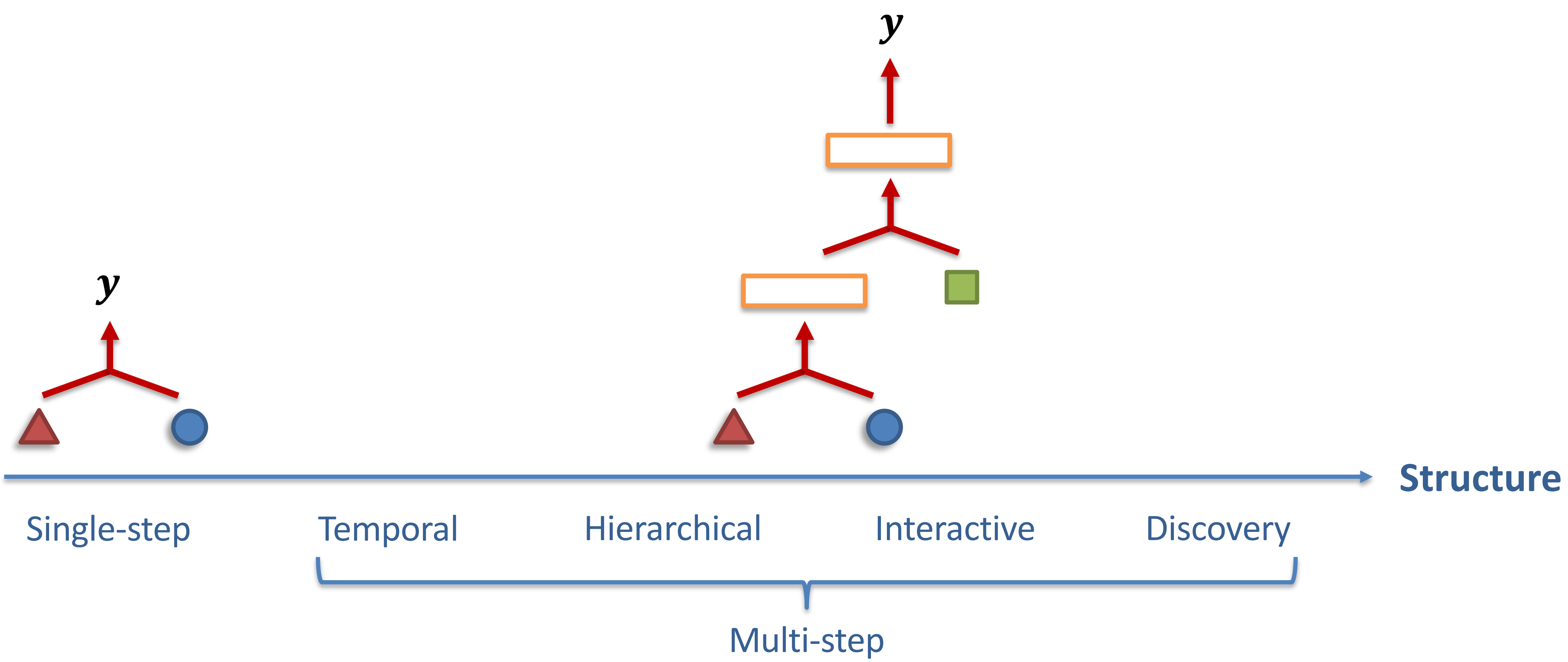
Compositional Generalization
to novel combinations outside of training data

1. Structure: <subject> <verb> <object>
2. Concepts: ‘plants’, ‘lightbulb’
3. Inference: ‘surrounding’ – spatial relation
4. Knowledge: from humans!

[Thrush et al., Winoground: Probing Vision and Language Models for Visio-Linguistic Compositionality. CVPR 2022]

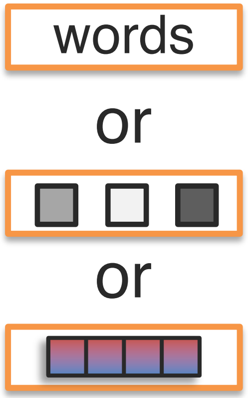
Sub-Challenge 3a: Structure Modeling

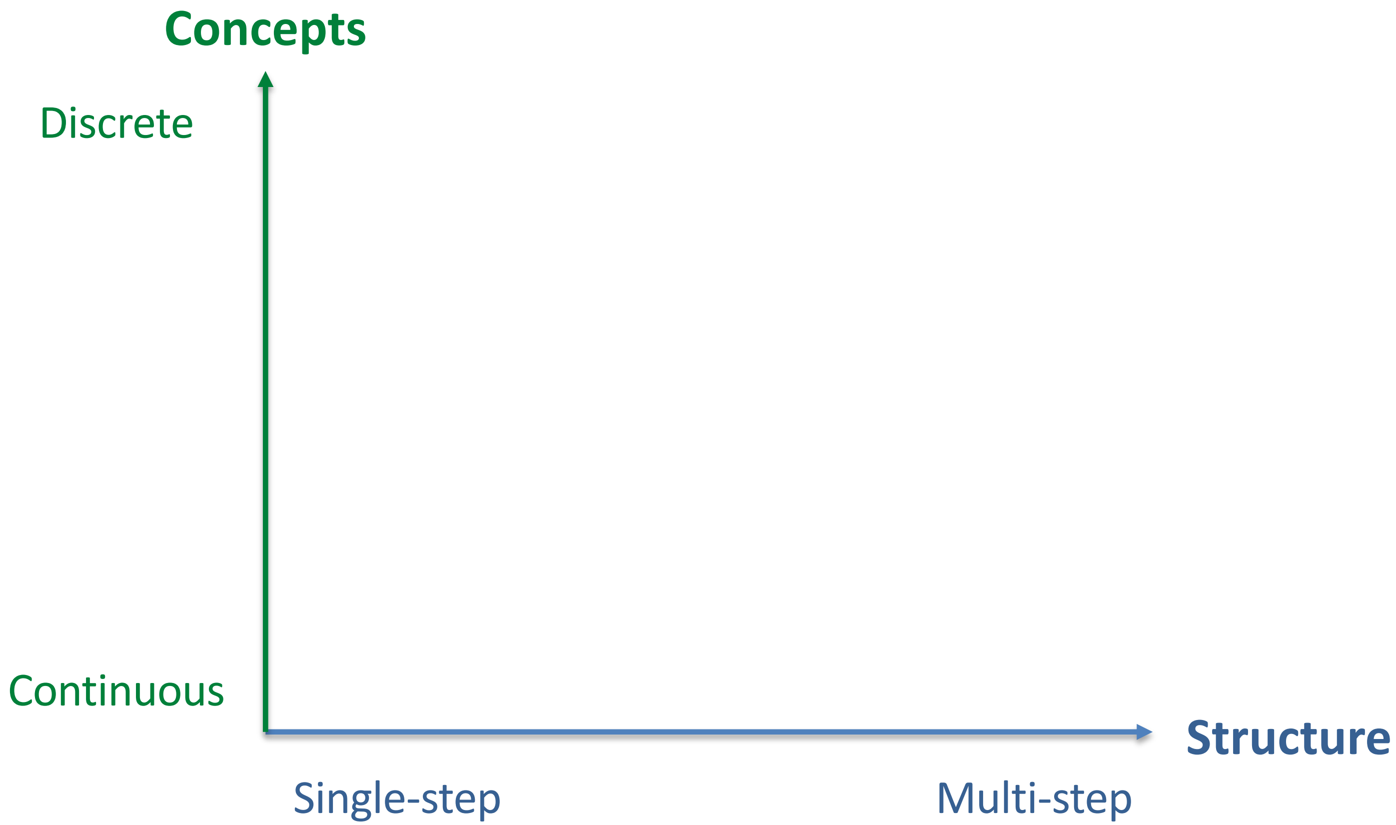
Definition: Defining or learning the relationships over which reasoning occurs.



Sub-Challenge 3b: Intermediate Concepts

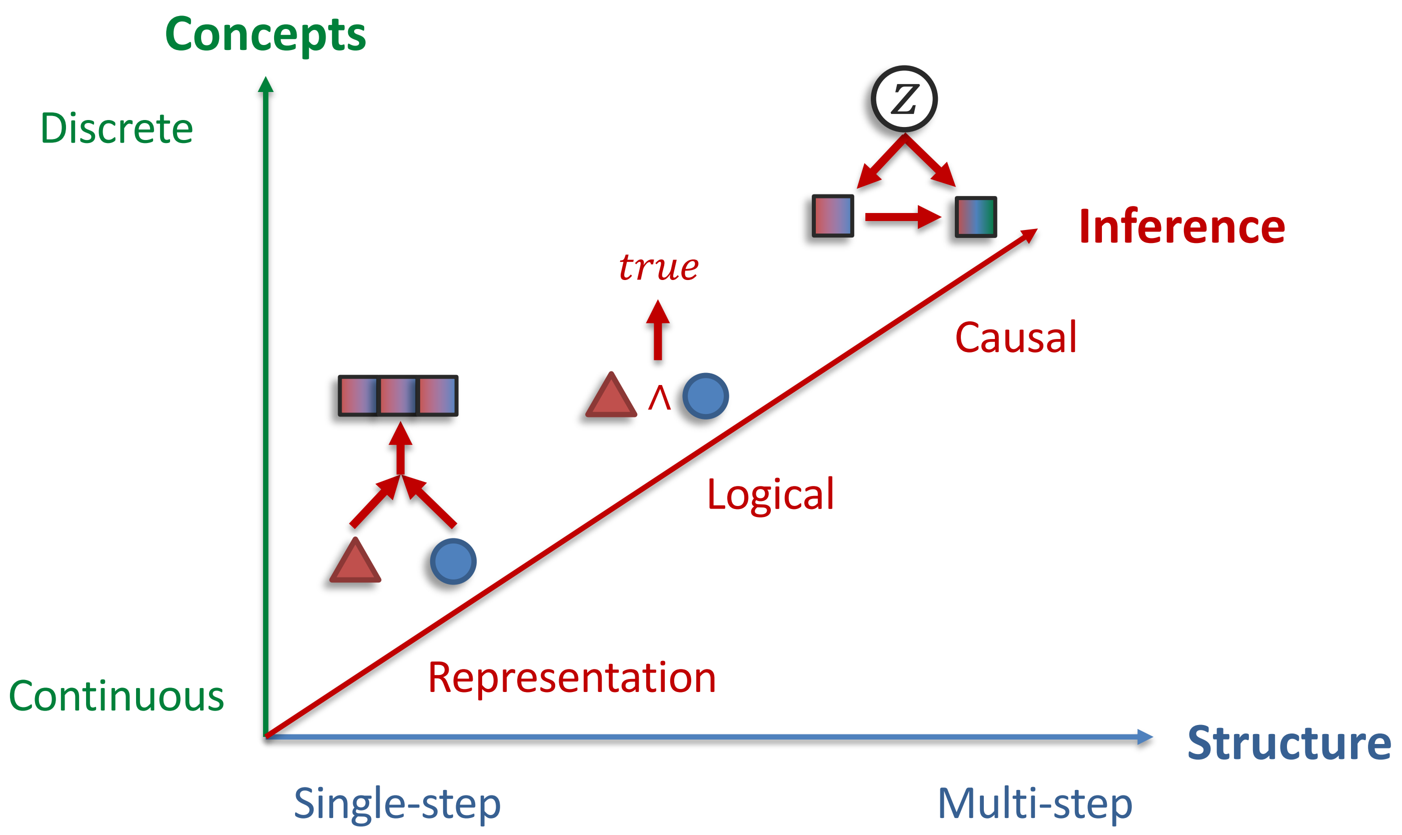
Definition: The parameterization of individual multimodal concepts in the reasoning process.





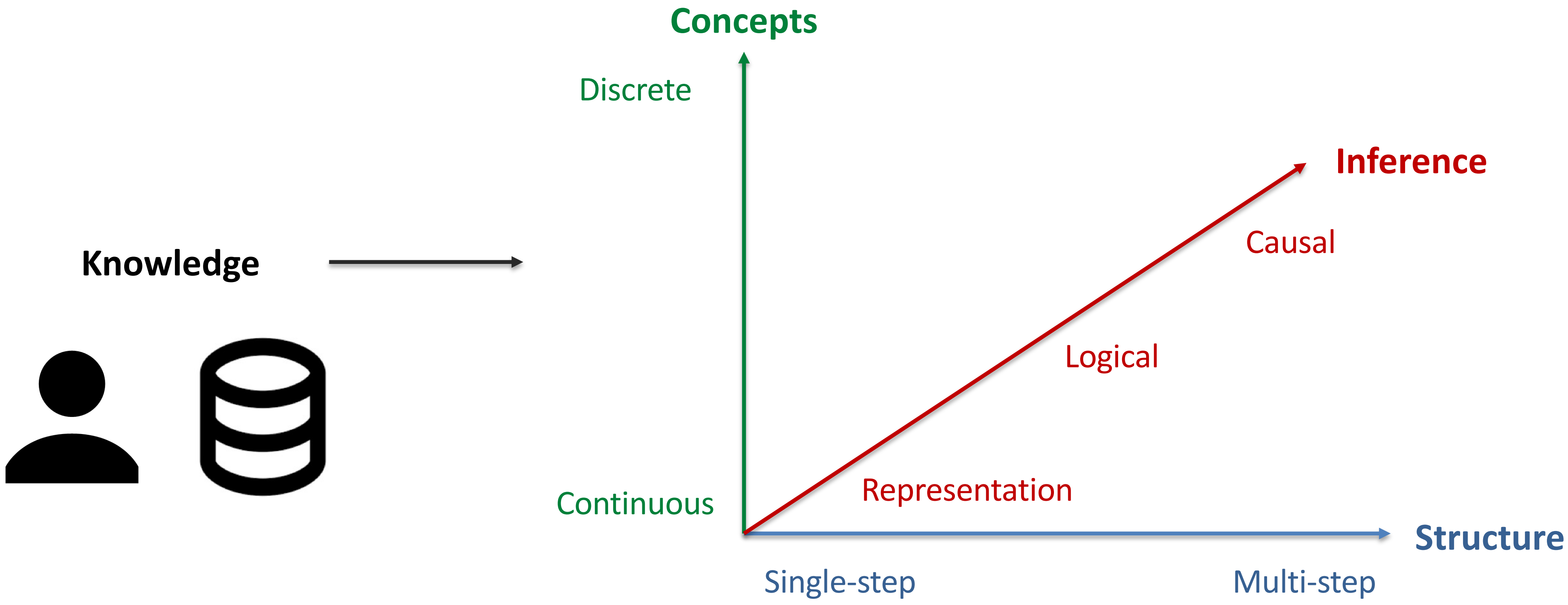
Sub-Challenge 3c: Inference Paradigm

Definition: How increasingly abstract concepts are inferred from individual multimodal evidences.



Sub-Challenge 3d: External Knowledge

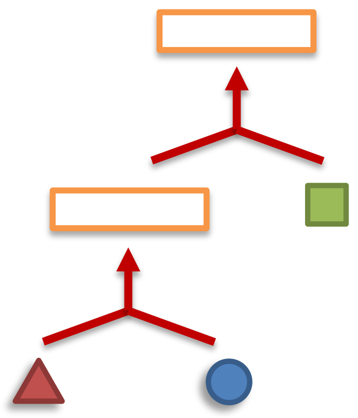
Definition: Leveraging external knowledge in the study of structure, concepts, and inference.



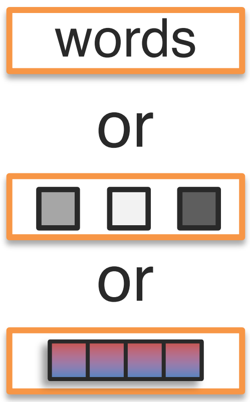
Reasoning

Definition: Combining knowledge, usually through multiple inferential steps, exploiting multimodal alignment and problem structure.

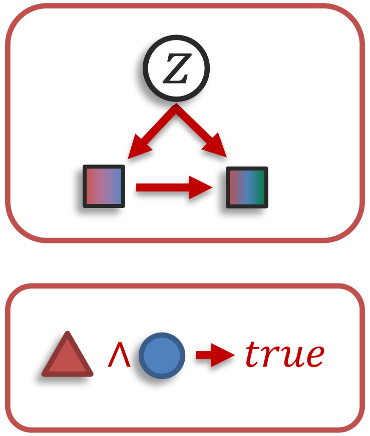
A Structure modeling



B Intermediate concepts



C Inference paradigm



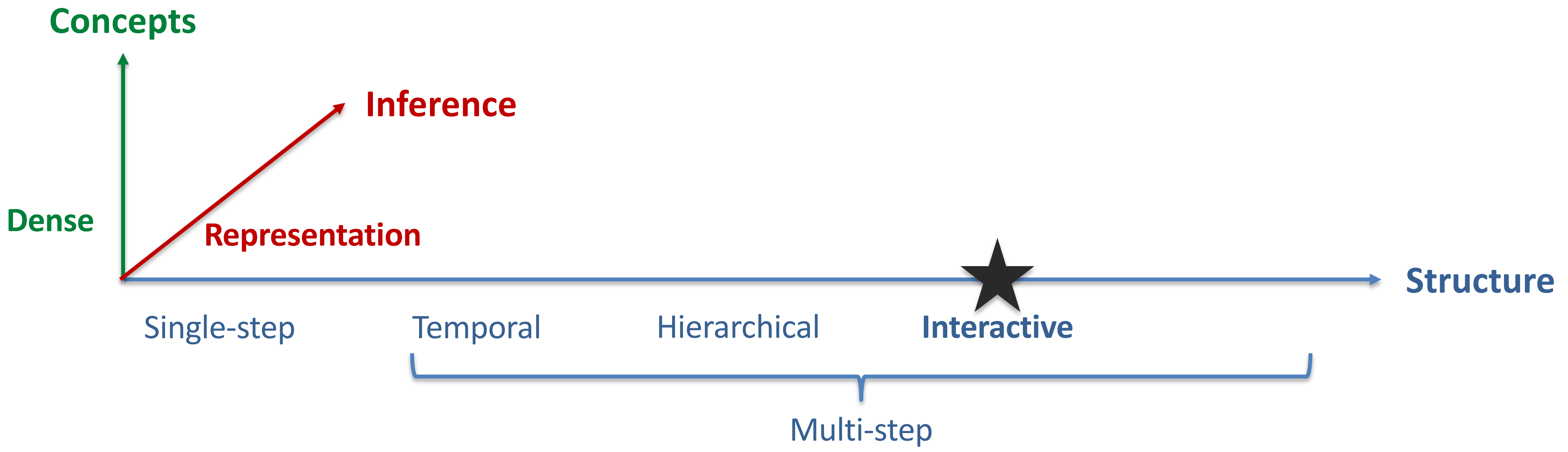
D External knowledge

Roadmap

Definition: Combining knowledge, usually through multiple inferential steps, exploiting multimodal alignment and problem structure.

	(A) Structure modeling	(B) Intermediate concepts	(C) Inference paradigm	(D) External knowledge
Last Week	Temporal Hierarchical	Continuous		
Today	Interactive			
Thursday	Discovery	Discrete	Causal Logical	Knowledge Commonsense

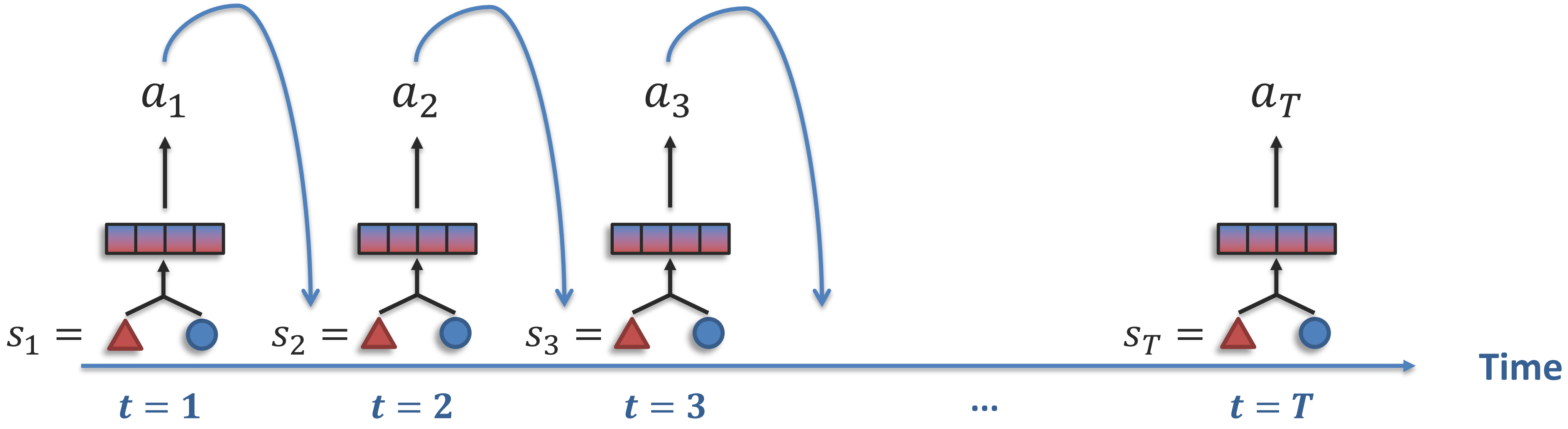
Sub-Challenge 3a: Structure Modeling



Interactive Structure

Structure defined through interactive environment
Main difference from temporal - actions taken at previous time steps affect future states

Integrates multimodality into the reinforcement learning framework



[Luketina et al., A Survey of Reinforcement Learning Informed by Natural Language. IJCAI 2019]

Interactive Structure

Structure defined through interactive environment
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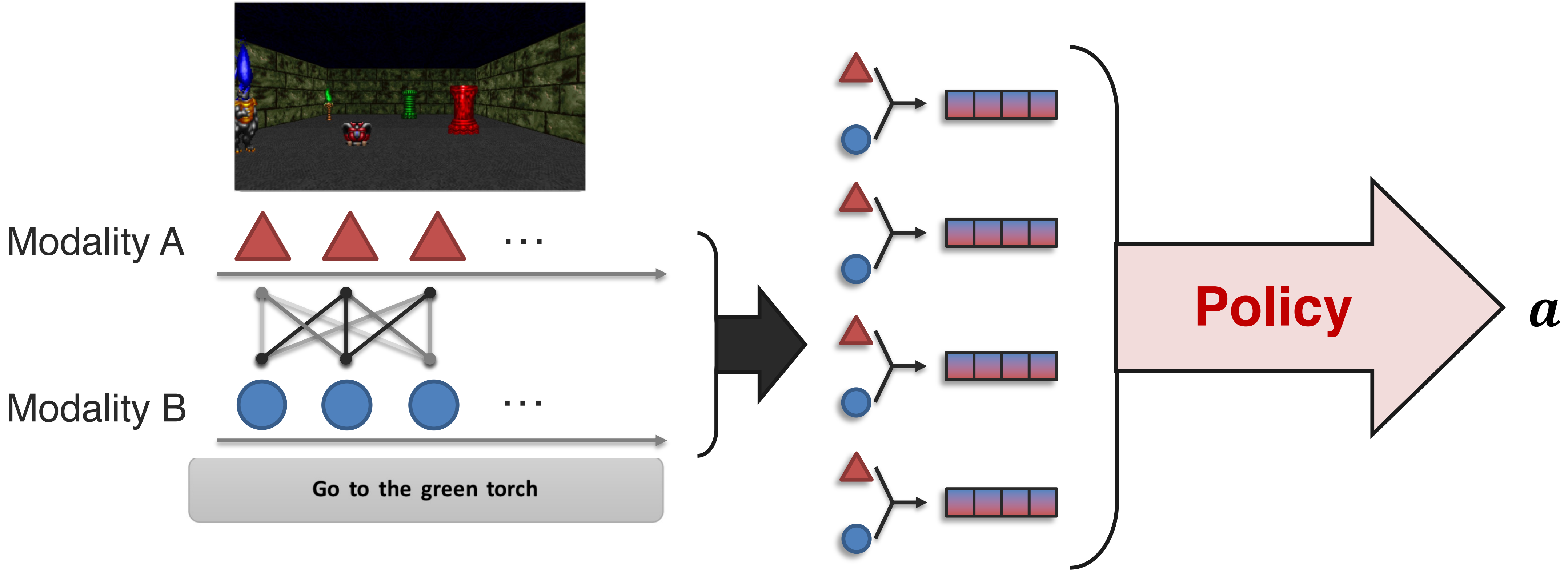


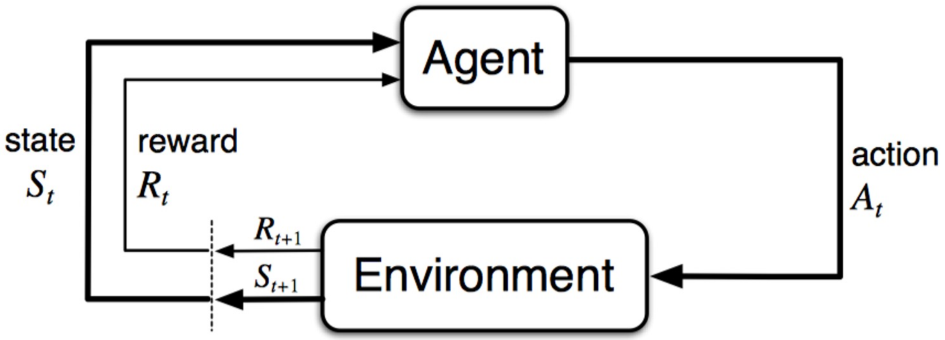
Local representation
+ Aligned representation

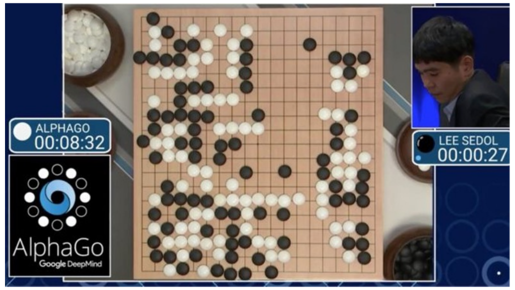
Reasoning

Learning a Policy – RL basics

Reinforcement learning

- Introduction to RL
- Markov Decision Processes (MDPs)
- Solving known MDPs using value and policy iteration
- Solving unknown MDPs using function approximation and Q-learning

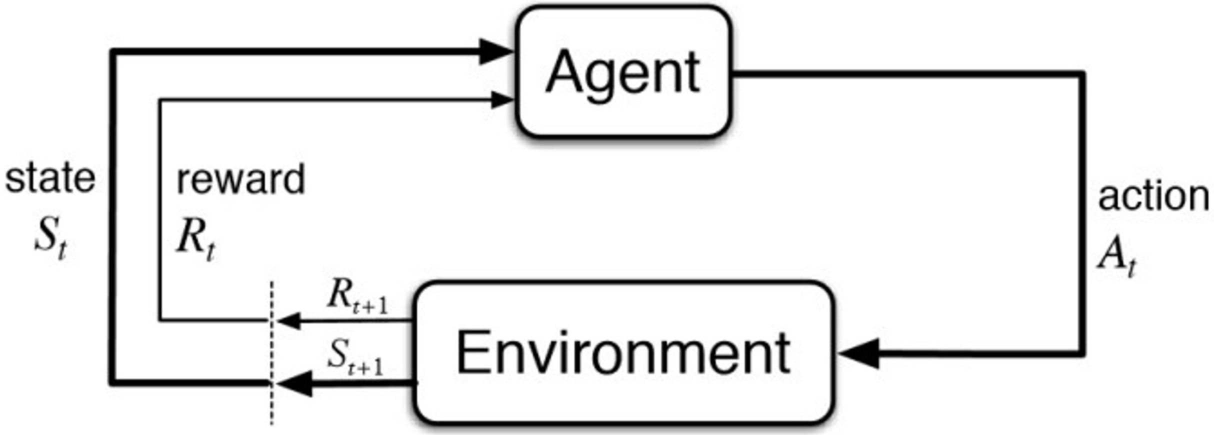




Learning a Policy – RL basics

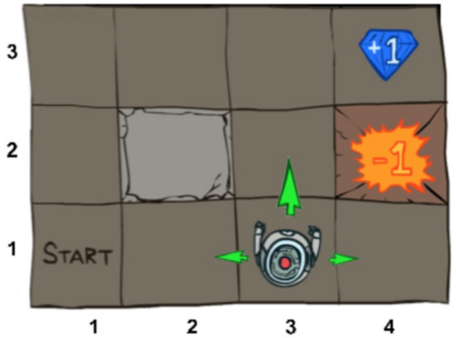
An MDP is defined by:

- Set of states S.
- Set of actions A.
- Transition function $P(s'|s,a)$.
- Reward function $r(s,a,s')$.
- Start state s_0.
- Discount factor γ.
- Horizon H.

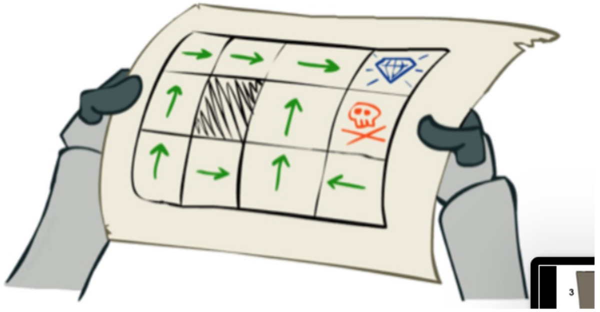


Return:

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$



π:



Policy: $\pi(a|s) = \Pr(A_t = a | S_t = s) \quad \forall t$

Goal: $\arg\max_{\pi} \mathbb{E}\left[\sum_{t=0}^{H} \gamma^t R_t | \pi\right]$

RL vs Supervised Learning

Reinforcement Learning

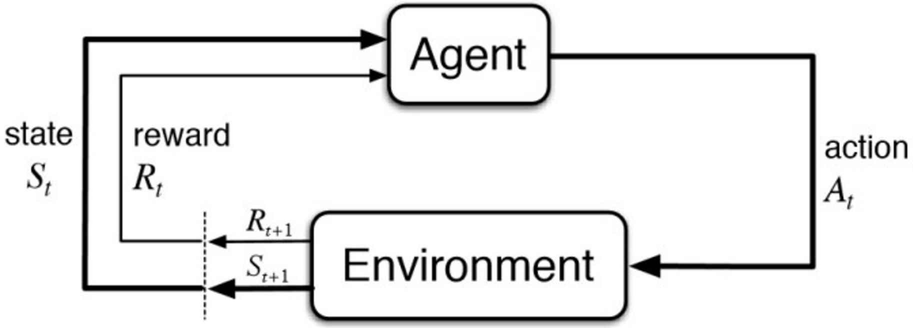
- Sequential decision making
- Maximize cumulative reward
- Sparse rewards
- Environment maybe unknown

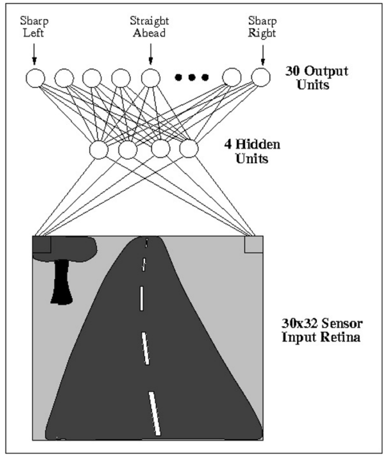
Supervised Learning

- One-step decision making
- Maximize immediate reward
- Dense supervision
- Environment always known

Intersection Between RL and Supervised Learning

Imitation learning





Obtain expert trajectories (e.g. human driver/video demonstrations):

$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \dots$$

Perform supervised learning by predicting expert action

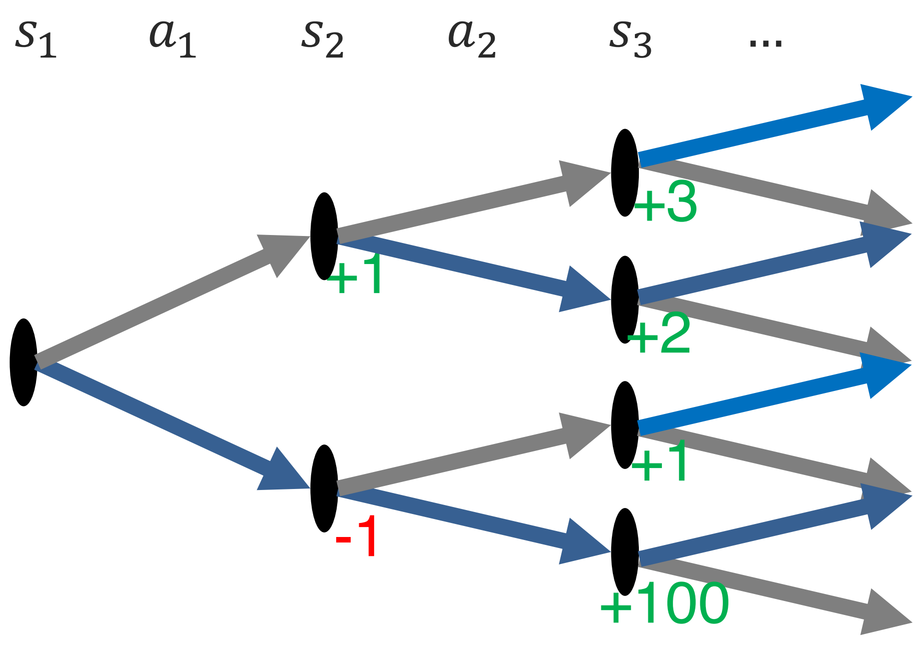
$$D = \{(s_0, a_0^*), (s_1, a_1^*), (s_2, a_2^*), \dots\}$$

But: distribution mismatch between training and testing

Hard to recover from sub-optimal states

Sometimes not safe/possible to collect expert trajectories

RL as Exploring a Tree



π which action to take from each s

$$V^\pi(s) = \mathbb{E}_\pi \left[G_t | S_t = s \right] \qquad V^*(s) = \max_\pi V^\pi(s)$$

State-value function: how much total reward should I expect following π from s?

$V^\pi(s_1) = 99$ $\qquad$ $V^\pi(s_1) = 99$

$$Q^\pi(s,a) = \mathbb{E}_\pi \left[G_t | S_t = s, A_t = a \right] \quad Q^*(s,a) = \max_\pi Q^\pi(s,a)$$

Action-value function: how much total reward should I expect taking a, then following π, from s?

$Q^\pi(s_1, up) = 3$ $\qquad$ $Q^*(s_1, up) = 4$

Relationships Between State and Action Values

State value functions | **Action value functions**

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\boxed{V^{\pi}(s)} \arrow[d, "{V^*(s) = \max_{\pi} V^{\pi}(s)}"'] & \boxed{Q^{\pi}(s,a)} \arrow[l, "{V^{\pi}(s) = \sum_a \pi(a|s) Q^{\pi}(s,a)}"'] \arrow[d, "{Q^*(s,a) = \max_{\pi} Q^{\pi}(s,a)}"] \\
\boxed{V^*(s)} & \boxed{Q^*(s,a)} \arrow[l, "{V^*(s) = \max_a Q^*(s,a)}"]
\end{tikzcd}
$$

Value-based Methods

- Value Based
  - Learned Value Function
  - Implicit policy (e.g. ε-greedy)

State value functions	Action value functions
$V^{\pi}(s)$	$Q^{\pi}(s,a)$
$V^{*}(s)$	$Q^{*}(s,a)$

Optimal policy can be found by maximizing over Q*(s,a)

$$\pi^*(a|s) = \begin{cases} 1-\epsilon, & \text{if } a = \arg\max_a \; Q^*(s,a) \\ \epsilon, & \text{else} \end{cases}$$

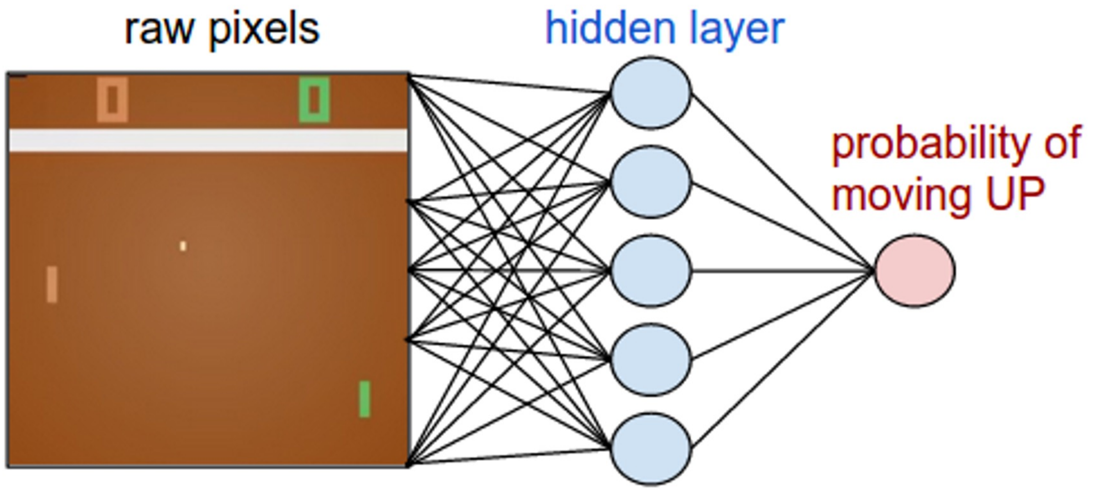
Optimal policy can also be found by maximizing over V*(s')
with **one-step look ahead**

$$\pi^*(a|s) = \begin{cases} 1-\epsilon, & \text{if } a = \arg\max_a \mathbb{E}_{s'}\left[r(s,a,s') + \gamma V^*(s')\right] \\ \epsilon, & \text{else} \end{cases}$$

Policy-based Methods

- Policy Based
  - No Value Function
  - Learned Policy

$$\pi_\theta(s, a) = \mathbb{P}\left[a \mid s, \theta\right]$$



- Often π can be simpler than Q or V
  - E.g., robotic grasp

Q(s,a) and V(s) very high-dimensional
But policy could be just 'open/close hand'

- V: doesn't prescribe actions
  - Would need dynamics model (+ compute 1 Bellman back-up)
- Q: need to be able to efficiently solve $\arg\max_a \; Q^*(s, a)$
  - Challenge for continuous / high-dimensional action spaces

Value-based vs Policy-based

$$Q^*(s,a)$$

$$\pi^*(a|s) = \begin{cases} 1-\epsilon, & \text{if } a = \arg\max_a \; Q^*(s,a) \\ \epsilon, & \text{else} \end{cases}$$

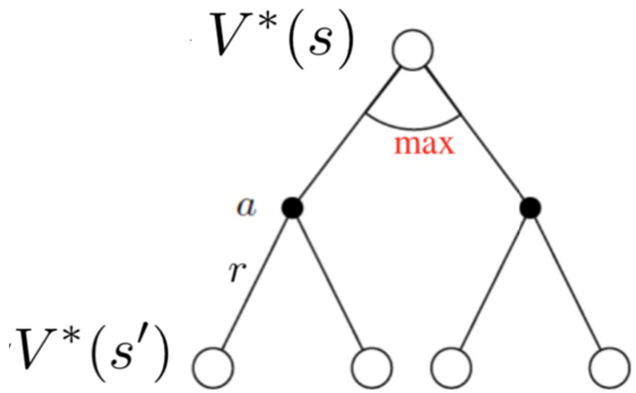
$$\pi_\theta(s,a) = \mathbb{P}\left[a \mid s, \theta\right]$$

Value-based
- More sample efficient, respects MDP structure
- Easier to add human knowledge about states and actions
- More complex algorithm
- Can't handle continuous argmax, harder to understand, sometimes values are more complex than policies

Policy-based
- Less sample efficient, more akin to trial-and-error
- Harder to add human knowledge
- Simpler algorithm
- Directly learns policy, can be more interpretable

Policy-based RL in 15 minutes

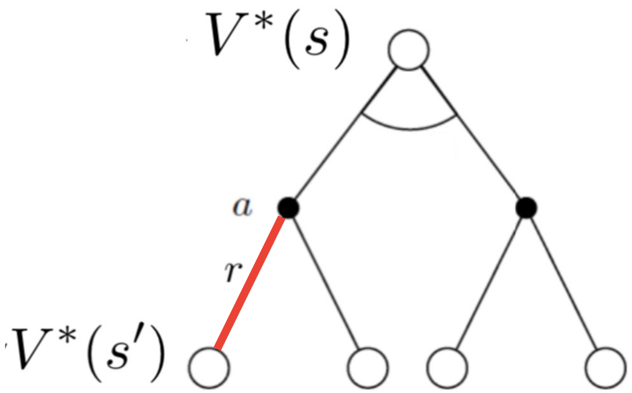
Recursive definition



$$V^*(s) = \max_a Q^*(s,a)$$

Bellman Optimality for State Value Functions

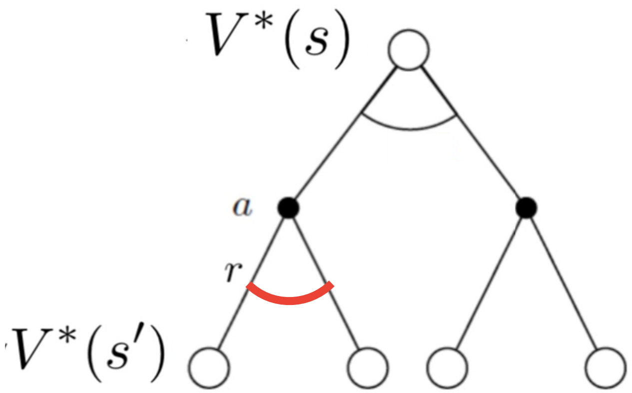
Recursive definition



$$
\begin{aligned}
V^*(s) &= \max_a Q^*(s,a) \\
&= \max_a \mathbb{E}_{s'}\left[r(s,a,s') + \gamma V^*(s')\right]
\end{aligned}
$$

Bellman Optimality for State Value Functions

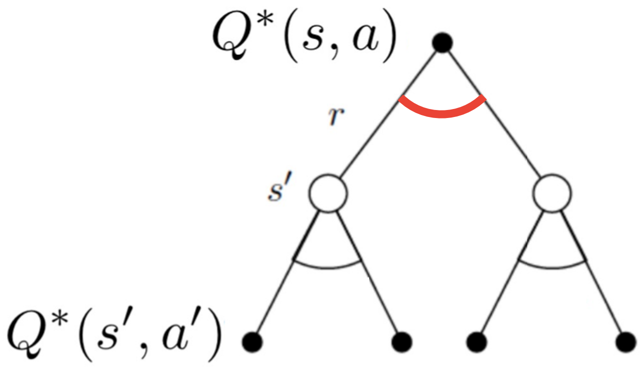
Recursive definition



$$
\begin{aligned}
V^*(s) &= \max_a Q^*(s,a) \\
&= \max_a \mathbb{E}_{s'}\left[r(s,a,s') + \gamma V^*(s')\right] \\
&= \max_a \left[\sum_{s'} p(s'|s,a)(r(s,a,s') + \gamma V^*(s'))\right]
\end{aligned}
$$

Bellman Optimality for Action Value Functions

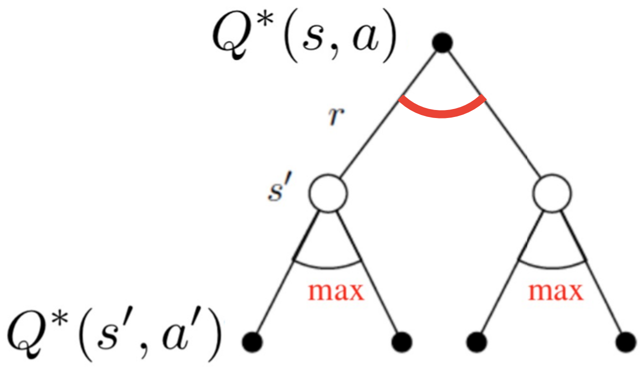
Recursive definition



$$Q^*(s,a) = \mathbb{E}_{s'}\left[r(s,a,s') + \gamma V^*(s')\right]$$

Bellman Optimality for Action Value Functions

Recursive definition



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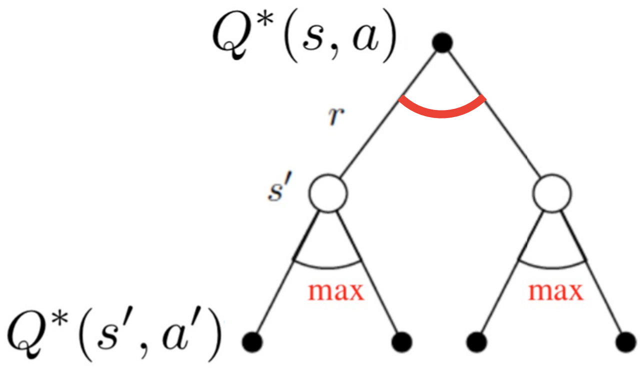
Bellman Optimality for Action Value Functions

Recursive definition



$$
\begin{aligned}
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&= \mathbb{E}_{s'}\left[r(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right] \\
&= \sum_{s'} p(s'|s,a)\left(r(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right)
\end{aligned}
$$

Solving the Bellman Optimality Equations

Recursive definition

$$V^*(s) = \max_a \left[\sum_{s'} p(s'|s,a)(r(s,a,s') + \gamma V^*(s')) \right]$$

Solve by iterative methods

$$V^*_{[k+1]}(s) = \max_a \left[\sum_{s'} p(s'|s,a)(r(s,a,s') + \gamma V^*_{[k]}(s')) \right]$$

[Slides from Fragkiadaki, 10-703 CMU]

Value Iteration

Algorithm:

Start with $V_0^*(s) = 0$ for all s.

For k = 1, ... , H:

For all states s in S:

$$V_k^*(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)\left(R(s,a,s') + \gamma V_{k-1}^*(s')\right)$$

[Slides from Fragkiadaki, 10-703 CMU]

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For k = 1, … , H:

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$$V_k^*(s) \leftarrow \max_a \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s')\right)$$

$$\pi_k^*(s) \leftarrow \arg\max_a \sum_{s'} P(s'|s,a) \left(R(s,a,s') + \gamma V_{k-1}^*(s')\right)$$

Find the best action according to one-step look ahead

This is called a value update or Bellman update/back-up

Repeat until policy converges. Guaranteed to converge to optimal policy.

[Slides from Fragkiadaki, 10-703 CMU]

Q-Value Iteration

$Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

Bellman Equation:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$

Q-Value Iteration:

$$Q^*_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*_k(s',a'))$$

[Slides from Fragkiadaki, 10-703 CMU]

Summary: Exact Methods

Fully known MDP
states
transitions
rewards

Bellman optimality equations

$Q^*(s, a)$ — Q-value iteration

$V^*(s)$ — Value iteration

Bellman expectation equations

$Q^\pi(s, a)$ — Q-policy iteration

$V^\pi(s)$ — Policy iteration

Repeat until policy converges. Guaranteed to converge to optimal policy.

Limitations:
Iterate over and storage for all states and actions: requires small, discrete state and action space
Update equations require fully observable MDP and known transitions

Unknown MDPs?

$Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

Bellman Equation:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$

Q-Value Iteration:

$$Q^*_{k+1}(s,a) \leftarrow \boxed{\sum_{s'} P(s'|s,a)}(R(s,a,s') + \gamma \max_{a'} Q^*_k(s',a'))$$

This is problematic when do not know the transitions

[Slides from Fragkiadaki, 10-703 CMU]

Tabular Q-learning

- Q-value iteration: $Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$
- Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$

[Slides from Fragkiadaki, 10-703 CMU]

Tabular Q-learning

- Q-value iteration: $Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$
- Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')\right]$
- (Tabular) Q-Learning: replace expectation by samples
  - For an state-action pair (s,a), receive: $s' \sim P(s'|s,a)$ simulation and exploration
  - Consider your old estimate: $Q_k(s,a)$
  - Consider your new sample estimate:

$$\text{target}(s') = r(s,a,s') + \gamma \max_{a'} Q_k(s',a')$$

$$\text{error}(s') = \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a)\right)$$

[Slides from Fragkiadaki, 10-703 CMU]

Tabular Q-learning

learning
rate

$$Q_{k+1}(s,a) = Q_k(s,a) + \alpha \; \text{error}(s')$$
$$= Q_k(s,a) + \alpha \left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a) \right)$$

Key idea: implicitly estimate the transitions via simulation

Tabular Q-learning

Bellman optimality

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r(s,a,s') + \gamma \max_{a'} Q^*(s',a')\right]$$

Algorithm:

Start with $Q_0(s,a)$ for all s, a.

Get initial state s

For k = 1, 2, … till convergence

Sample action a, get next state s'

If s' is terminal:

$$\text{target} = r(s,a,s')$$

Sample new initial state s'

else:

$$\text{target} = r(s,a,s') + \gamma \max_{a'} Q_k(s',a')$$

$$Q_{k+1}(s,a) = Q_k(s,a) + \alpha\left(r(s,a,s') + \gamma \max_{a'} Q_k(s',a') - Q_k(s,a)\right)$$

$$s \leftarrow s'$$

[Slides from Fragkiadaki, 10-703 CMU]

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$$Q_{k+1}(s, a) = Q_k(s, a) + \alpha \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$$

$$s \leftarrow s'$$

- Choose random actions?
- Choose action that maximizes $Q_k(s, a)$ (i.e. greedily)?
- ε-Greedy: choose random action with prob. ε, otherwise choose action greedily

[Slides from Fragkiadaki, 10-703 CMU]

Exploration and Exploitation

Poor estimates of Q(s,a) at the start:

Bad initial estimates in the first few cases can drive policy into sub-optimal region, and never explore further.

$$\pi(s) = \begin{cases} \max_a \hat{Q}(s,a) & \text{with probability } 1-\epsilon \\ \text{random action} & \text{otherwise} \end{cases}$$

Gradually decrease epsilon as policy is learned.

[Slides from Fragkiadaki, 10-703 CMU]

Tabular Q-learning

Algorithm:

Start with $Q_0(s, a)$ for all s, a.

Get initial state s

For k = 1, 2, … till convergence

$\boxed{\text{Sample action a, get next state s'}}$

If s' is terminal:

$$\text{target} = r(s, a, s')$$

Sample new initial state s'

else:

$$\text{target} = r(s, a, s') + \gamma \max_{a'} Q_k(s', a')$$

$$Q_{k+1}(s, a) = Q_k(s, a) + \alpha \left(r(s, a, s') + \gamma \max_{a'} Q_k(s', a') - Q_k(s, a) \right)$$

$$s \leftarrow s'$$

Tabular: keep a |S| x |A| table of Q(s,a)

Still requires small and discrete state and action space

How can we generalize to unseen states?

- ε-Greedy: choose random action with prob. ε, otherwise choose action greedily

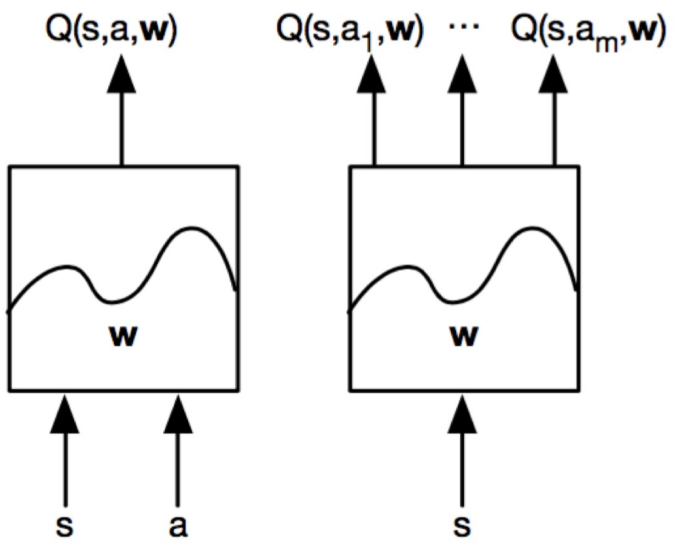
[Slides from Fragkiadaki, 10-703 CMU]

Deep Q-learning

Q-learning with function approximation to **extract informative features** from **high-dimensional** input states.

Represent value function by Q network with weights **w**

$$Q(s, a, \mathbf{w}) \approx Q^*(s, a)$$



+ high-dimensional, continuous states
+ generalization to new states

[Slides from Fragkiadaki, 10-703 CMU]

Deep Q-learning

- Optimal Q-values should obey Bellman equation

$$Q^*(s,a) = \mathbb{E}_{s'}\left[r + \gamma \max_{a'} Q(s',a')^* \mid s,a\right]$$

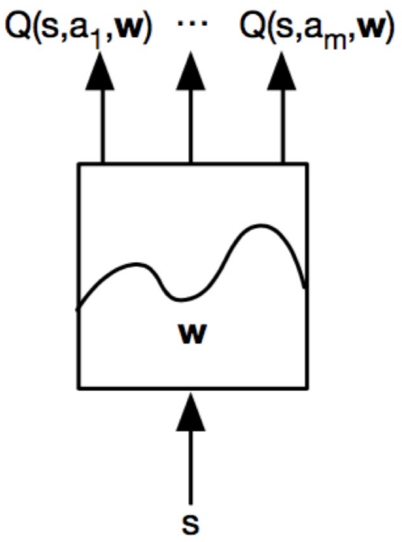
- Treat right-hand $r + \gamma \max_{a'} Q(s', a', \mathbf{w})$ as as a target
- Minimize MSE loss by stochastic gradient descent

$$l = \left(r + \gamma \max_{a} Q(s', a', \mathbf{w}) - Q(s, a, \mathbf{w})\right)^2$$

Deep Q-learning Challenges

- Minimize MSE loss by stochastic gradient descent

$$l = \left(r + \gamma \max_{a} Q(s', a', \mathbf{w}) - Q(s, a, \mathbf{w})\right)^2$$



- Converges to Q* using table lookup representation
- But diverges using neural networks due to:
  - Correlations between samples
  - Non-stationary targets

Deep Q-learning: Experience Replay

- To remove correlations, build data-set from agent's own experience

s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\ldots$
$s_t, a_t, r_{t+1}, s_{t+1}$

$\rightarrow$ s, a, r, s'

exploration, epsilon greedy is important!

- Sample random mini-batch of transitions (s,a,r,s') from **D**

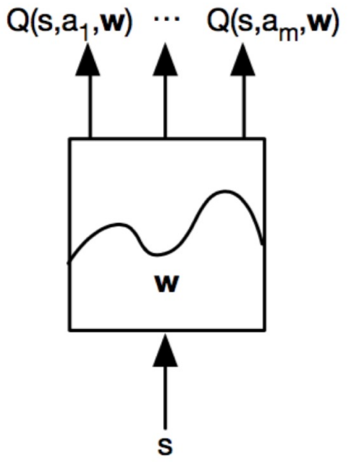
Deep Q-learning: Fixed Q-targets

- Sample random mini-batch of transitions (s,a,r,s') from D
- Compute Q-learning targets w.r.t. old fixed parameters **w-**

- Optimize MSE between Q-network and Q-learning targets

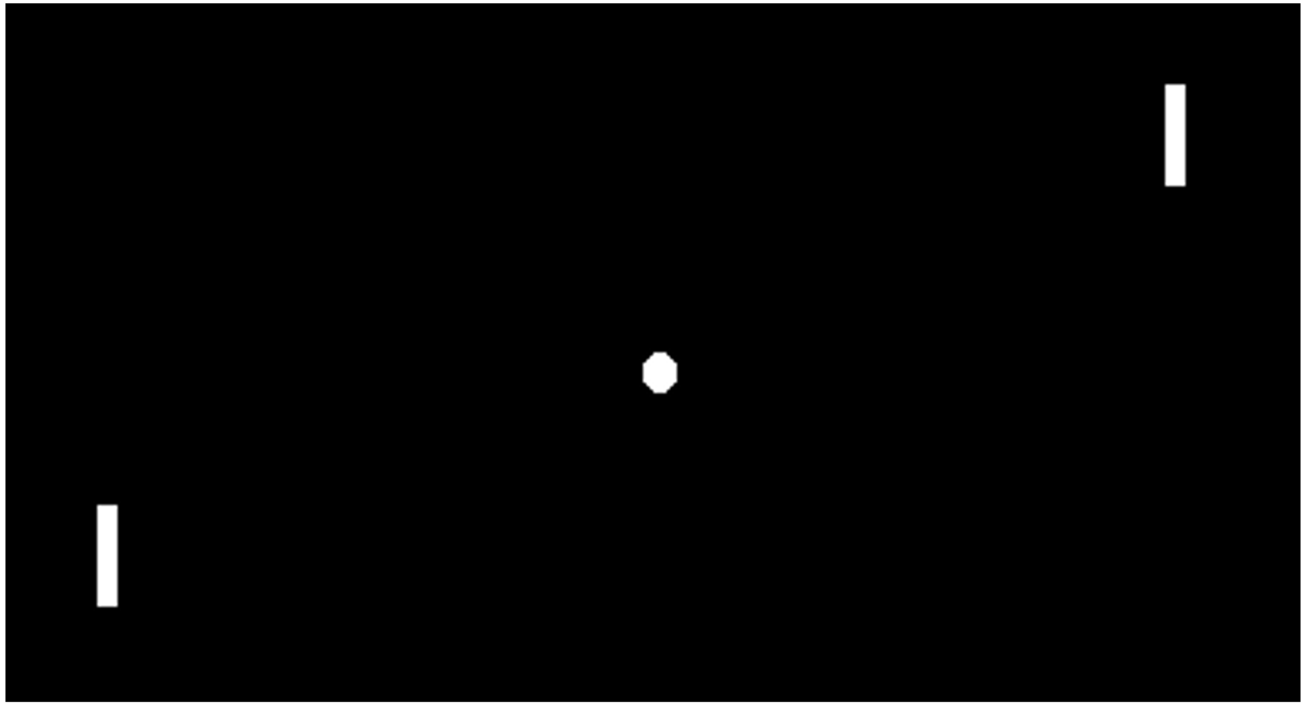
s_1, a_1, r_2, s_2
s_2, a_2, r_3, s_3
s_3, a_3, r_4, s_4
$\ldots$
$s_t, a_t, r_{t+1}, s_{t+1}$

$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}_i}\left[\left(\underbrace{r + \gamma \max_{a'} Q(s', a'; w_i^-)}_{\text{Q-learning target}} - \underbrace{Q(s, a; w_i)}_{\text{Q-network}}\right)^2\right]$$



- Use stochastic gradient descent
- Update **w-** with updated **w** every ~1000 iterations

Policy-based RL in 15 minutes

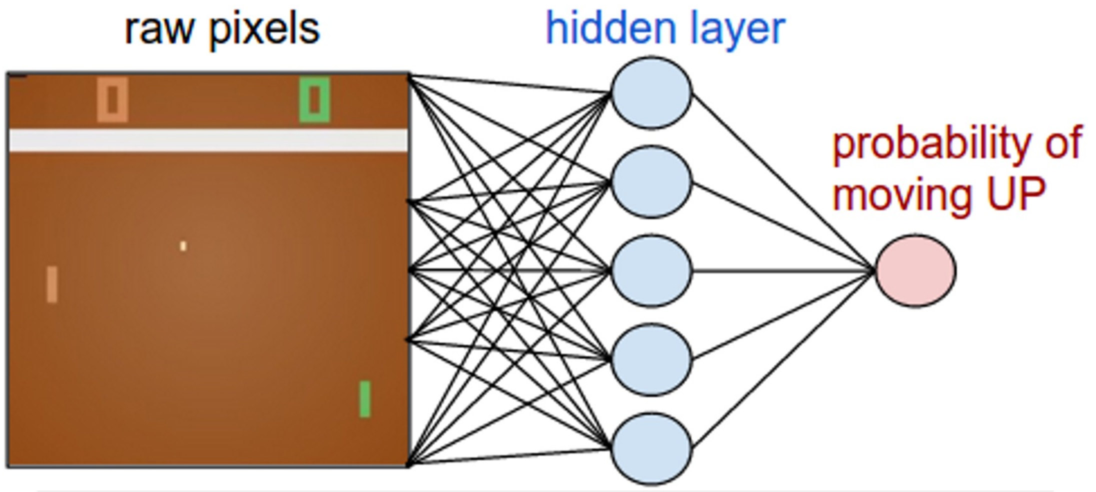


Pong from Pixels

e.g.,

height width

[80 x 80]
array of



Network sees +1 if it scored a point, and -1 if it was scored against.
How do we learn these parameters?

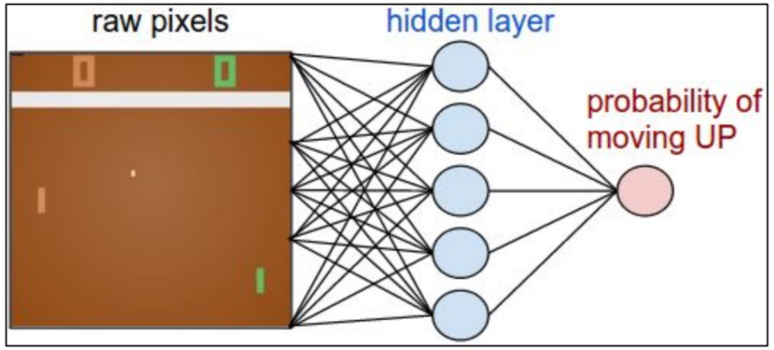
Pong from Pixels

Suppose we had the training labels…
(we know what to do in any state)

(x1,UP)
(x2,DOWN)
(x3,UP)
...

maximize:

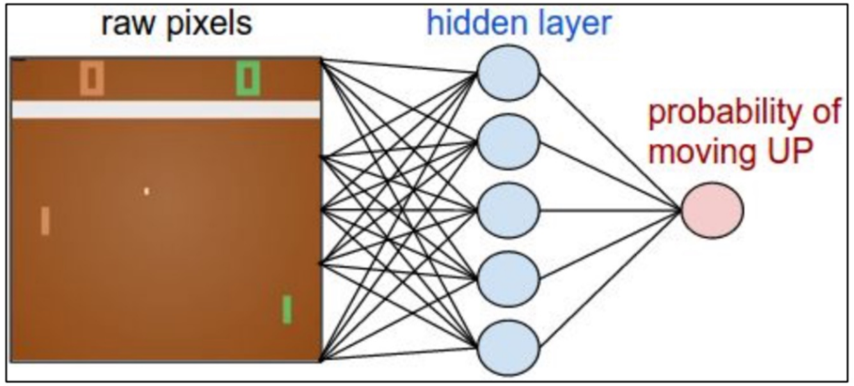
$$\sum_i \log p(y_i | x_i)$$



[Slides from Karpathy]

Pong from Pixels

Except, we don't have labels...

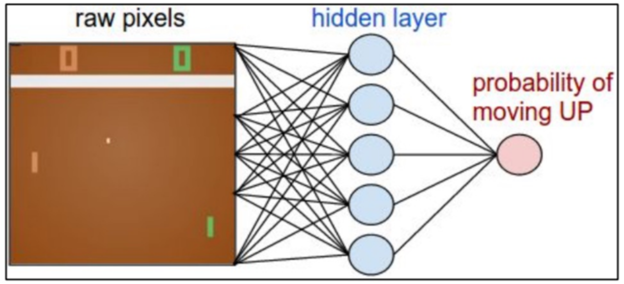


Should we go UP or DOWN?

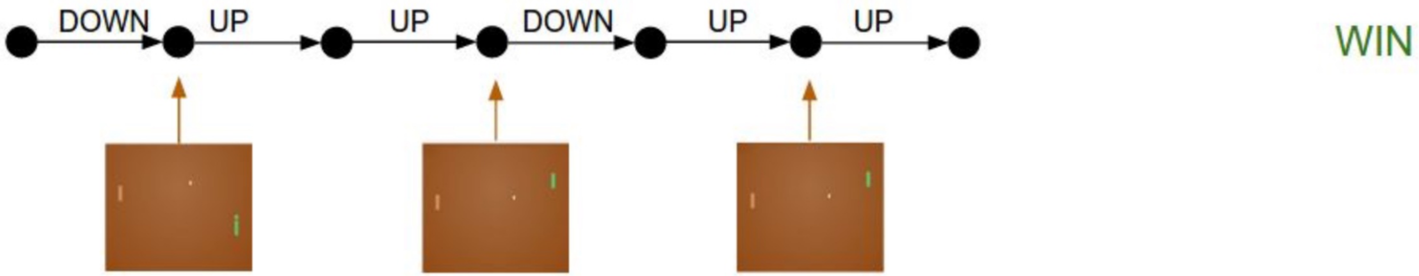
[Slides from Karpathy]

Pong from Pixels

Let’s just act according to our current policy...



Rollout the policy and collect an episode

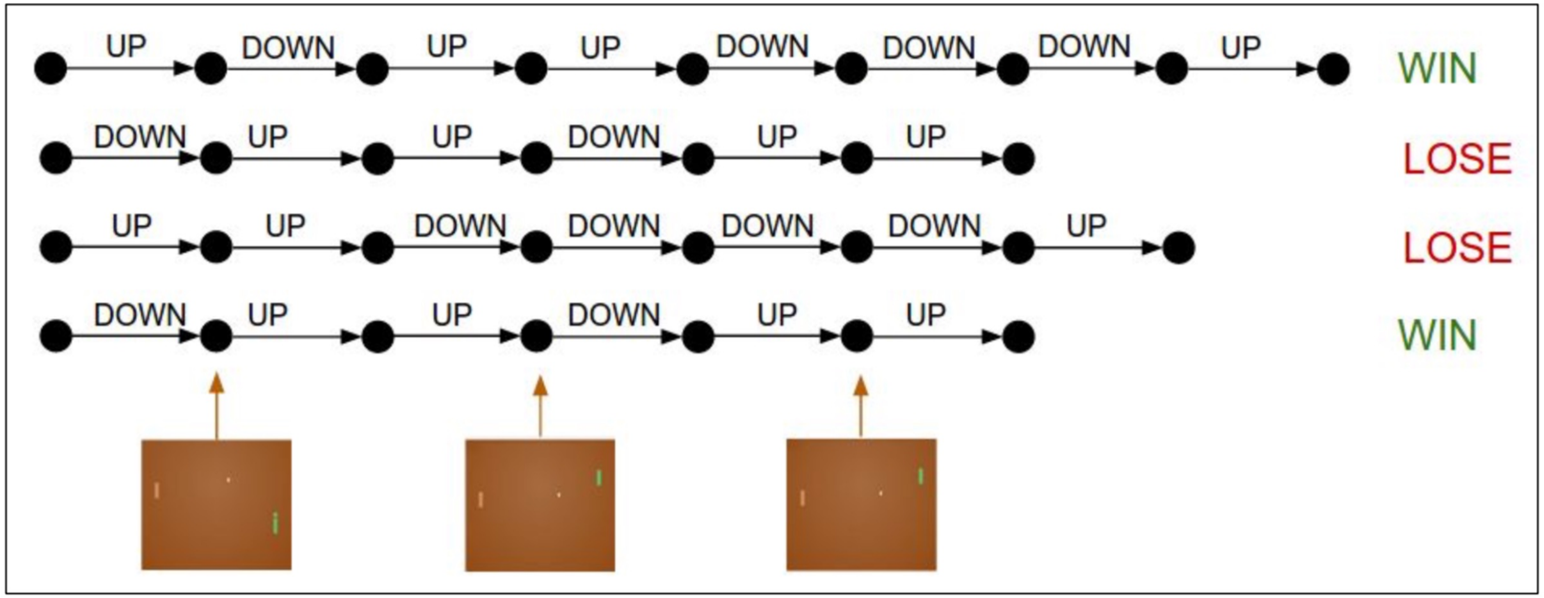


[Slides from Karpathy]

Pong from Pixels

Collect many rollouts...

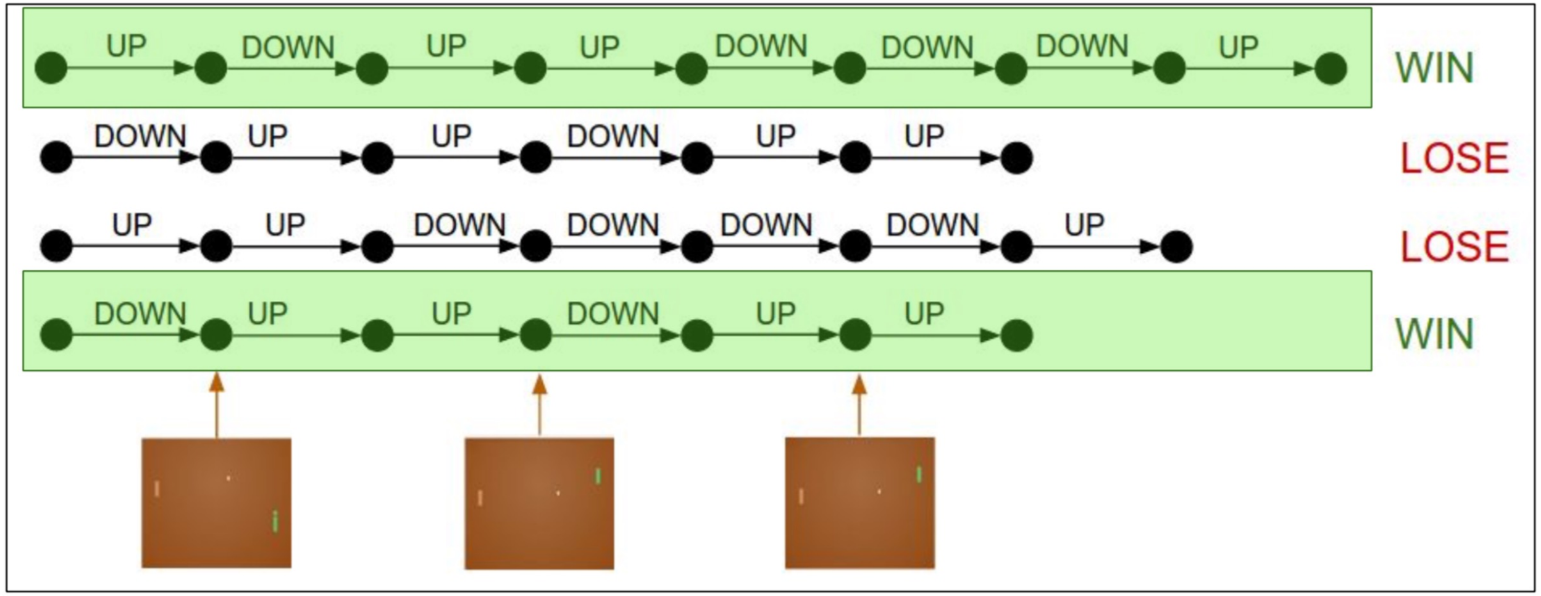
4 rollouts:



[Slides from Karpathy]

Pong from Pixels

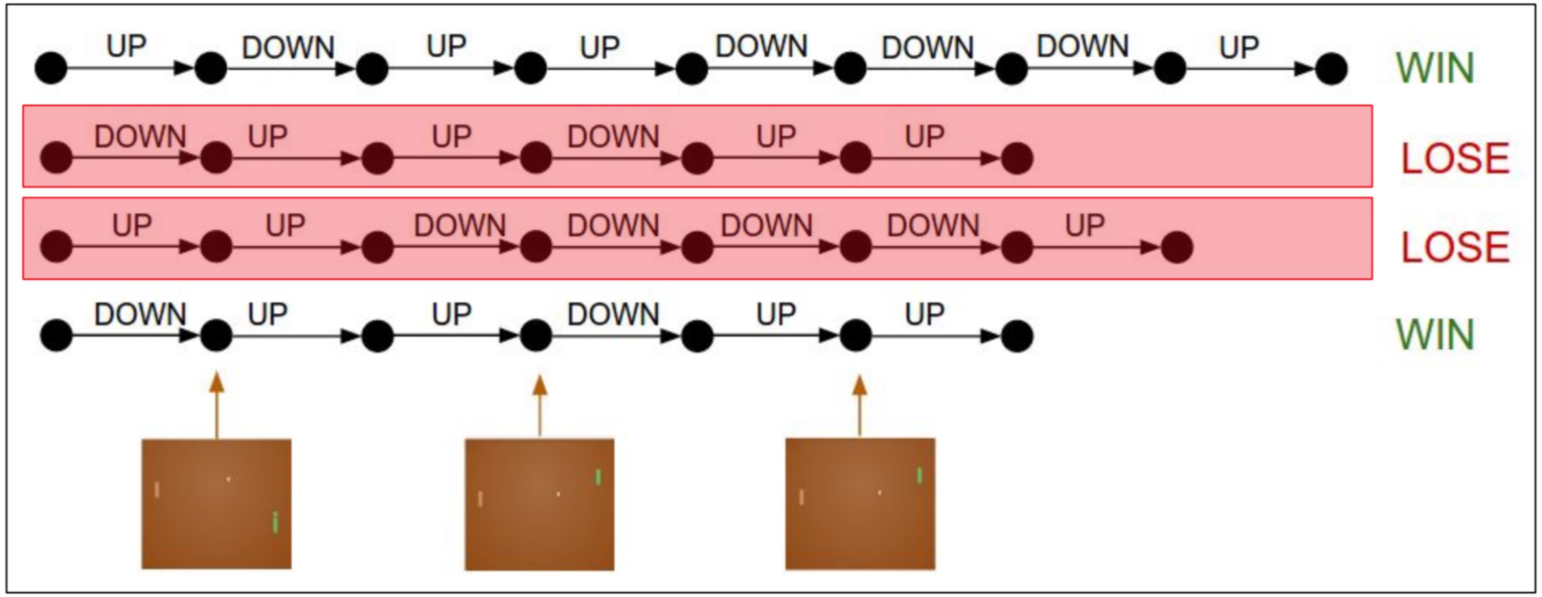
Not sure whatever we did here, but apparently it was good.



[Slides from Karpathy]

Pong from Pixels

Not sure whatever we did here, but it was bad.



[Slides from Karpathy]

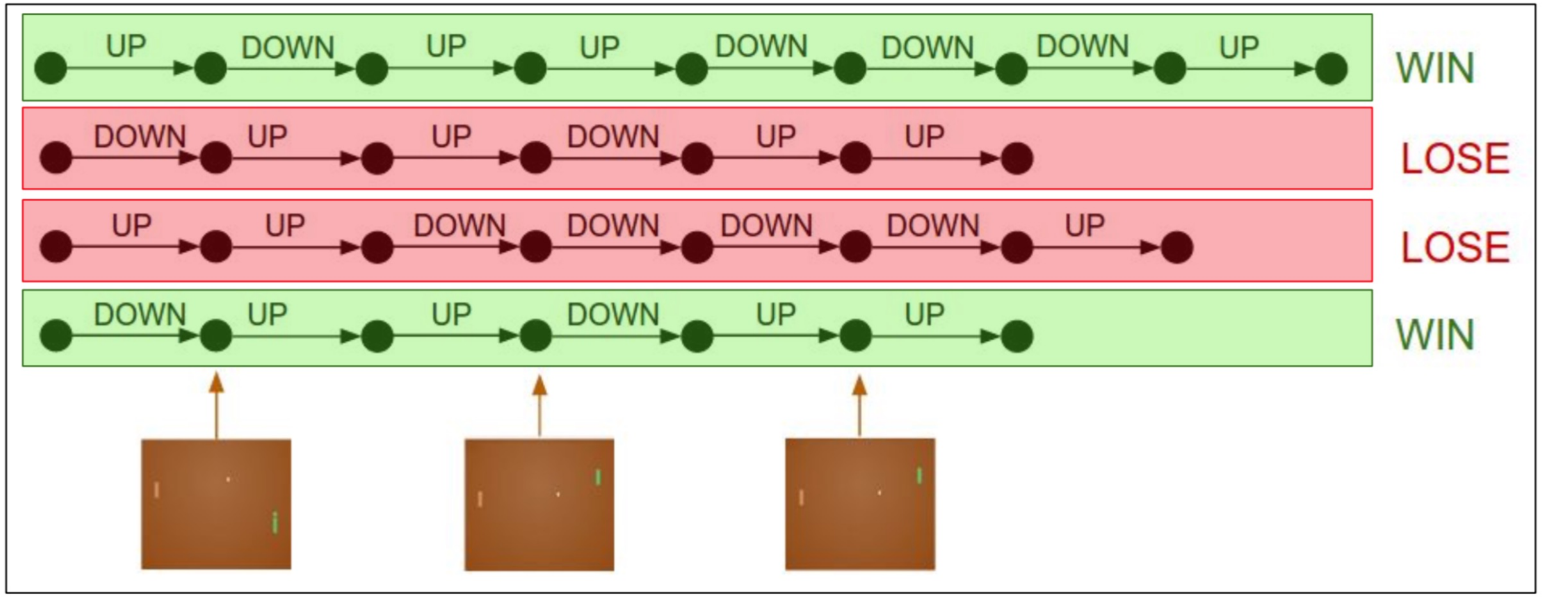
Pong from Pixels

Pretend every action we took here was the correct label.

maximize: $\log p(y_i \mid x_i)$

Pretend every action we took here was the wrong label.

maximize: $(-1) * \log p(y_i \mid x_i)$



[Slides from Karpathy]

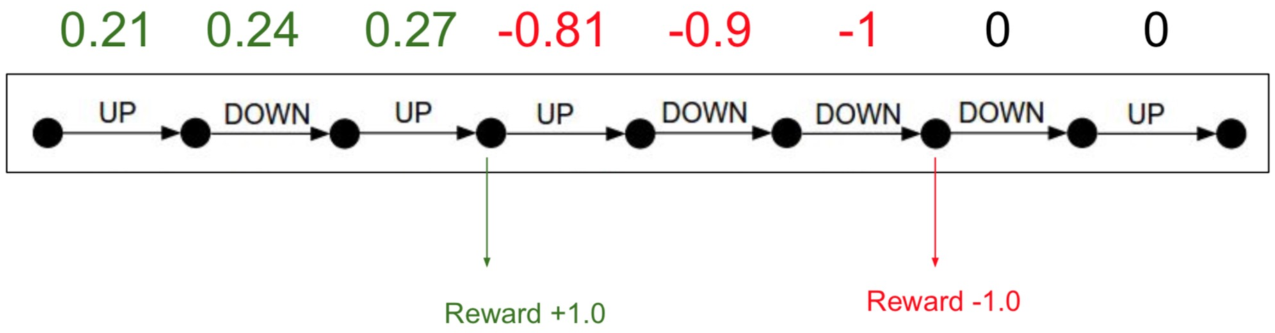
Pong from Pixels

Discounting

Blame each action assuming that its effects have exponentially decaying impact into the future.

$$\sum_i \boxed{A_i} * \log p(y_i|x_i)$$

Discounted rewards



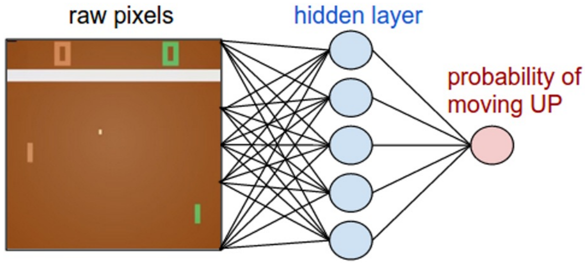
\gamma = 0.9

[Slides from Karpathy]

Pong from Pixels

$\pi(a \mid s)$

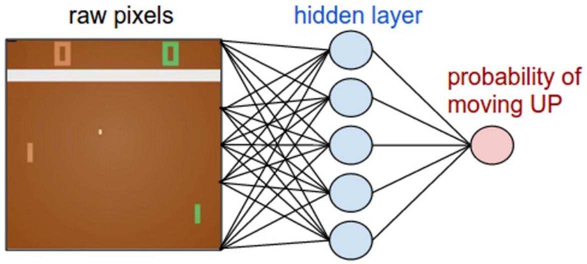
1. `Initialize a policy network at random`



[Slides from Karpathy]

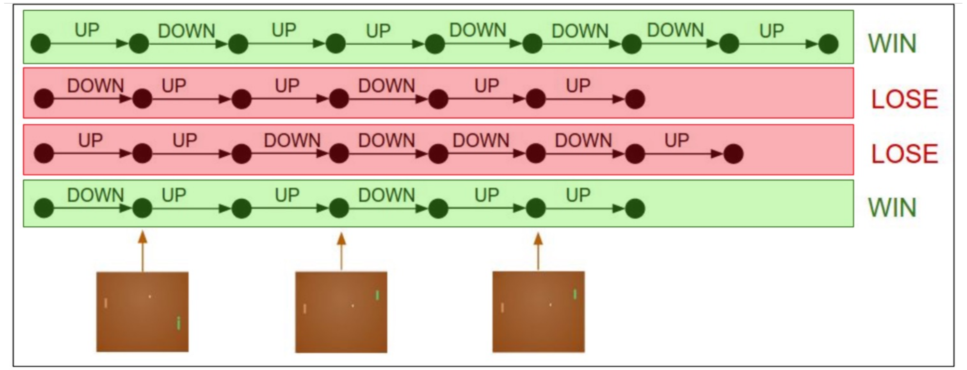
Pong from Pixels

$\pi(a\,|\,s)$




```
1.   Initialize a policy network at random
2.   Repeat Forever:
3.        Collect a bunch of rollouts with the policy
```

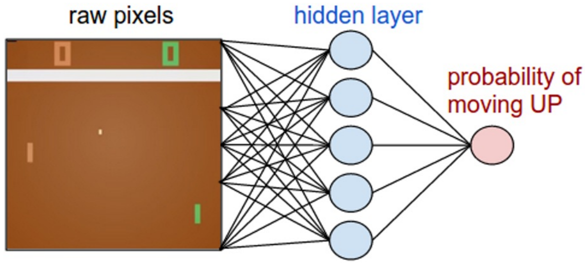
epsilon greedy!



[Slides from Karpathy]

Pong from Pixels

$\pi(a \mid s)$




```
1.  Initialize a policy network at random
2.  Repeat Forever:
3.       Collect a bunch of rollouts with the policy
4.       Increase the probability of actions that worked well
```

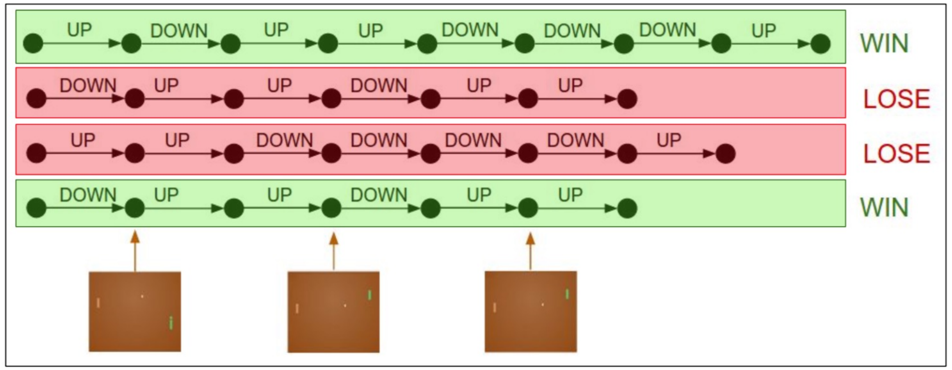
epsilon greedy!

Pretend every action we took here was the correct label.

maximize: $\log p(y_i \mid x_i)$

Pretend every action we took here was the wrong label.

maximize: $(-1) * \log p(y_i \mid x_i)$



$$\sum_i A_i * \log p(y_i | x_i)$$

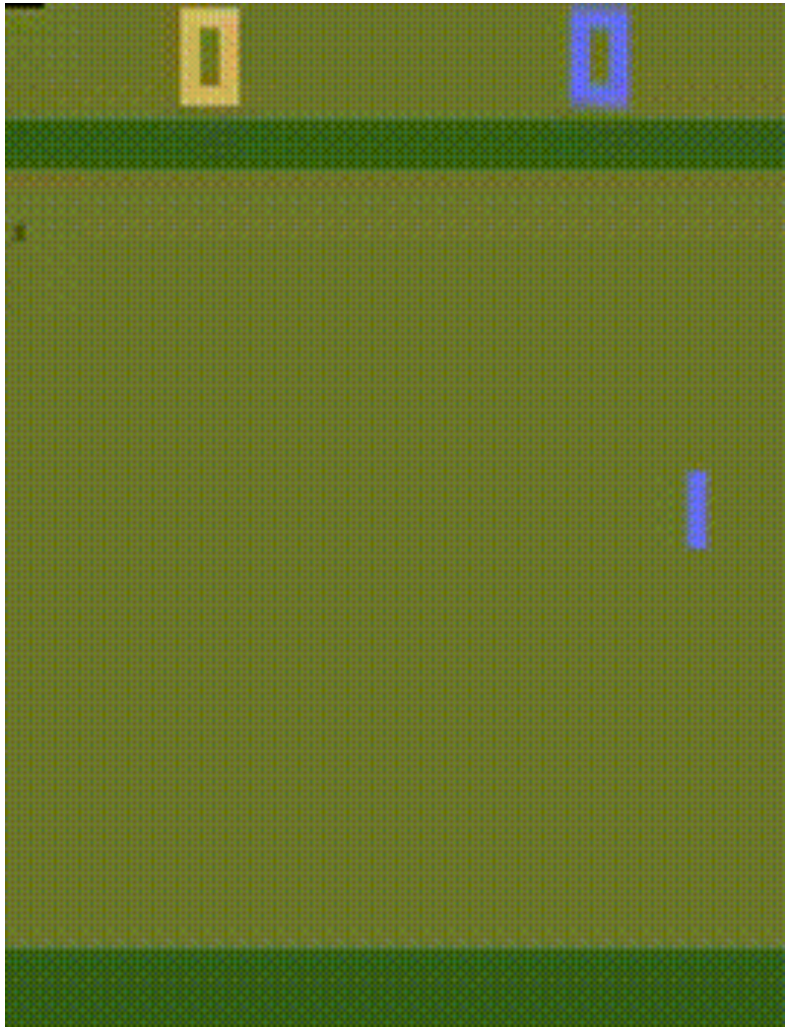
Does not require transition probabilities

Does not estimate Q(), V()

Predicts policy directly

[Slides from Karpathy]

Pong from Pixels



[Slides from Karpathy]

Policy Gradients

Why does this work?

1. Initialize a policy network at random
2. **Repeat Forever:**
3. Collect a bunch of rollouts with the policy
4. Increase the probability of actions that worked well

$$\boxed{\sum_i A_i * \log p(y_i|x_i)}$$

[Slides from Karpathy]

Policy Gradients

Formally, let's define a class of parameterized policies $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi_\theta\right]$$

Policy Gradients

Writing in terms of trajectories $\tau = (s_0, a_0, r_0, s_1, a_1, r_1, \ldots)$

Probability of a trajectory

$$\begin{aligned} p(\tau;\theta) &= \pi_\theta(a_0|s_0)p(s_1|s_0,a_0) \\ &\times \pi_\theta(a_1|s_1)p(s_2|s_1,a_1) \\ &\times \pi_\theta(a_2|s_2)p(s_3|s_2,a_2) \\ &\times \ldots \\ &= \prod_{t\geq 0} p(s_{t+1}|s_t,a_t)\pi_\theta(a_t|s_t) \end{aligned}$$

Reward of a trajectory

$$r(\tau) = \sum_{t\geq 0} \gamma^t r_t$$

$$\boxed{J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi_\theta\right] = \mathbb{E}_{\tau\sim p(\tau;\theta)}\left[r(\tau)\right]}$$

Policy Gradients

Formally, let's define a class of parameterized policies $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$\boxed{J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi_\theta\right] = \mathbb{E}_{\tau\sim p(\tau;\theta)}\left[r(\tau)\right]}$$

We want to find the optimal policy $\theta^* = \arg\max_\theta J(\theta)$

How can we do this?

Gradient ascent on policy parameters

REINFORCE Algorithm

Expected reward: $$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right] = \int_{\tau} r(\tau)p(\tau;\theta)\ d\tau$$

REINFORCE Algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau)]$

$$= \int_\tau r(\tau) p(\tau;\theta)\ d\tau$$

$$\boxed{p(\tau;\theta) = \prod_{t \geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)}$$

Now let's differentiate this: $\nabla_\theta J(\theta) = \int_\tau r(\tau) \nabla_\theta p(\tau;\theta)\ d\tau$

Intractable

REINFORCE Algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau)]$

$$= \int_\tau r(\tau) p(\tau;\theta) \; d\tau$$

$$\boxed{p(\tau;\theta) = \prod_{t \geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)}$$

Now let's differentiate this: $\nabla_\theta J(\theta) = \int_\tau r(\tau) \nabla_\theta p(\tau;\theta) \; d\tau$

Intractable

However, we can use a nice trick: $\nabla_\theta p(\tau;\theta) = p(\tau;\theta) \frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta) \nabla_\theta \log p(\tau;\theta)$

REINFORCE Algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$

$$= \int_\tau r(\tau)p(\tau;\theta)\; d\tau$$

$$\boxed{p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)}$$

Now let's differentiate this: $\nabla_\theta J(\theta) = \int_\tau r(\tau)\nabla_\theta p(\tau;\theta)\; d\tau$

Intractable

However, we can use a nice trick: $\nabla_\theta p(\tau;\theta) = p(\tau;\theta)\dfrac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta)\nabla_\theta \log p(\tau;\theta)$

If we inject this back:

$$\nabla_\theta J(\theta) = \int_\tau \left(r(\tau)\nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta)\; d\tau$$

$$= \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\nabla_\theta \log p(\tau;\theta)\right]$$

REINFORCE Algorithm

Can we compute these without knowing the transition probabilities?

We have: $$p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$$

REINFORCE Algorithm

Can we compute these without knowing the transition probabilities?

We have: $$p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$$

Thus: $$\log p(\tau;\theta) = \sum_{t\geq 0} (\log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t))$$

REINFORCE Algorithm

Can we compute these without knowing the transition probabilities?

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Thus: $$\log p(\tau;\theta) = \sum_{t\geq 0} \left(\log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)\right)$$

And when differentiating: $$\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Doesn't depend on transition probabilities

REINFORCE Algorithm

Can we compute these without knowing the transition probabilities?

We have: $$p(\tau;\theta) = \prod_{t \geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$$

Thus: $$\log p(\tau;\theta) = \sum_{t \geq 0} \left(\log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)\right)$$

And when differentiating: $$\nabla_\theta \log p(\tau;\theta) = \sum_{t \geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Doesn't depend on transition probabilities

Therefore when sampling a trajectory, we can estimate gradients:

$$\boxed{\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\nabla_\theta \log p(\tau;\theta)\right] \approx \sum_{t \geq 0} r(\tau)\nabla_\theta \log \pi_\theta(a_t|s_t)}$$

Policy Gradients

Gradient estimator: $$\nabla_\theta J(\theta) \approx \sum_{t\geq 0} r(\tau)\nabla_\theta \log \pi_\theta(a_t|s_t)$$

Interpretation:

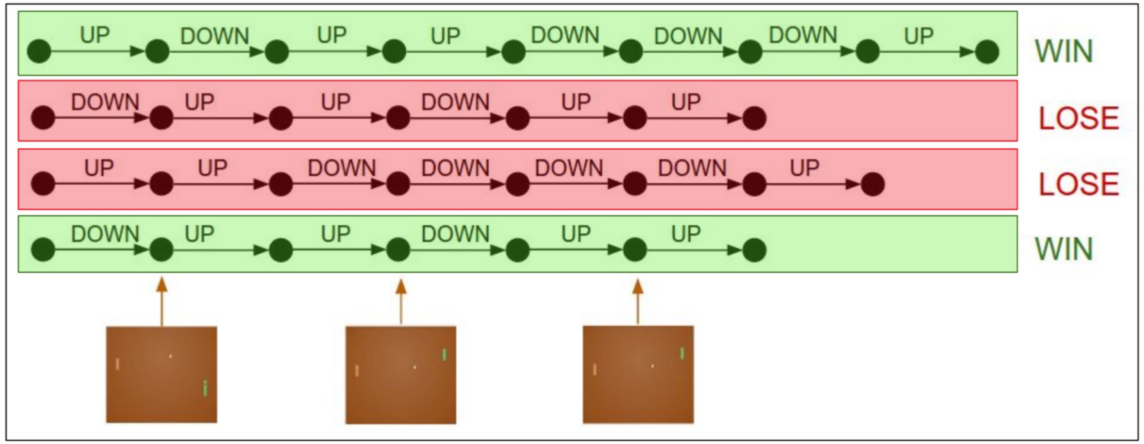
- If **r(trajectory)** is high, push up the probabilities of the actions seen
- If **r(trajectory)** is low, push down the probabilities of the actions seen

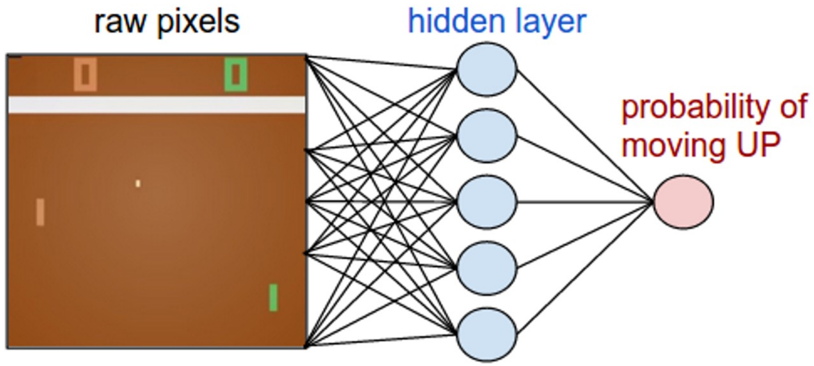
Pretend every action we took here was the correct label.

maximize: $\log p(y_i \mid x_i)$

Pretend every action we took here was the wrong label.

maximize: $(-1) * \log p(y_i \mid x_i)$





$$\sum_i A_i * \log p(y_i|x_i)$$

Policy Gradients

Gradient estimator:

$$\nabla_\theta J(\theta) \approx \sum_{t\geq 0} r(\tau)\nabla_\theta \log \pi_\theta(a_t|s_t)$$

Interpretation:

- If **r(trajectory)** is high, push up the probabilities of the actions seen
- If **r(trajectory)** is low, push down the probabilities of the actions seen

REINFORCE, A Monte-Carlo Policy-Gradient Method (episodic)

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta}), \forall a \in \mathcal{A}, s \in \mathcal{S}, \boldsymbol{\theta} \in \mathbb{R}^n$
Initialize policy weights $\boldsymbol{\theta}$
Repeat forever:
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$ $\boxed{\text{following } \pi(\cdot|\cdot,\boldsymbol{\theta})}$
  For each step of the episode $t = 0, \ldots, T-1$:
    $G_t \leftarrow$ return from step t
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha\gamma^t G_t \nabla_{\boldsymbol{\theta}} \log \pi(A_t|S_t,\boldsymbol{\theta})$

epsilon greedy

[Slides from Fragkiadaki, 10-703 CMU]

Policy Gradients

Gradient estimator:

$$\boxed{\nabla_\theta J(\theta) \approx \sum_{t\geq 0} r(\tau)\nabla_\theta \log \pi_\theta(a_t|s_t)}$$

Interpretation:

- If **r(trajectory)** is high, push up the probabilities of the actions seen
- If **r(trajectory)** is low, push down the probabilities of the actions seen

Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!

However, this also suffers from high variance because credit assignment is really hard - can we help this estimator?

[Slides from Fragkiadaki, 10-703 CMU]

Variance Reduction with a Baseline

Problem: The raw reward of a trajectory isn't necessarily meaningful. E.g. if all rewards are positive, you keep pushing up probabilities of all actions.

What is important then? Whether a reward is higher or lower than what you expect to get.

Idea: Introduce a baseline function dependent on the state, which gives us an estimator:

$$\boxed{\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(r(\tau) - b(s_t)\right) \nabla_\theta \log \pi_\theta(a_t|s_t)}$$

e.g. exponential moving average of the rewards.

[Slides from Fragkiadaki, 10-703 CMU]

Actor-Critic Methods

A better baseline: want to push the probability of an action from a state, if this action was better than the expected value of what we should get from that state

Recall: **Q and V - action and state value functions!**

We are happy with an action **a** in a state **s** if **Q(s,a) - V(s)** is large.
Otherwise we are unhappy with an action if it's small.

Using this, we get the estimator:

$$\boxed{\nabla_\theta J(\theta) \approx \sum_{t\geq 0} \left(Q^{\pi_\theta}(s_t, a_t) - V^{\pi_\theta}(s_t)\right) \nabla_\theta \log \pi_\theta(a_t|s_t)}$$

Actor-Critic Methods

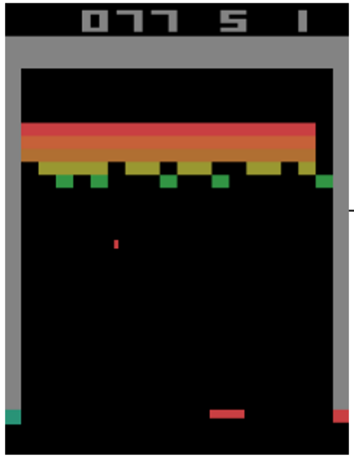
Problem: we don't know Q and V - can we learn them?

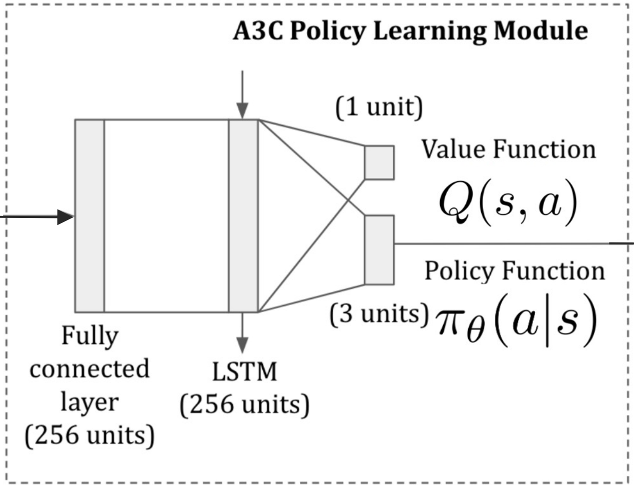
Yes, using Q-learning! We can combine Policy Gradients and Q-learning by training both an **actor** (the policy) and a **critic** (the Q function)

Exploration + experience replay
Decorrelate samples
Fixed targets

Critic: evaluates how good the action is

$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s'\sim\mathcal{D}_i}\left[\left(\underbrace{r+\gamma\max_{a'}Q(s',a';w_i^-)}_{\text{Q-learning target}} - \underbrace{Q(s,a;w_i)}_{\text{Q-network}}\right)^2\right]$$





$\pi_\theta(a|s)$

Actor: decides what actions to take

$$\nabla_\theta J(\theta) \approx \sum_{t\geq 0} \left(Q^{\pi_\theta}(s_t,a_t) - V^{\pi_\theta}(s_t)\right)\nabla_\theta \log \pi_\theta(a_t|s_t)$$

Variance reduction with a baseline

[Minh et al., Asynchronous Methods for Deep Reinforcement Learning. ICML 2016]

Summary: RL Methods

Value iteration
Policy iteration
(Deep) Q-learning

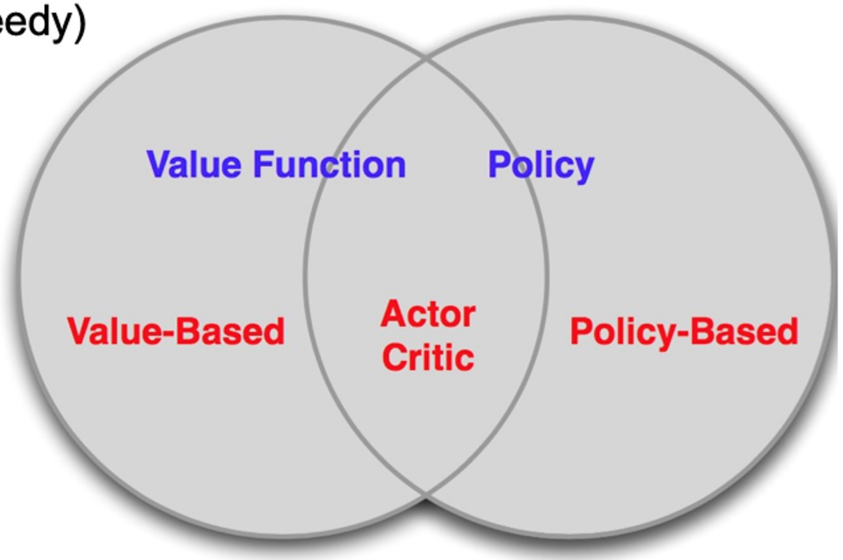
- Value Based
  - Learned Value Function
  - Implicit policy (e.g. ε-greedy)

Policy gradients

- Policy Based
  - No Value Function
  - Learned Policy

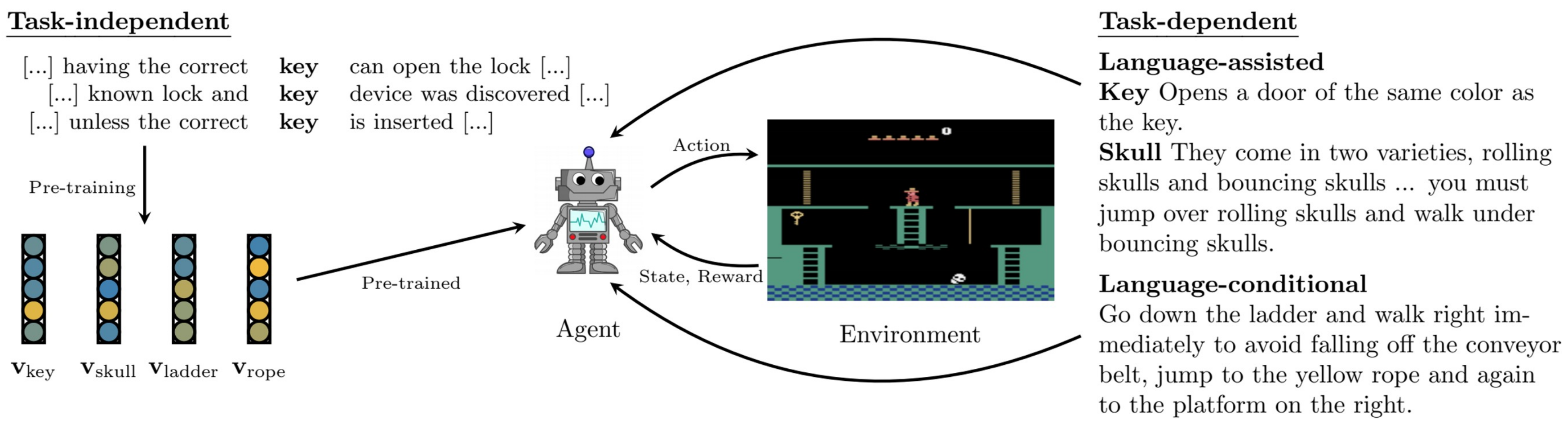
Actor (policy)
Critic (Q-values)

- Actor-Critic
  - Learned Value Function
  - Learned Policy



[Slides from Fragkiadaki, 10-703 CMU]

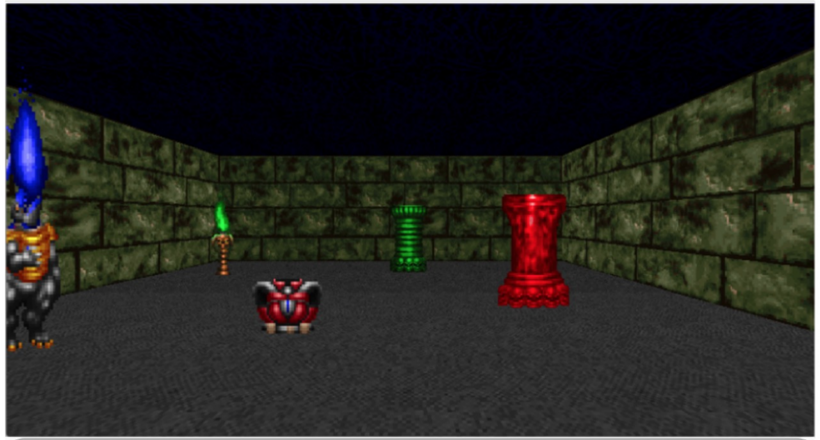
Back to Reasoning: Interactive Reasoning



[Luketina et al., A Survey of Reinforcement Learning Informed by Natural Language. IJCAI 2019]

Language-conditional RL: Instruction Following

Language specifies the task



Go to the green torch

Train

Go to the short red torch
Go to the blue keycard
Go to the largest yellow object
Go to the green object

Test

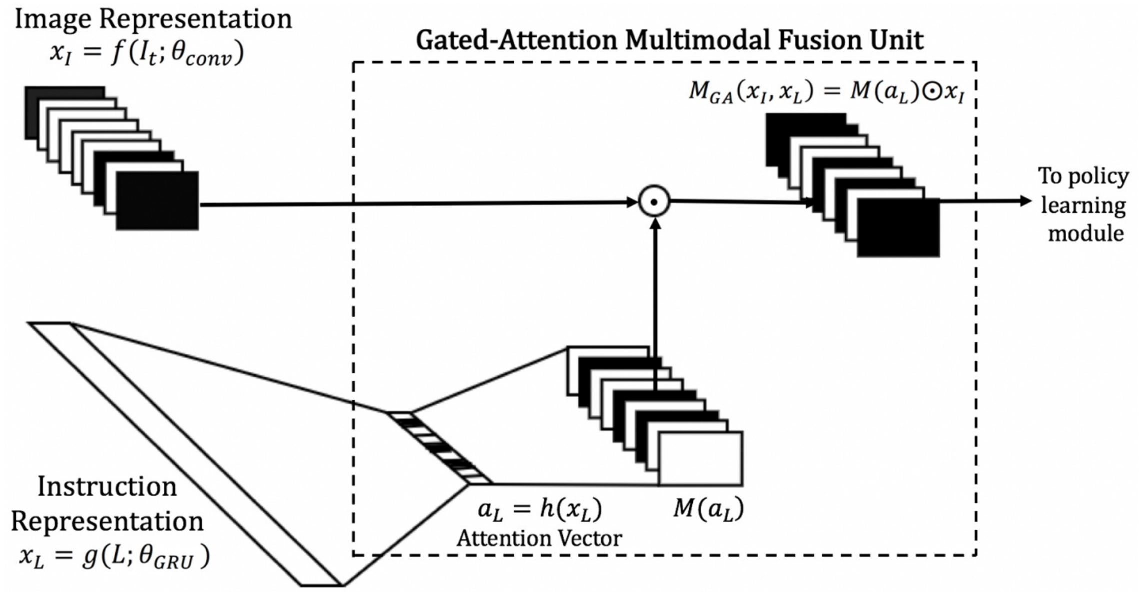
Go to the tall green torch
Go to the red keycard
Go to the smallest blue object

Fusion
Alignment
Ground language
Recognize objects
Navigate to objects
Generalize to unseen objects

[Misra et al., Mapping Instructions and Visual Observations to Actions with Reinforcement Learning. EMNLP 2017]
[Chaplot et al., Gated-Attention Architectures for Task-Oriented Language Grounding. AAAI 2018]

Language-conditional RL: Instruction Following

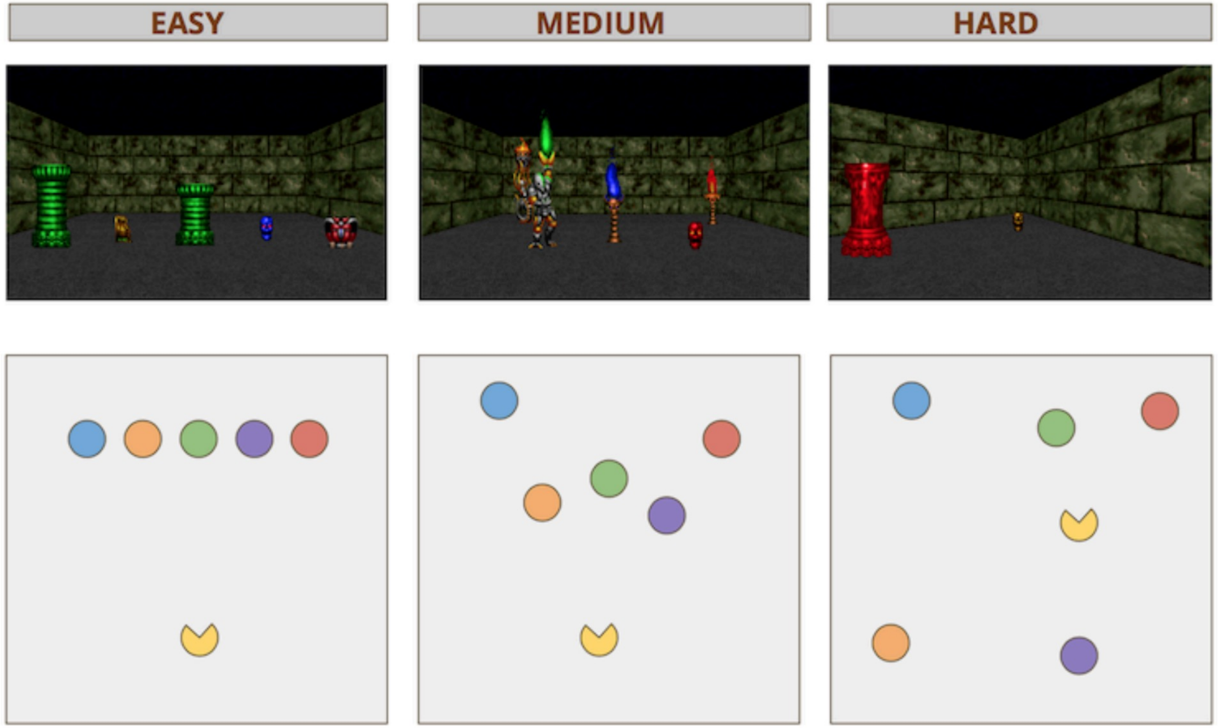
- Gated attention via element-wise product



Fusion
Alignment
Ground language
Recognize objects

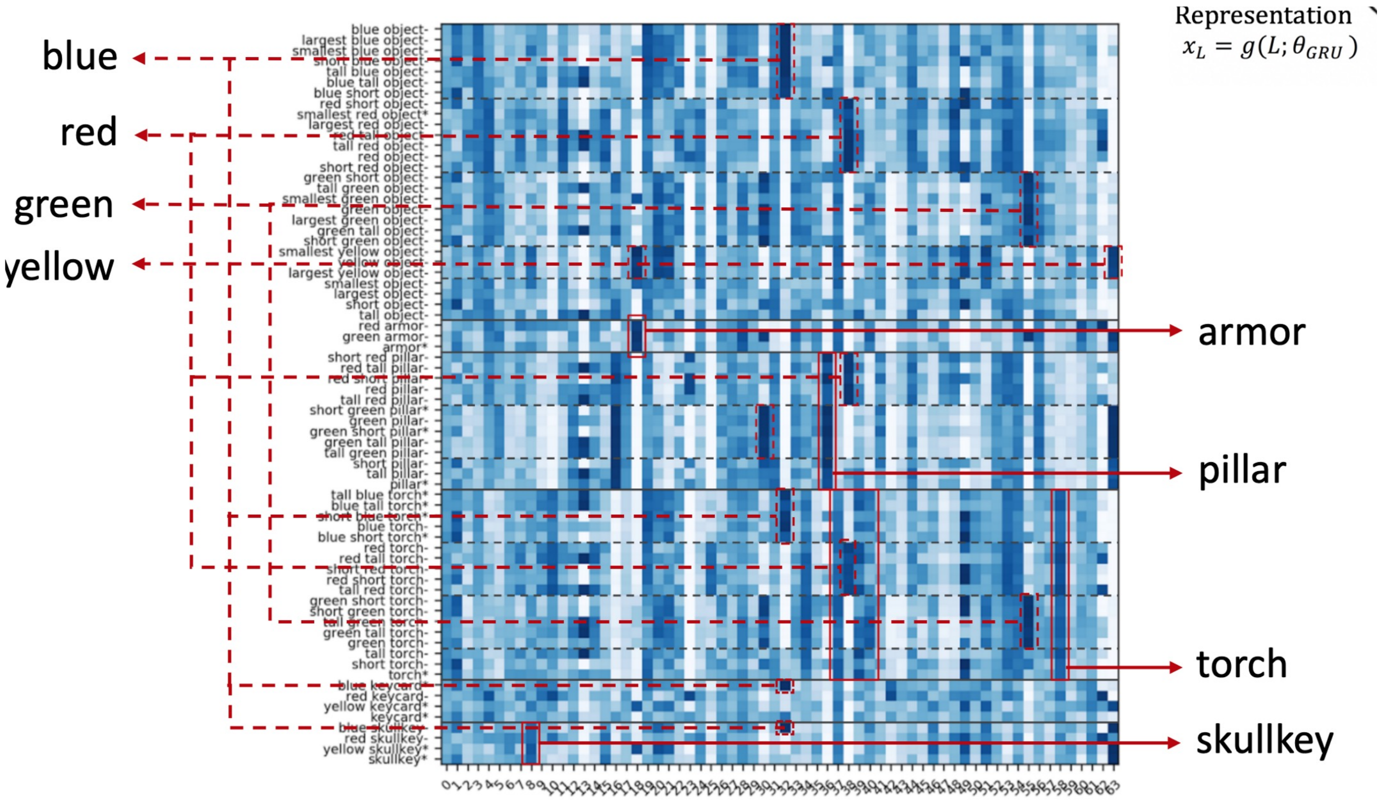
[Chaplot et al., Gated-Attention Architectures for Task-Oriented Language Grounding. AAAI 2018]

Language-conditional RL: Instruction Following



[Chaplot et al., Gated-Attention Architectures for Task-Oriented Language Grounding. AAAI 2018]

Language-conditional RL: Instruction Following



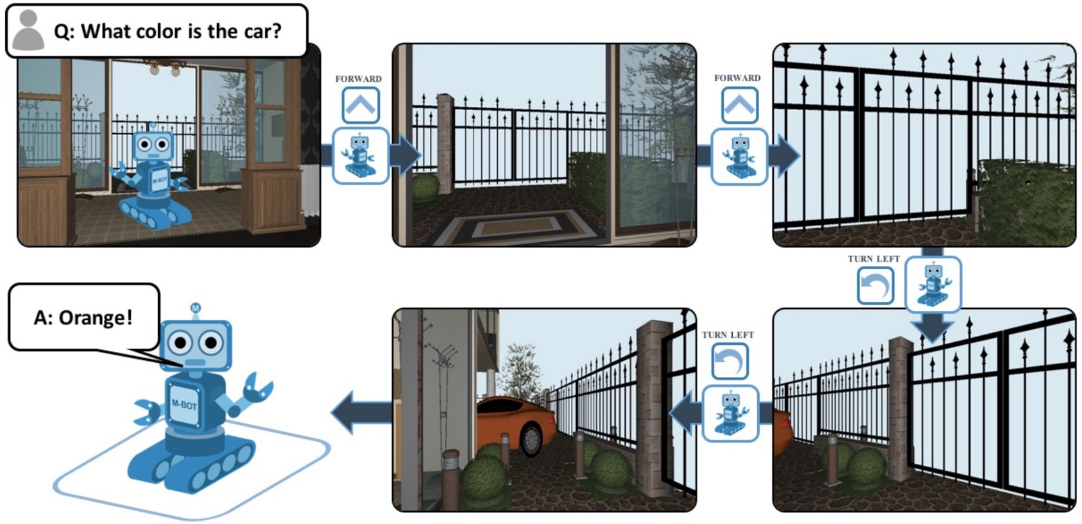
Grounding is important for generalization

blue armor, red pillar -> blue pillar

[Chaplot et al., Gated-Attention Architectures for Task-Oriented Language Grounding. AAAI 2018]

Language-conditional RL: Embodied QA

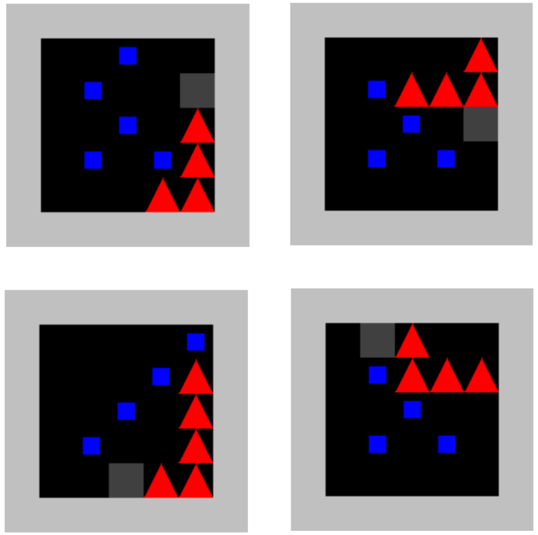
Navigation + QA



[Das et al., Embodied Question Answering. CVPR 2018]

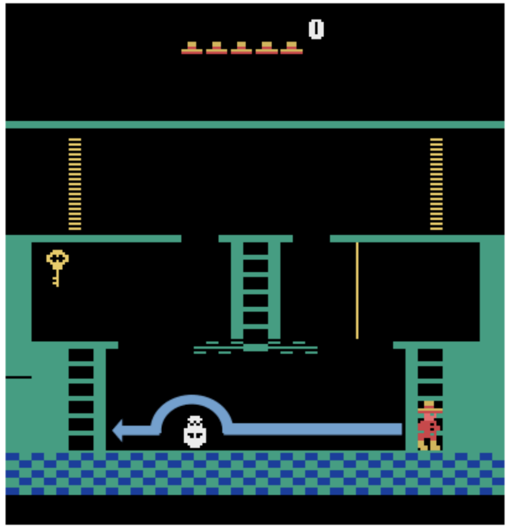
Language-assisted RL: Language to Rewards

Language specifies the rewards rather than actions



“build an L-like shape from red blocks”

Goal specification
(Bahdanau et al. 2019)



“Jump over the skull while going to the left”

Reward shaping
(Goyal et al. 2019)

“I prefer JetBlue, even if it’s expensive”

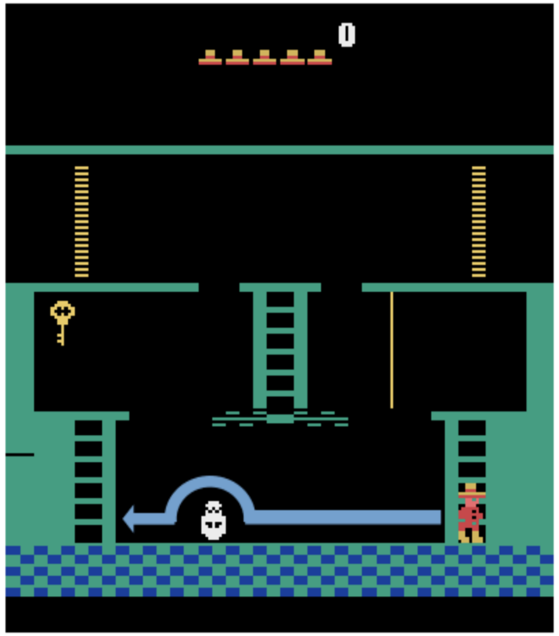
Preferences
(Lin et al. 2022)

[Goyal et al., Using Natural Language for Reward Shaping in Reinforcement Learning. IJCAI 2019]

https://arxiv.org/abs/1806.01946,
https://arxiv.org/abs/1902.07742,
https://www.ijcai.org/proceedings/2019/331,
https://arxiv.org/abs/2204.02515

Language-assisted RL: Language to Rewards

Language specifies the rewards rather than actions



Montezuma's revenge

Sparse, long-term reward problem
General solution: reward shaping via auxiliary rewards

Natural language for reward shaping

"Jump over the skull while going to the left"

from Amazon Mturk :-(
asked annotators to play the game and describe entities

Intermediate rewards to speed up learning

[Goyal et al., Using Natural Language for Reward Shaping in Reinforcement Learning. IJCAI 2019]

Language-assisted RL: Domain knowledge

Language as domain knowledge – instruction manuals

The natural resources available where a population settles affects its ability to produce food and goods. Build your city on a plains or grassland square with a river running through it if possible.

Figure 1: An excerpt from the user manual of the game Civilization II.

[Branavan et al., Learning to Win by Reading Manuals in a Monte-Carlo Framework. JAIR 2012]

Language-assisted RL: Domain knowledge

Language as domain knowledge – instruction manuals

The natural resources available where a population settles affects its ability to produce food and goods. Build your city on a plains or grassland square with a river running through it if possible.

Map tile attributes:
- Terrain type (e.g. grassland, mountain, etc)
- Tile resources (e.g. wheat, coal, wildlife, etc)

City attributes:
- City population
- Amount of food produced

Unit attributes:
- Unit type (e.g., worker, explorer, archer, etc)
- Is unit in a city ?

1. Choose **relevant** sentences
2. Label words into **action-description, state-description**, or **background**

[Branavan et al., Learning to Win by Reading Manuals in a Monte-Carlo Framework. JAIR 2012]

Language-assisted RL: Domain knowledge

Language as domain knowledge – instruction manuals

- Phalanxes are twice as effective at defending cities as warriors. ✔
- Build the city on plains or grassland with a river running through it. ✔
- You can rename the city if you like, but we'll refer to it as washington.
- There are many different strategies dictating the order in which advances are researched

Relevant sentences

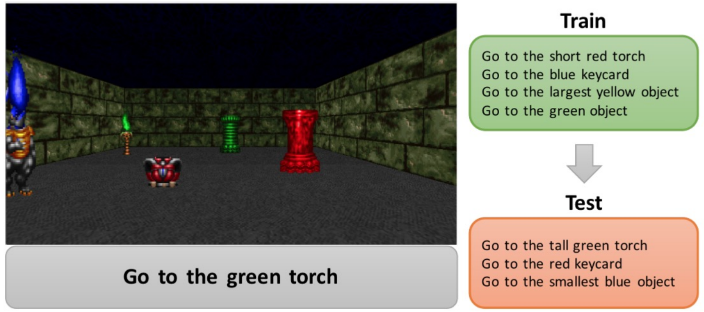
- After the road is built, use the settlers to start improving the terrain.
  S S S A A A A A
- When the settlers becomes active, chose build road.
  S S S A A A
- Use settlers or engineers to improve a terrain square within the city radius
  A S ✘ A A S A ✘ S S S S

A: action-description
S: state-description

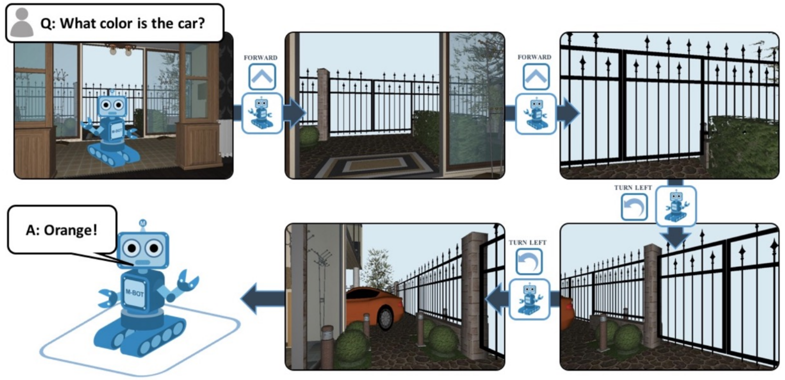
[Branavan et al., Learning to Win by Reading Manuals in a Monte-Carlo Framework. JAIR 2012]

Summary: Interactive Reasoning

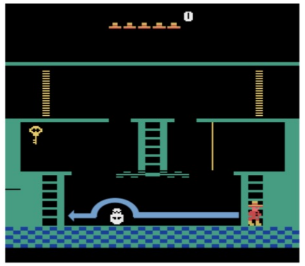
Instruction following



Embodied QA



Reward shaping



"Jump over the skull while going to the left"

Domain knowledge

The natural resources available where a population settles affects its ability to produce food and goods. Build your city on a plains or grassland square with a river running through it if possible.

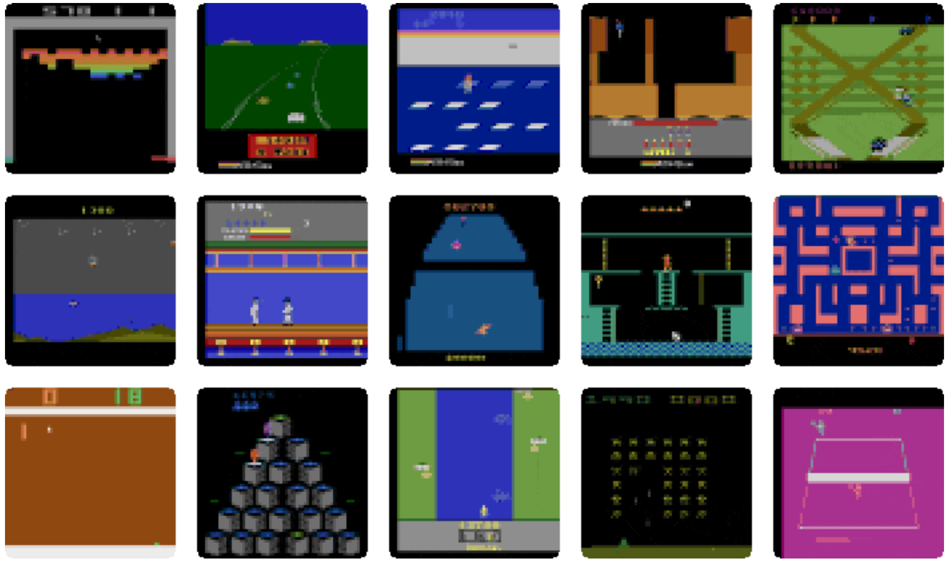
Figure 1: An excerpt from the user manual of the game Civilization II.

[Luketina et al., A Survey of Reinforcement Learning Informed by Natural Language. IJCAI 2019]

Interactive Reasoning Challenges

Open challenges

Learning from open-ended manuals



A L I E N
20th Century Fox
Games of the Century
(picture of the ALIEN movie poster)
"In space no one can hear you scream"
Game Instructions
Fox Video Games

A L I E N

TO SET UP: Set up your video computer system and left joystick controller as instructed in your manufacturer owner's manual. Move the Color/B-W lever to the correct setting. Turn the power OFF and insert the Alien game cartridge.

(Screen shot of the ALIEN maze setup: Alien, Alien Egg, Human, Pulsar and Play Level-demo mode only)

TO BEGIN: Turn the power ON. Use the Game Select lever and Difficulty Switches to choose a play level. Press the Game Reset lever and get ready to run for your life.

THE OBJECTIVE: Your job is to run through the hallways of your space ship and crush all the Alien Eggs which have been placed there. You must also avoid or destroy the adult Aliens and snatch up as many prizes as possible.

THE CONTROLS: Tilt the joystick forward, backward, left and right to maneuver through the hallways. To smash Eggs, simply run over them. You may travel off one side of the maze and back into the other using the "Hyperwarp Passage." Each Human is equipped with a Flame Thrower that is activated by the joystick button (see below).

SCREEN DISPLAY: The Play Level and Humans allowed per Play Level are displayed in the bottom left corner of the screen when Alien is not in play. During the game, the current score and Humans remaining are shown there.

LEVELS OF PLAY/DIFFICULTY SWITCHES/BONUS ROUNDS: Each game of Alien lasts until you run out of Humans. If you can clear all of the Eggs out of a playing screen, you get the chance to earn extra points in a "Bonus Round" and then are returned to a new and more difficult playing screen. All points and Humans remaining are carried over to the new screens.

Bonus Rounds: The object of the Bonus Round is to travel STRAIGHT UP to the top of the screen and grab the prize shown there. You have only eight seconds to do so. You do not lose a human if you fail, but you earn the point value of the prize if you succeed.

Left Difficulty Switch A: Aliens travel in random order about the screen.

Left Difficulty Switch B: Aliend travel in fixed patterns about the screen.

Right Difficult Switch B: Capturing a Pulsar has standard effect on the Aliens.

Right Difficulty Switch A: Capturing a Pulsar has no effect on the Aliens.

(Screen shot of ALIEN maze: Flame Thrower, Prize, Hyperwarp Passages, Humans Remaining and Current Score)

LEVEL 1 - NORMAL GAME PLAY: You begin with three Humans and receive a bonus Human after successfully clearing the second screen. Prizes appear in chart order.

LEVEL 2 - ADVANCED GAME PLAY: You begin with two Humans and receive no bonus Humans. Prizes appear in chart order.

LEVEL 3 - FOR EXPERTS ONLY: You begin with three Humans and receive no bonus Human after clearing the first screen. All Prizes in Level 3 are Saturns.

LEVEL 4 - EASY PRACTICE GAME: You begin with six Humans and receive 1 bonus Human after clearing the first sceen. All Prizes in Level 4 are also Saturns.

OBJECTS/SCORING: Each time an Alien catches you, one Human is lost. You score points for smashing Eggs and frying Aliens with the aid of your Flame Thrower or Pulsar. In addition, you can gain points for picking up Prizes. Be sure to record your high scores on the back of this booklet!

(Screen shot of the bonus round with the human at the bottom of the screen, the prize at the top of the screen and the horizontal moving Aliens in the centre portion -- similar to the road portion of Frogger.)

FLAME THROWER - 1 PER HUMAN: A spurt of flam from this contraption cause Aliens to turn away from you or become immobilized for a short period of time. Use the Throwers carefully. Each has only four secons of flame and the Thrower will not operate in the extreme left or right areas of the screen. You can also use the Flame Thrower to run over a Pulsar without picking it up, allowing you to save the Pulsar to use at a later time.

PULSARS - 3 PER MAZE: Capturing a Pulsar causes the Aliens to weaken and turn blue. Then, for a short period of time, you can destroy them by running over and touching them. The instant the Aliens return to their original colr, however, they once again become deadly.

PRIZES - 2 PER MAZE: Prizes appear in all levels of play and in the Bonus Rounds.

POINT CHART:

OBJECT	POINTS	PRIZES	POINTS
Eggs	10	Rocket	500
Pulsar	100	Saturn	1,000
1st Alien	500	Star Ship	2,000
2nd Alien	1,000	1st Surprise	2,000-3,000
3rd Alien	2,000	2nd Surprise	3,000
Completed Screen	1	3rd Surprise	5,000

HINTS FROM DALLAS NORTH...
A good playing strategy is to crush all of the Eggs in one area at a time, keeping within easy readh of a Pulsar. The best way to destroy Aliens is to sit near a Pulsar until the Aliens are almost upon you. Then grab that Pulsar and go get 'em !

Use the Hyperwarp Passage to ditch Aliens. Many times they won't follow you in.

If you're having trouble with the Bonus Rounds, try going between the Alien pairs rather than around them.

SUPER SMASHERS (a place to enter your high scores)

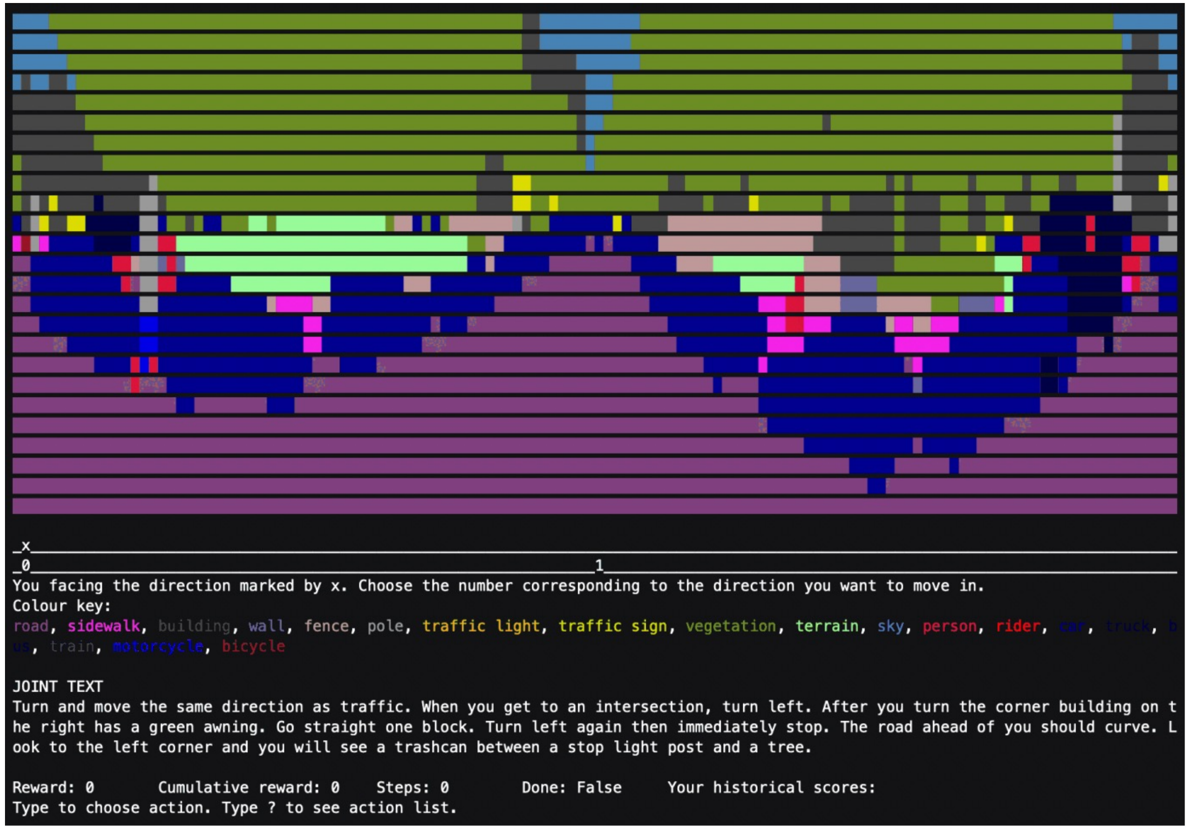
Name	Level	Score

[Atari Learning Environment]

Interactive Reasoning Challenges

Open challenges

Learning from text-based games



[Zhong et al., SILG: The Multi-environment Symbolic Interactive Language Grounding Benchmark. NeurIPS 2021]

Interactive Reasoning Challenges

Open challenges

Learning from lots of offline data

[Fan et al., MineDojo: Building Open-Ended Embodied Agents with Internet-Scale Knowledge. arXiv 2022]

Interactive Reasoning Challenges

Open challenges

Hard to specify reward, but only final goal

[Habitat Rearrangement Challenge 2022]

Summary

Definition: Combining knowledge, usually through multiple inferential steps, exploiting multimodal alignment and problem structure.

	A Structure modeling	B Intermediate concepts	C Inference paradigm	D External knowledge
Last Thursday	Temporal Hierarchical	Continuous		
Today	Interactive			

RL basics

Summary: RL Methods

Epsilon greedy + exploration
Experience replay
Decorrelate samples
Fixed targets

- Value Based
  - Learned Value Function
  - Implicit policy (e.g. ε-greedy)

Value iteration
Policy iteration
(Deep) Q-learning

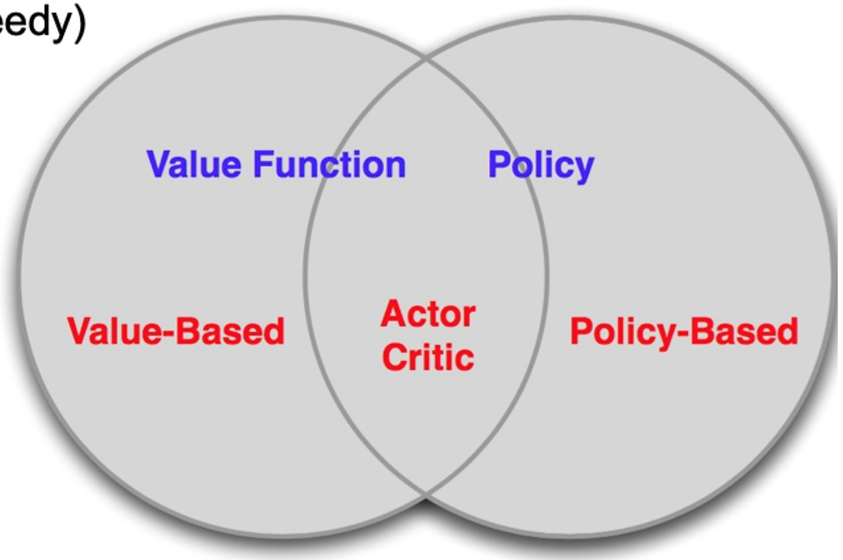
- Policy Based
  - No Value Function
  - Learned Policy

Policy gradients

Variance reduction with a baseline

- Actor-Critic
  - Learned Value Function
  - Learned Policy

Actor (policy)
Critic (Q-values)



[Slides from Fragkiadaki, 10-703 CMU]

Summary

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	A Structure modeling	B Intermediate concepts	C Inference paradigm	D External knowledge
Last Thursday	Temporal Hierarchical	Continuous		
Today	Interactive			
Thursday	Discovery	Discrete	Causal Logical	Knowledge Commonsense