



Language
Technologies
Institute

Carnegie
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University

Multimodal Machine Learning

Lecture 3.1: Multimodal Representation Fusion

Louis-Philippe Morency

** Original course co-developed with Tadas Baltrusaitis.
Major revision co-developed with Paul Liang. Co-lecturers
included Yonatan Bisk, Daniel Fried and Carlos Busso.*

Upcoming Course Assignments

Week 3 Reading Assignment (due Friday 9/11 at 8pm ET)

- Two papers: a first about unimodal and a second about multimodal

Pre-proposal (due Wednesday 9/9 at 8pm ET)

- All team matching should be completed
- Describe your team's dataset and present initial research ideas

Proposal, aka First Assignment (due Sunday 9/20 at 8pm ET)

- Literature review of your research problem
- Second revision of your research ideas (i.e., top-down approach)

Still looking for teammates?

See post on Piazza and add your follow-up post

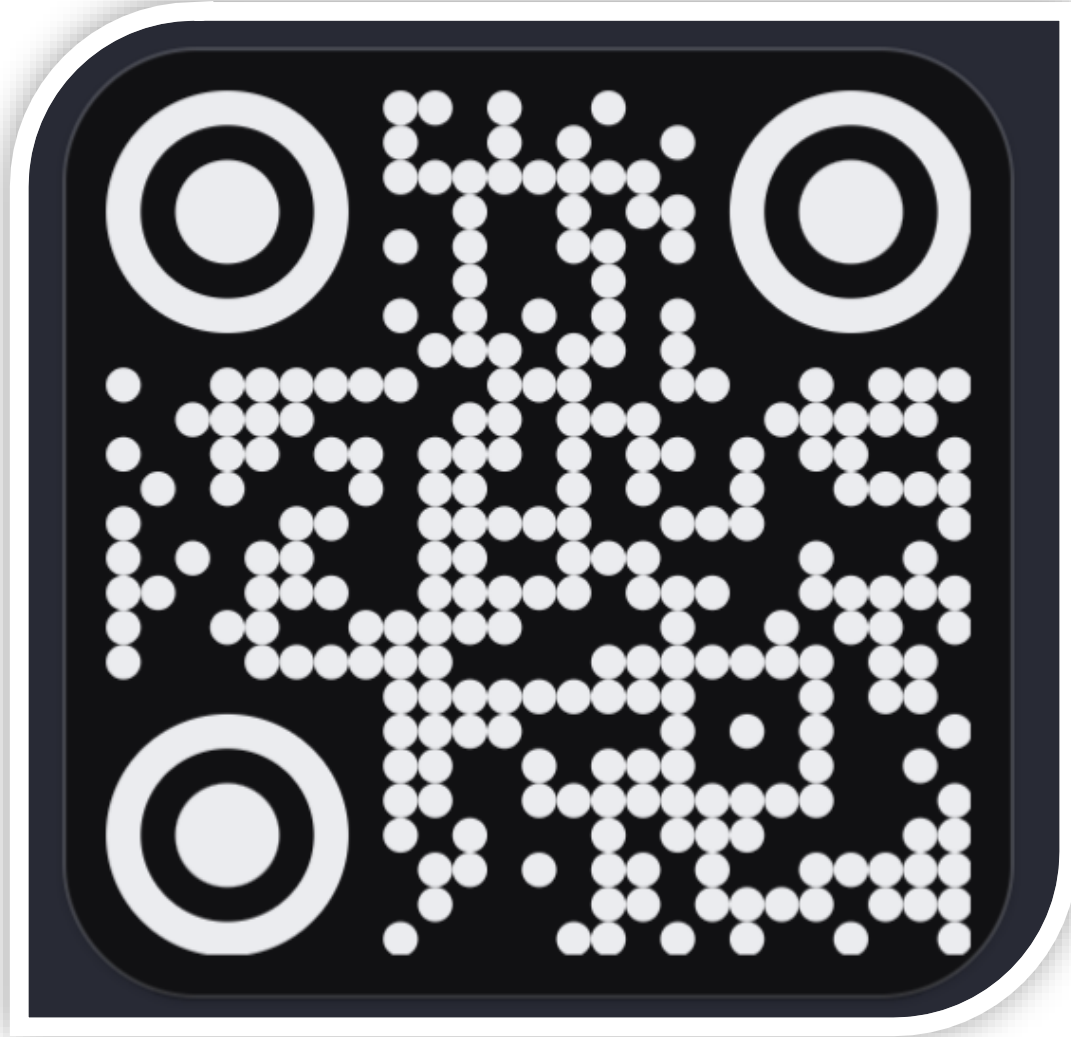
Primary TAs

- Each team will have one primary TA
- Meetings with primary TA will be scheduled for next week
 - Feedback about the pre-proposals
- Contact your primary TA anytime (piazza or email)
 - Groups will be created in Piazza for each team
- Some projects may have a secondary TA, with complementary expertise

NEW: Asynchronous Lecture Questions on Piazza

- Questions live during the lecture are still the preferred approach
 - Ask them by raising you hand 😊
- If you prefer to ask asynchronously your question:
 - You can post them, as follow-discussion post, in Piazza
 - Up to 6 hours after the end of the lecture, if possible
 - With the help of the TA team, we plan to respond these questions asynchronously (within 24 hours, if possible)

Attendance using Poll Everywhere



- 1 Scan QR Code
or
<https://pe.app/mmm1>
- 2 Enter your AndrewID
(as your “name tag”)
- 3 Click “Share location”

✗ *Do not try to sign-in*



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Lecture Objectives

- Vision Transformers
 - CNN with self-attention
 - Masked auto-encoder
- Multimodal representations
 - Cross-modal interactions
- Representation fusion
 - Additive and multiplicative fusion
 - Gated fusion
 - Fusion on raw modalities
 - Heterogeneity-aware fusion
- Measuring non-additive interactions

Vision Transformers

Convolution in 2D – Example



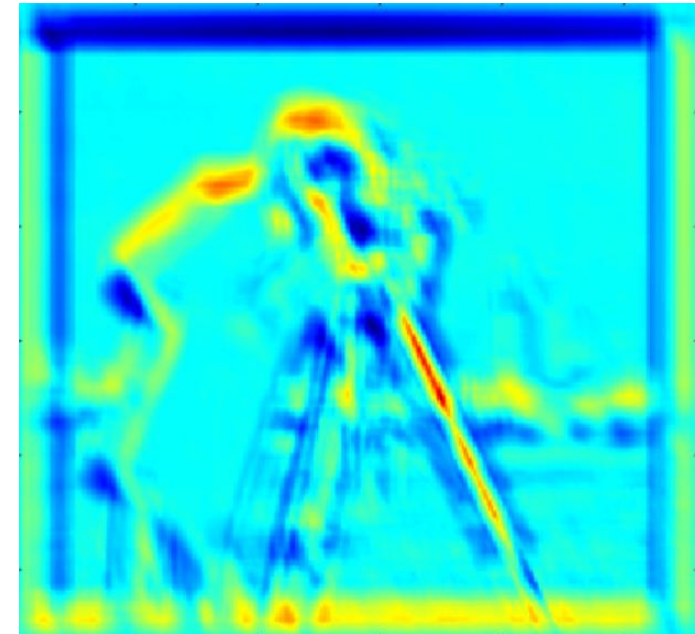
Input image

*



Convolution
kernel

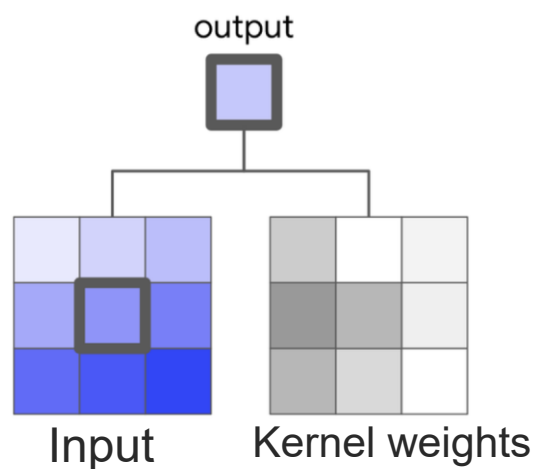
=



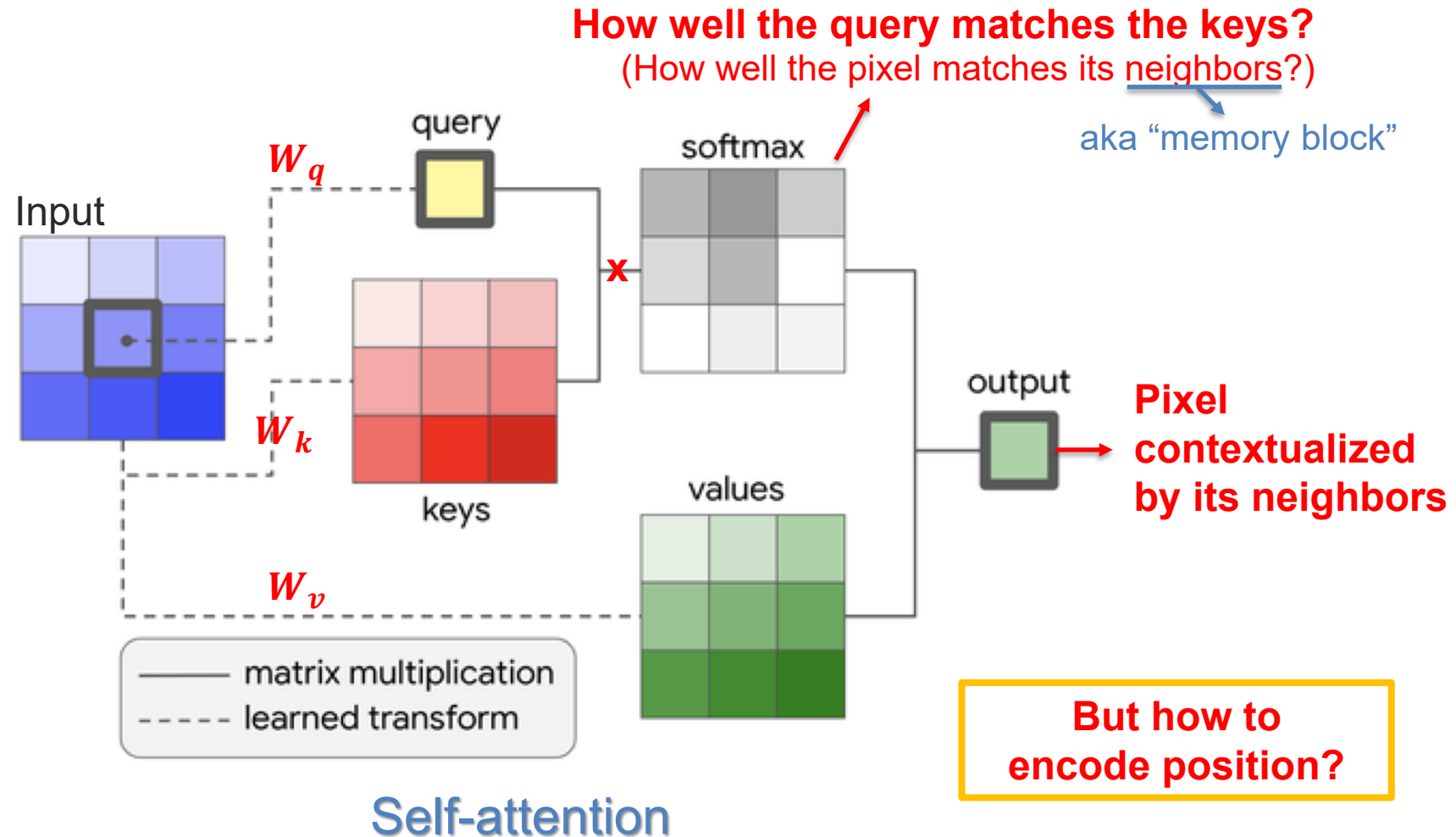
Response map

CNN

Replacing a CNN w/ Self-Attention



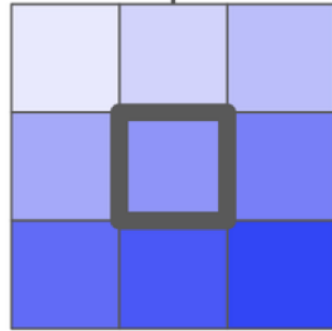
Convolution



Self-attention

Replacing a CNN w/ Self-Attention

Image patch



2D relative position embedding

-1, -1	-1, 0	-1, 1	-1, 2
0, -1	0, 0	0, 1	0, 2
1, -1	1, 0	1, 1	1, 2
2, -1	2, 0	2, 1	2, 2

Position embedding is added to the key:

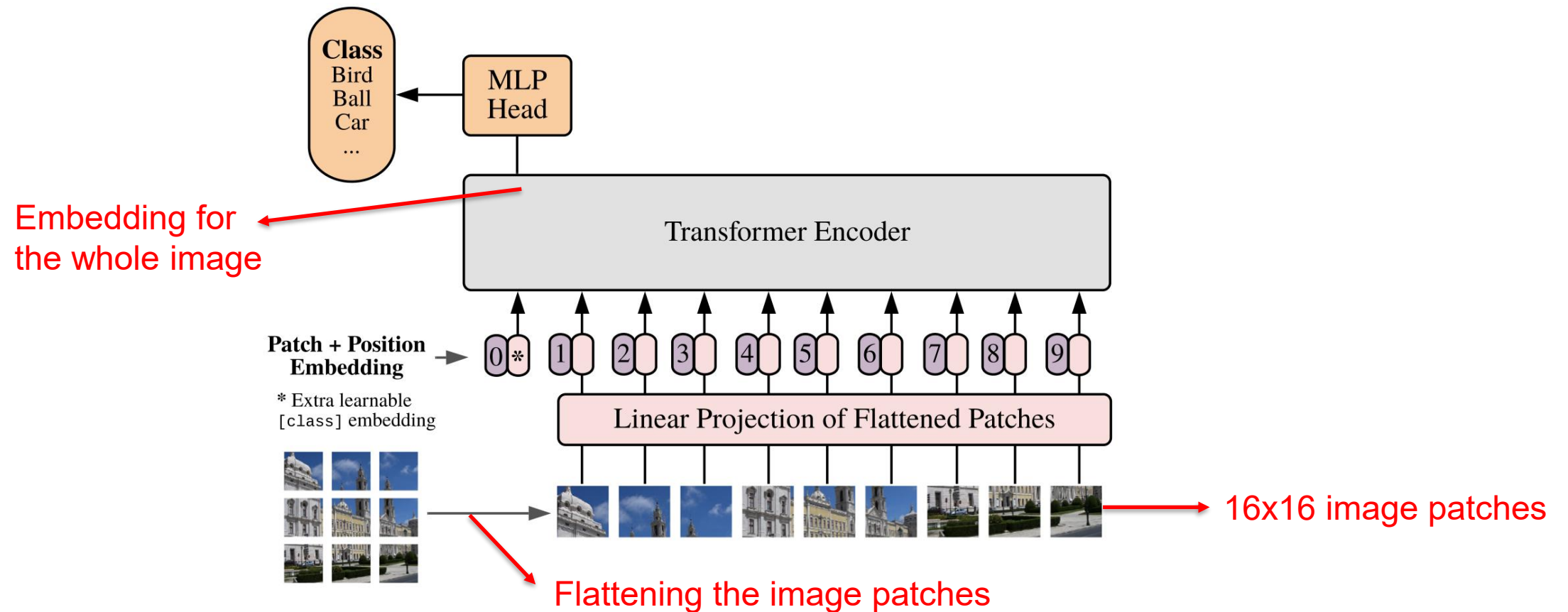
$$y_{ij} = \sum_{a,b \in \mathcal{N}_k(i,j)} \text{softmax}_{ab} \left(q_{ij}^\top k_{ab} + q_{ij}^\top r_{a-i,b-j} \right) v_{ab}$$

Vision Transformer (ViT)



Dosovitskiy, Alexey, et al. "An image is worth 16x16 words: Transformers for image recognition at scale." *arXiv* (2020).

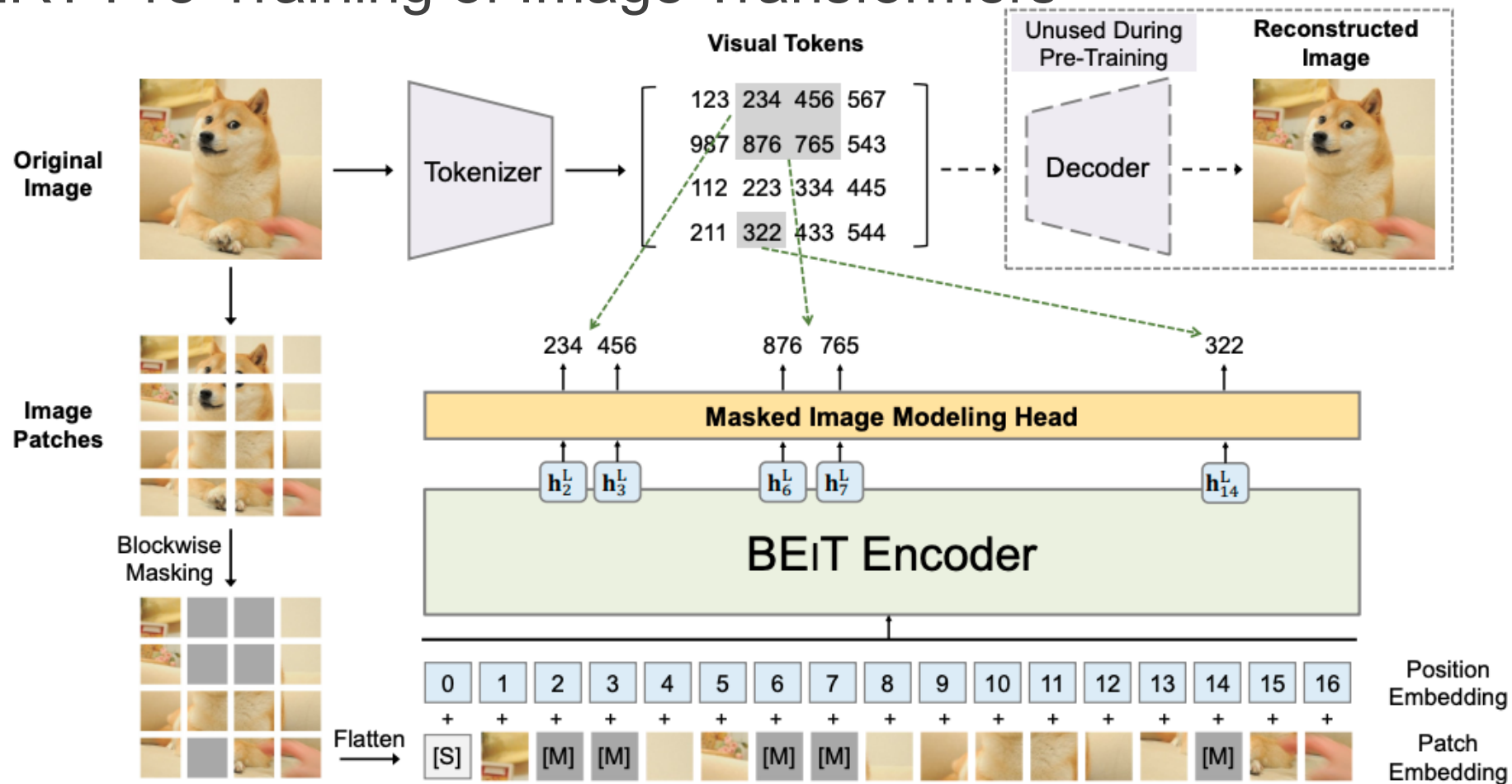
Vision Transformer (ViT)



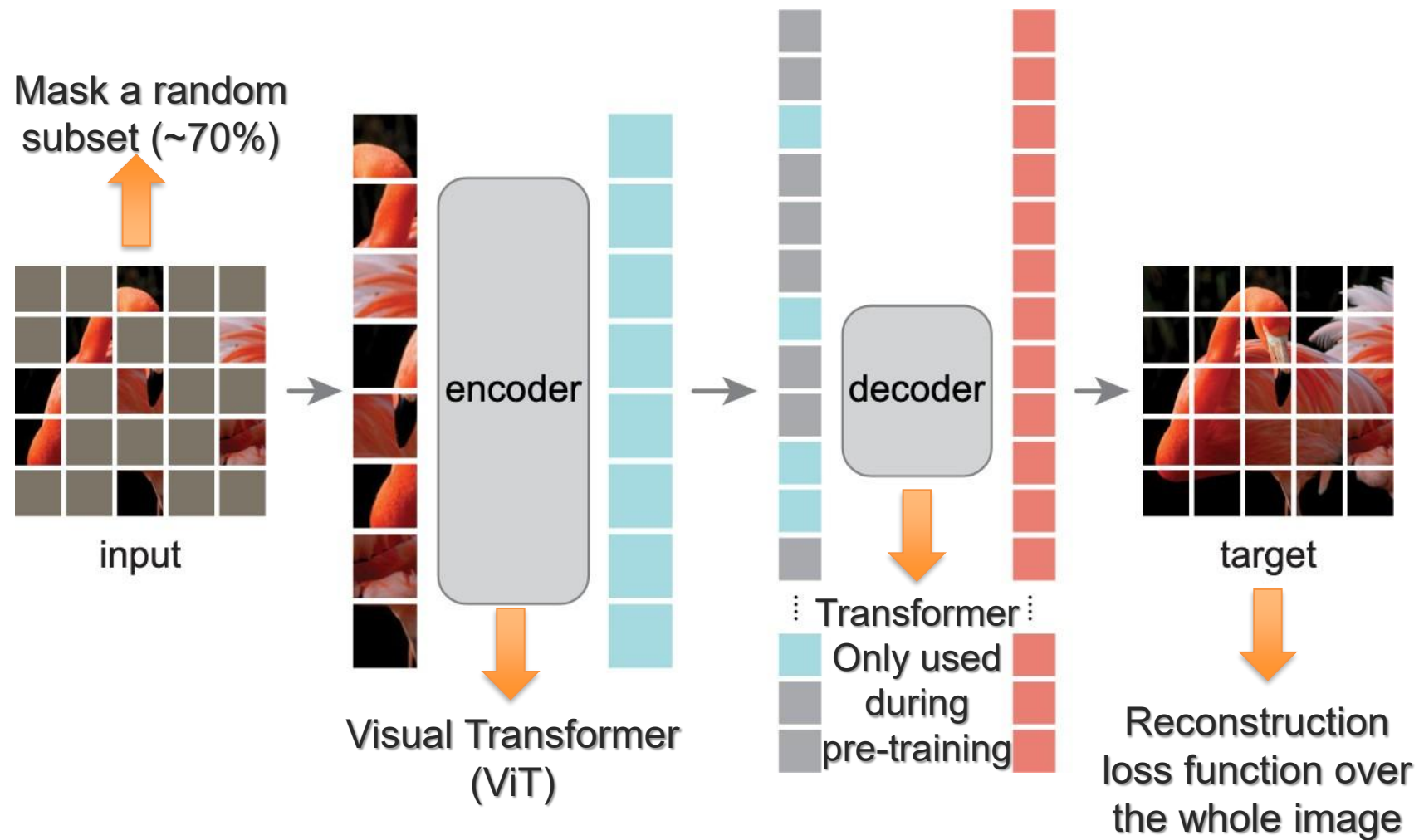
Dosovitskiy, Alexey, et al. "An image is worth 16x16 words: Transformers for image recognition at scale." *arXiv* (2020).

Visual Tokens

BeIT: BERT Pre-Training of Image Transformers

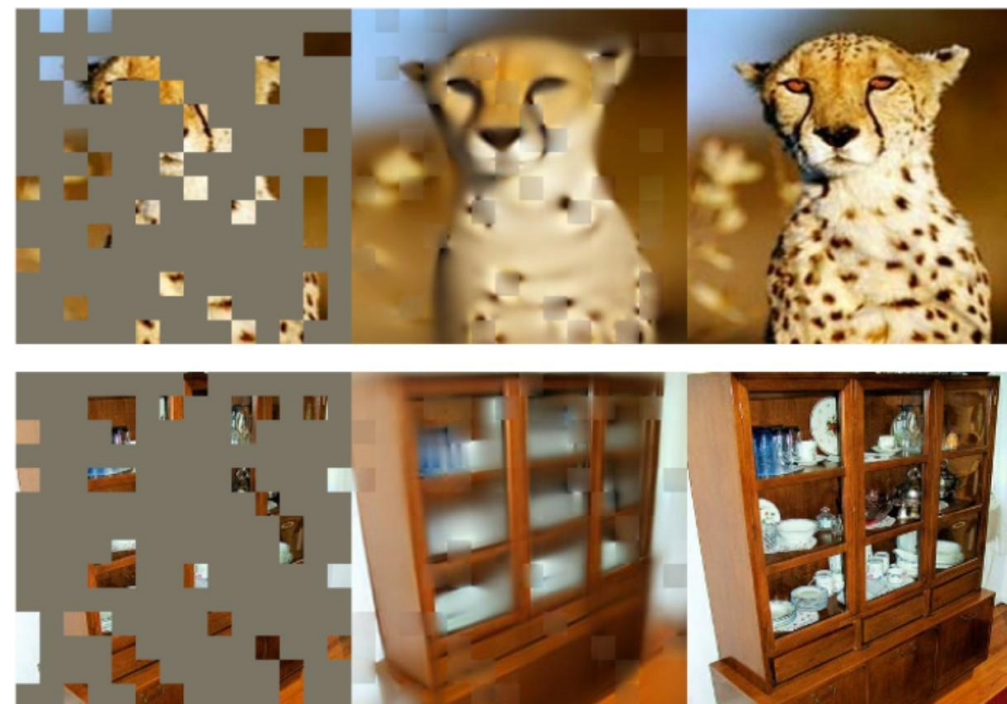
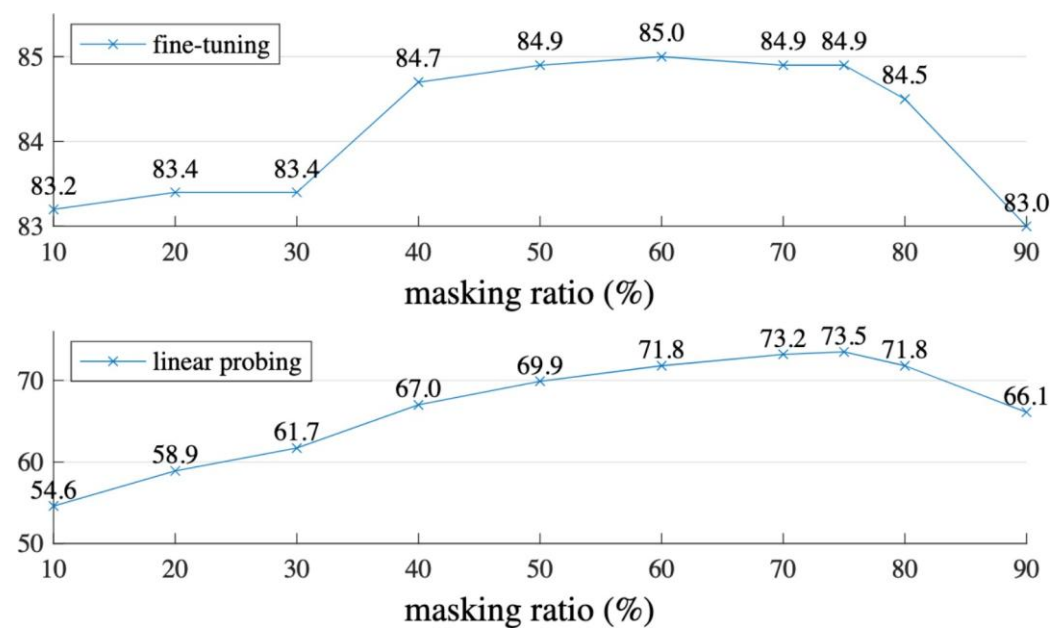


Masked Auto-Encoder (MAE)



He et al., Masked Autoencoders Are Scalable Vision Learners, CVPR 2022

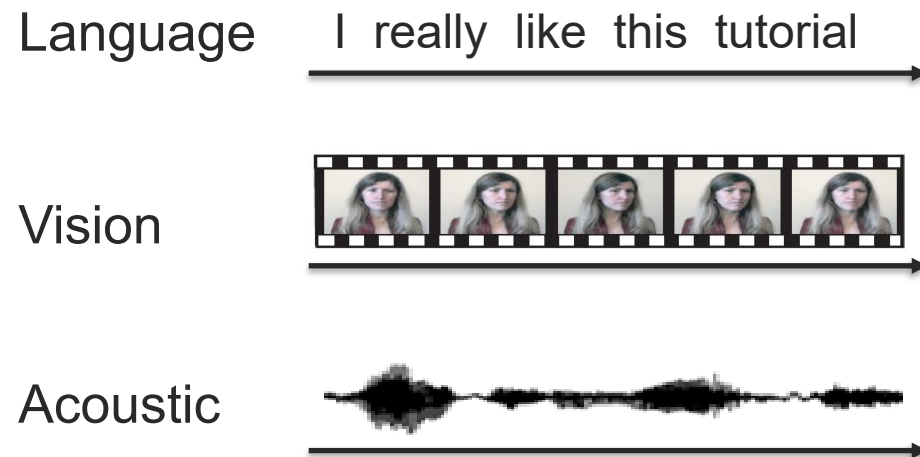
Masked Auto-Encoder (MAE)



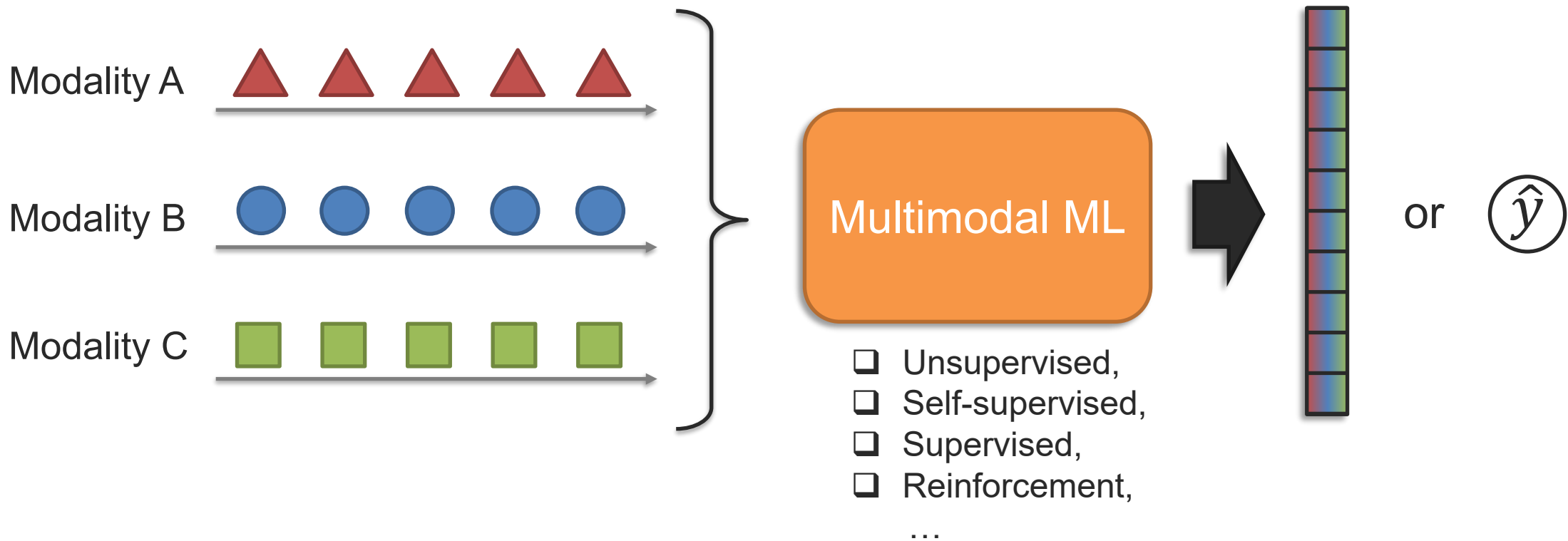
He et al., Masked Autoencoders Are Scalable Vision Learners, CVPR 2022

Multimodal Representation

Multimodal Machine Learning



Multimodal Machine Learning

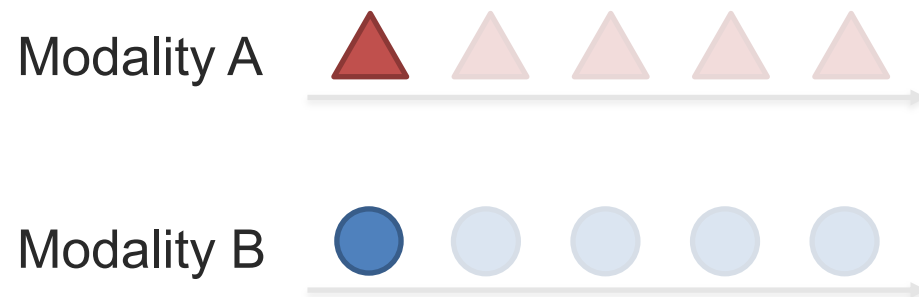


Challenge 1: Representation

Definition: Learning representations that reflect cross-modal interactions between individual elements, across different modalities

➡ This is a core building block for most multimodal modeling problems!

Individual elements:



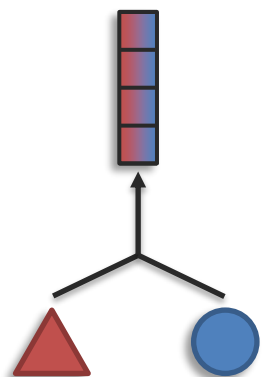
*It can be seen as a “local” representation
or
representation using holistic features*

Challenge 1: Representation

Definition: Learning representations that reflect cross-modal interactions between individual elements, across different modalities

Sub-challenges:

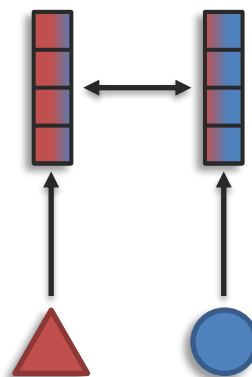
Fusion



modalities $\gt$ # representations

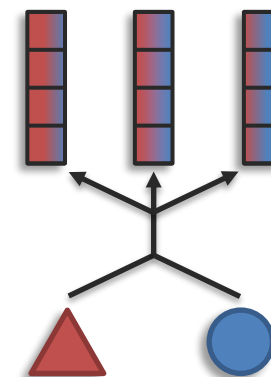
Today

Coordination



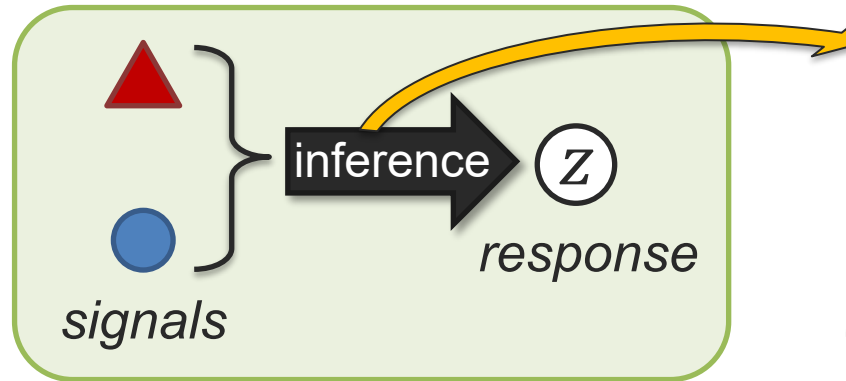
modalities $=$ # representations

Fission



modalities $\lt$ # representations

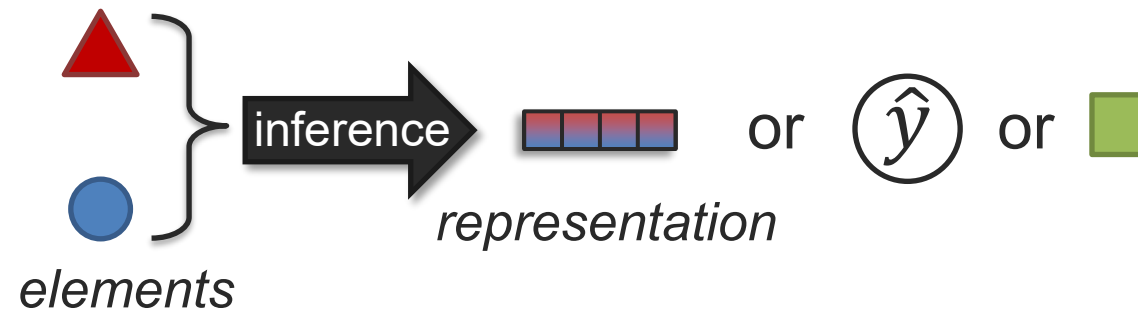
Cross-modal Interactions



Interactions happen during inference!

“Inference” examples:

- Representation fusion
- Prediction task
- Modality translation



Interacting Modalities

Interactions: How multimodal information changes when modalities are combined for a response.

*Is this
indoors?*



*A teacup on the right of a
laptop in a clean room.*

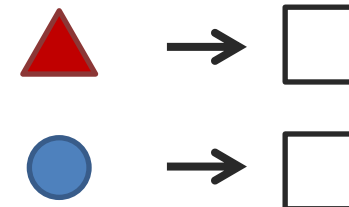


Yes!



Yes!

Redundant



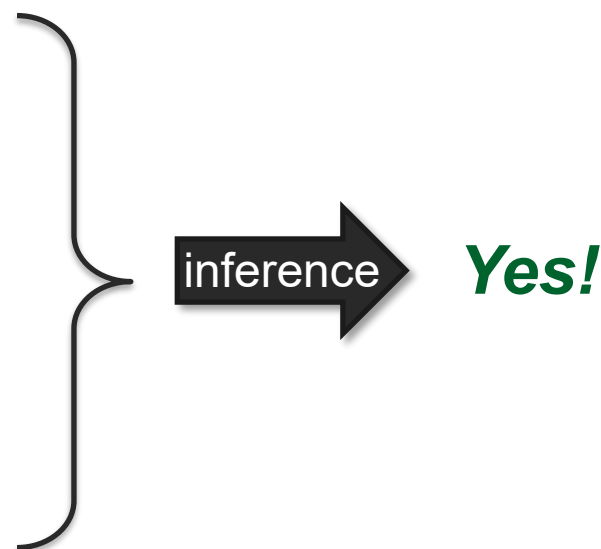
Interacting Modalities

Interactions: How multimodal information changes when modalities are combined for a response.

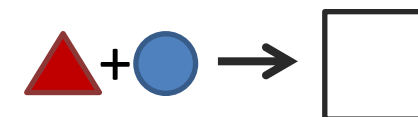
*Is this
indoors?*



*A teacup on the right of a
laptop in a clean room.*



Redundant



Enhancement

Interacting Modalities

Interactions: How multimodal information changes when modalities are combined for a response.

*Is this a
living
room?*



*A teacup on the right of a
laptop in a clean room.*

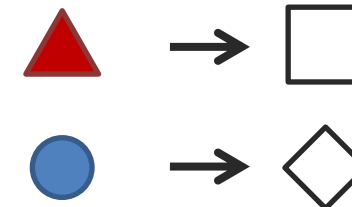


Yes!



*No, probably
study room.*

Non-redundant



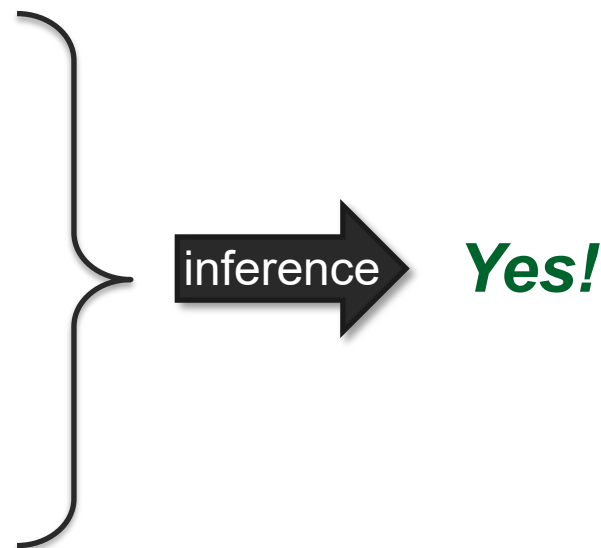
Interacting Modalities

Interactions: How multimodal information changes when modalities are combined for a response.

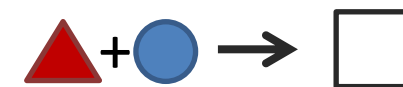
*Is this a
living
room?*



*A teacup on the right of a
laptop in a clean room.*



Non-redundant



Dominance

Interacting Modalities

Interactions: How multimodal information changes when modalities are combined for a response.

*Should I
work here?*



*A teacup on the right of a
laptop in a clean room.*

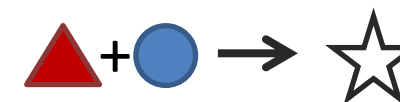


*Maybe? Comfy
sofa but table's
too small.*



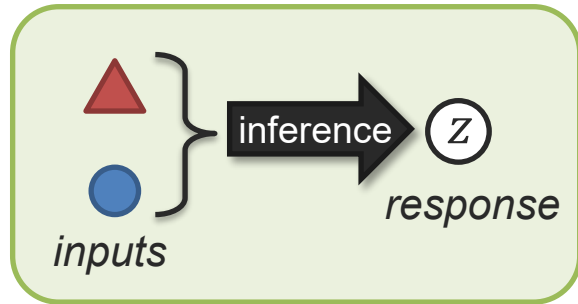
*Maybe? Clean
and there's tea.*

Non-redundant



Emergence

Taxonomy of Interaction Responses – A Behavioral Science View



Multimodal Communication



Redundancy

signal response

a → □

b → □

signal response

a+b → □

a+b → □

Equivalence

Enhancement

Nonredundancy

a → □

b → ○

a+b → □ and ○

Independence

a+b → □

Dominance

a+b → □ (or □)

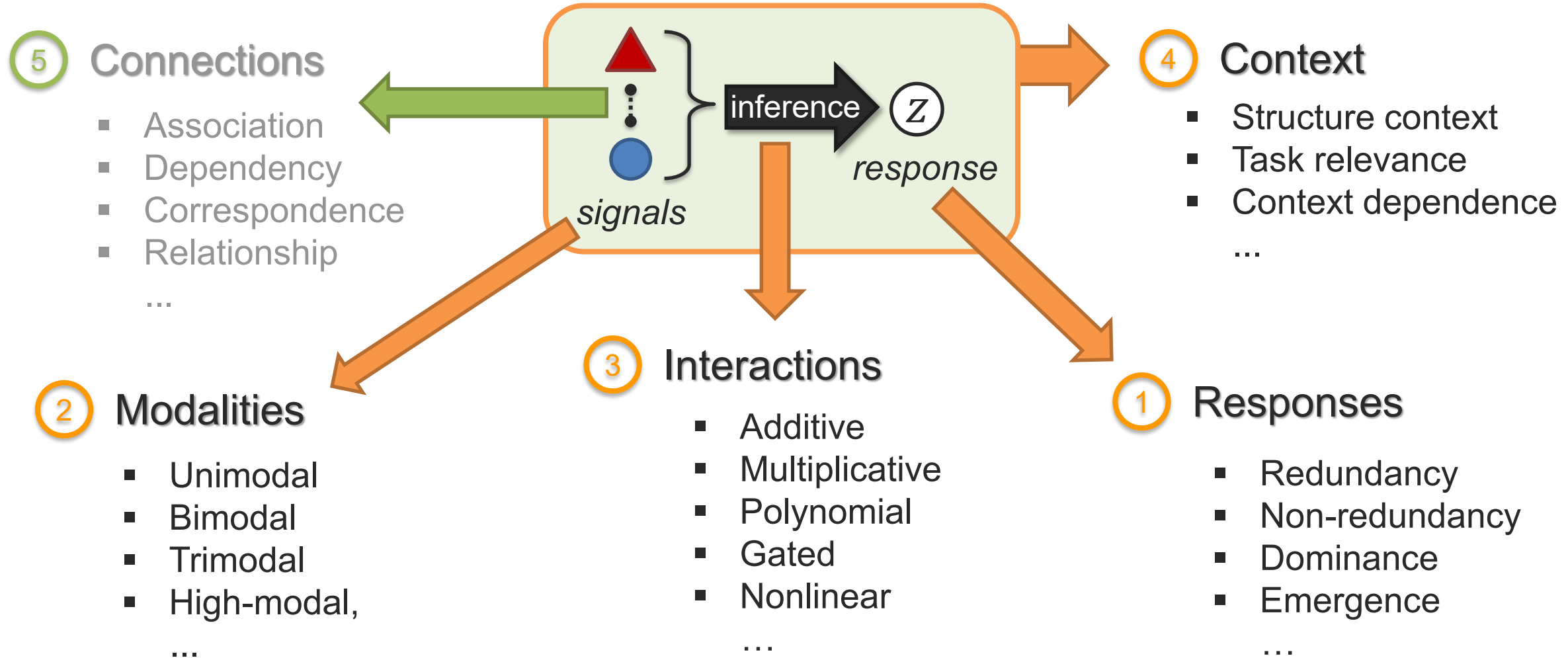
Modulation

a+b → △

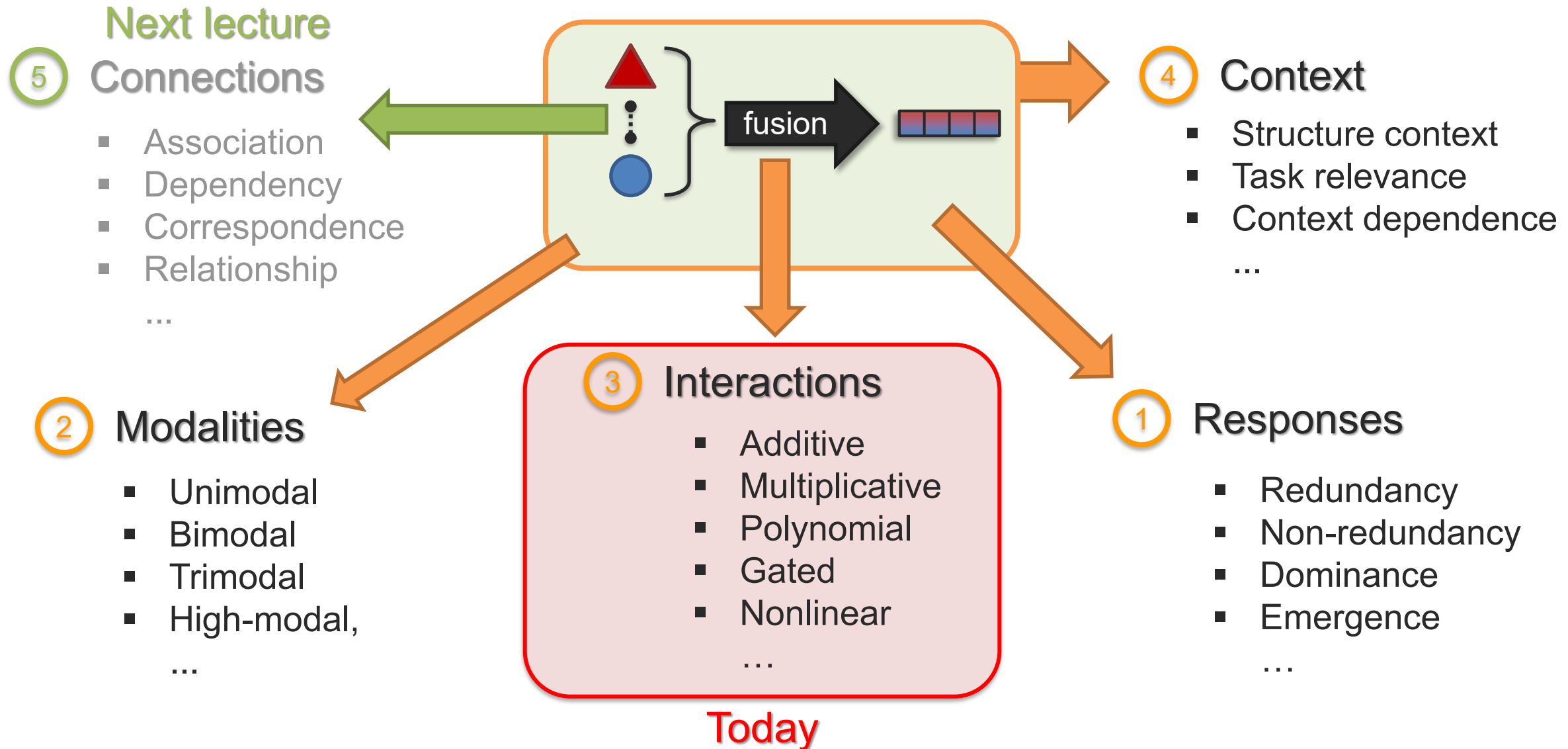
Emergence

Partan and Marler (2005). *Issues in the classification of multimodal communication signals*. American Naturalist, 166(2)

Cross-modal Interactions – A Taxonomy

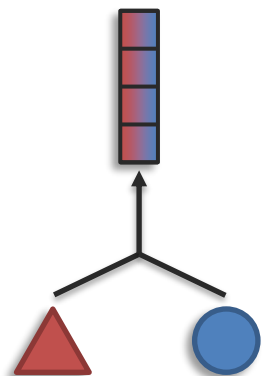


Cross-modal Interactions – Representation Fusion



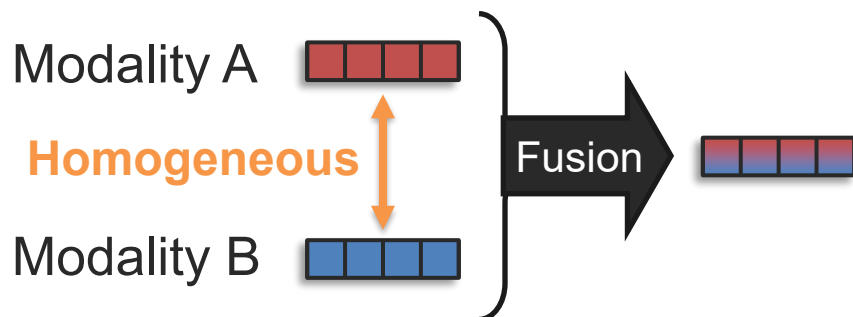
Representation Fusion

Sub-Challenge 1a: Representation Fusion

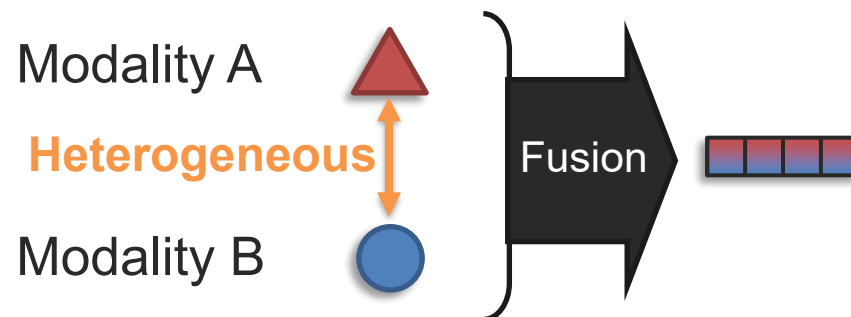


Definition: Learn a joint representation that models cross-modal interactions between individual elements of different modalities

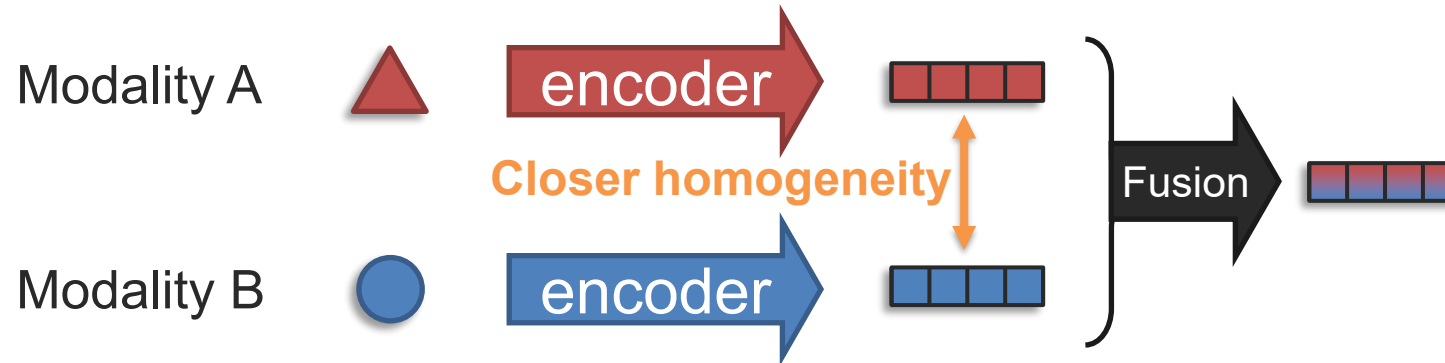
Basic fusion:



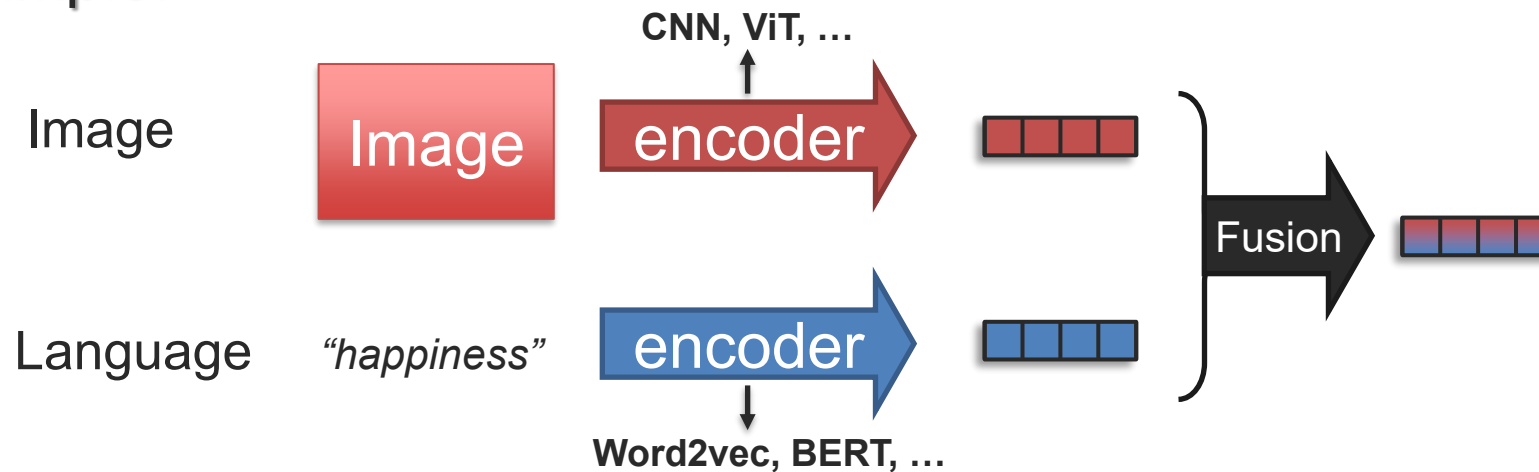
Raw-modality fusion:



Fusion with Unimodal Encoders



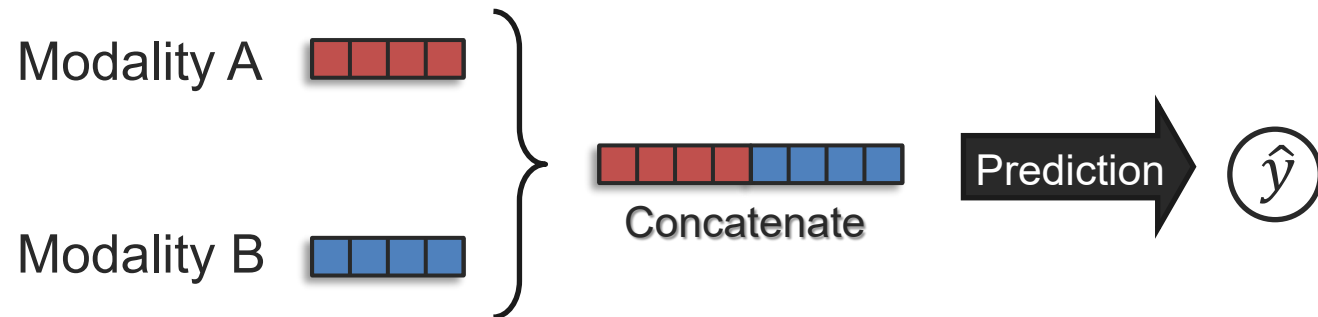
Example:



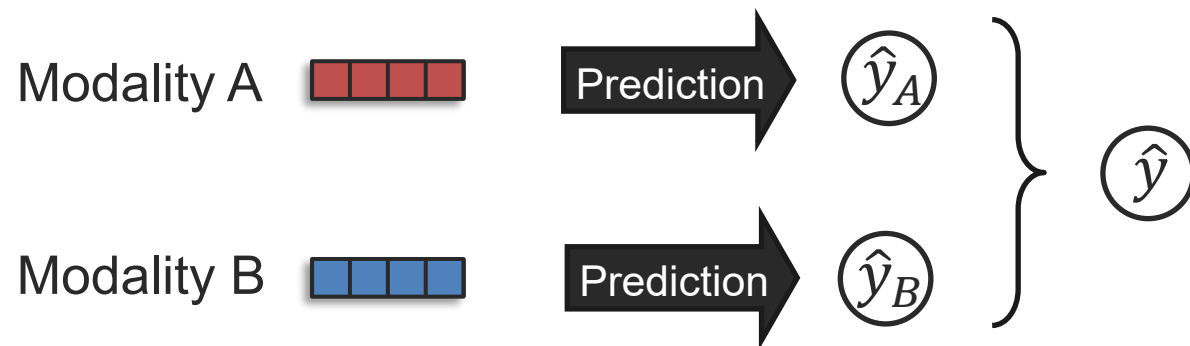
➡ Unimodal encoders can be jointly learned with fusion network, or pre-trained

Early and Late Fusion – A historical View

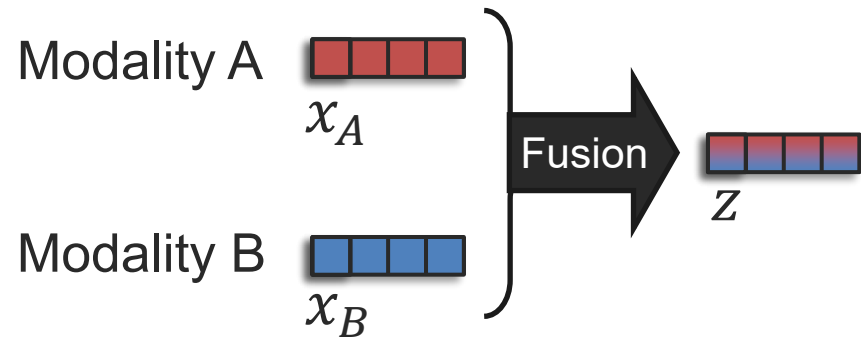
Early fusion:



Late fusion:



Basic Concepts for Representation Fusion (aka, Basic Fusion)



Goal: Model *cross-modal interactions* between the multimodal elements

→ Let's study the **univariate case first**
↳ (only 1-dimensional features)

Linear regression:

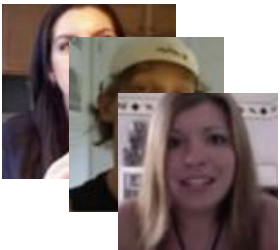
$$Z = w_0 + w_1 x_A + w_2 x_B + w_3 (x_A \times x_B) + \epsilon$$

intercept (bias term) Additive terms Multiplicative term error (residual term)

Linear Regression

Linear regression is used to test research hypotheses, over a whole dataset

300 book reviews



y : audience score

x_A : percentage of smiling

x_B : professional status
(0=non-critic, 1=critic)

H1: Does smiling reveal what the audience score was?

H2: Does the effect of smiling depend on professional status?

Linear regression:

$$y = w_0 + w_1 x_A + w_2 x_B + w_3 (x_A \times x_B) + \epsilon$$

Diagram illustrating the components of the linear regression equation:

- w_0 : intercept (bias term)
- $w_1 x_A + w_2 x_B$: Additive terms
- $w_3 (x_A \times x_B)$: Multiplicative term
- ϵ : error (residual term)

w_0 : average score when x_A and x_B are zero

w_1 : effect from x_A variable only

w_2 : effect from x_B variable only

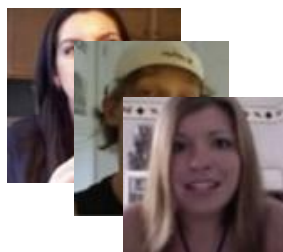
w_3 : effect from x_A and x_B interaction only

ϵ : residual not modeled by w_0 , w_1 , w_2 or w_3

Linear Regression

Linear regression is used to test research hypotheses, over a whole dataset

300 book reviews



y : audience score

x_A : percentage of smiling

x_B : professional status
(0=non-critic, 1=critic)

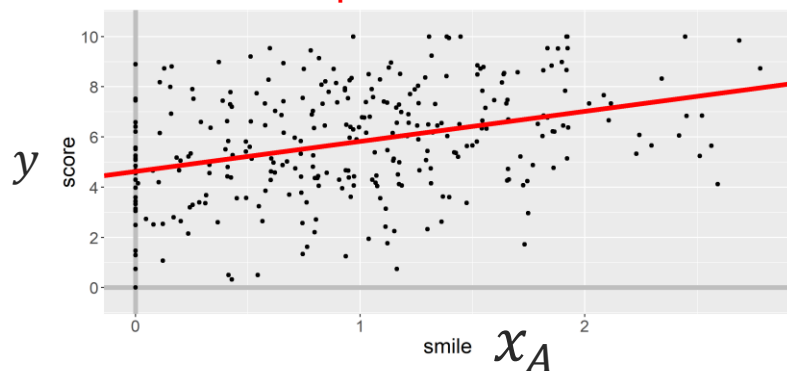
H1: Does smiling reveal what the audience score was?

H2: Does the effect of smiling depend on professional status?

Linear regression:

$$Z = w_0 + \boxed{w_1} x_A + \epsilon$$

slope



Confidence interval: “95% confident that w parameter is contained within this interval”

	Estimate	95% CI
w_0	4.63	[4.20, 5.06]
w_1	1.20	[0.83, 1.57]

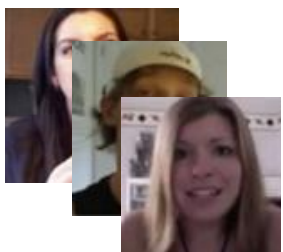
p-values would be another way to test hypothesis

Confidence interval does not contain 0, so effect is significant

Linear Regression

Linear regression is used to test research hypotheses, over a whole dataset

300 book reviews



y : audience score

x_A : percentage of smiling

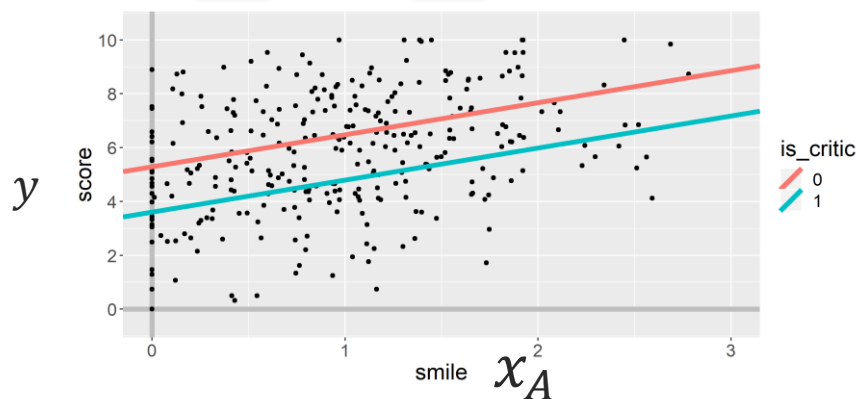
x_B : professional status
(0=non-critic, 1=critic)

H1: Does smiling reveal what the audience score was?

H2: Does the effect of smiling depend on professional status?

Linear regression:

$$Z = w_0 + w_1 x_A + w_2 x_B + \epsilon$$



	Estimate	95% CI
w_0	5.29	[4.86, 5.73]
w_1	1.19	[0.85, 1.53]
w_2	-1.69	[-2.14, -1.24]

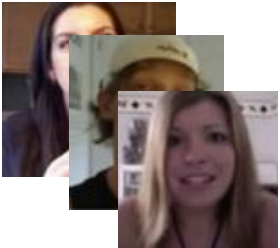
➡ Positive effect

➡ Negative effect

Linear Regression

Linear regression is used to test research hypotheses, over a whole dataset

300 book reviews



y : audience score

x_A : percentage of smiling

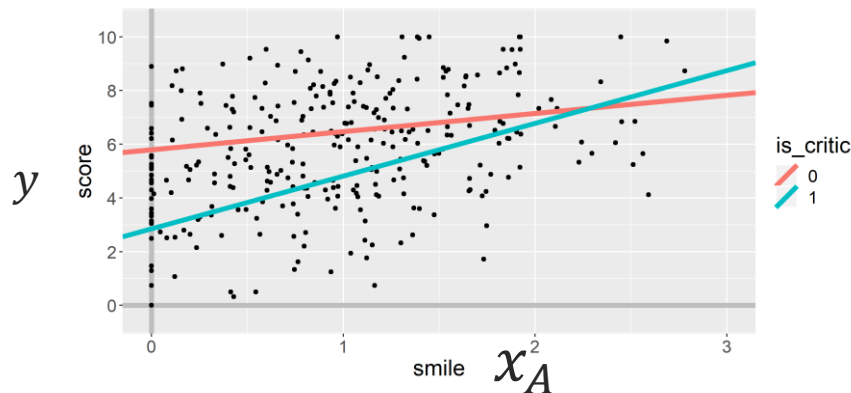
x_B : professional status
(0=non-critic, 1=critic)

H1: Does smiling reveal what the audience score was?

H2: Does the effect of smiling depend on professional status?

Linear regression:

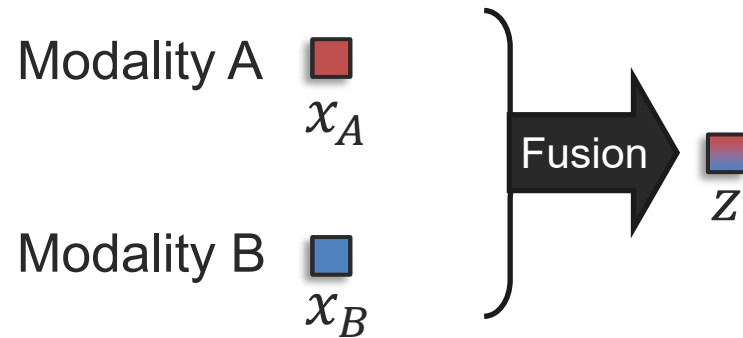
$$Z = w_0 + \boxed{w_1}x_A + \boxed{w_2}x_B + \boxed{w_3}(x_A \times x_b) + \epsilon$$



	Estimate	95% CI
w_0	5.79	[5.29, 6.29]
w_1	0.68	[0.25, 1.11]
w_2	-2.94	[-3.73, -2.15]
w_3	1.29	[0.61, 1.97]

➡ **Multiplicative interaction!**

Basic Concepts for Representation Fusion (aka, Basic Fusion)



Goal: Model *cross-modal interactions* between the multimodal elements

→ Let's study the **univariate case first**
↳ (only 1-dimensional features)

Linear regression:

$$Z = w_0 + w_1 x_A + w_2 x_B + w_3 (x_A \times x_B) + \epsilon$$

intercept (bias term) Additive terms Multiplicative term error (residual term)

① Additive terms:

$$Z = w_1 x_A + w_2 x_B + \epsilon$$

② Multiplicative “interaction” term:

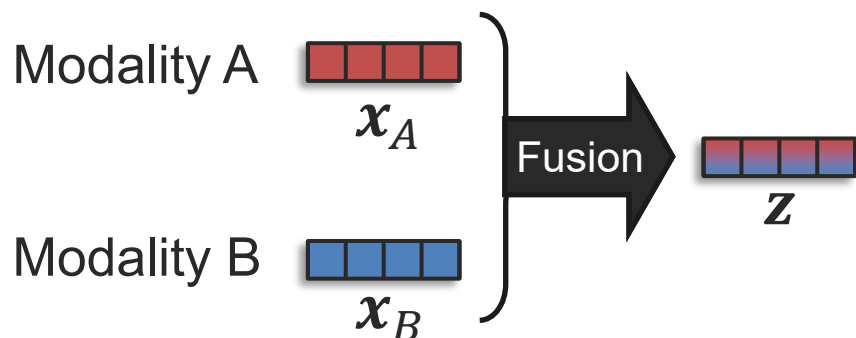
$$Z = w_3 (x_A \times x_B) + \epsilon$$

③ Additive and multiplicative terms:

$$Z = w_1 x_A + w_2 x_B + w_3 (x_A \times x_B) + \epsilon$$

Additive Fusion → Back to multivariate case!

↳ (multi-dimensional features)

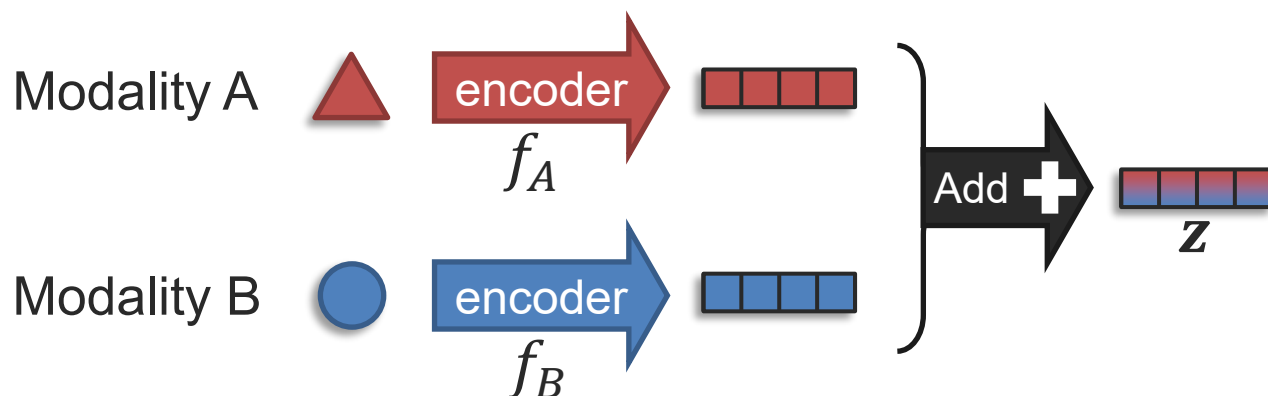


Additive fusion:

$$z = w_1 x_A + w_2 x_B$$

→ 1-layer neural network
can be seen as additive

With unimodal encoders:

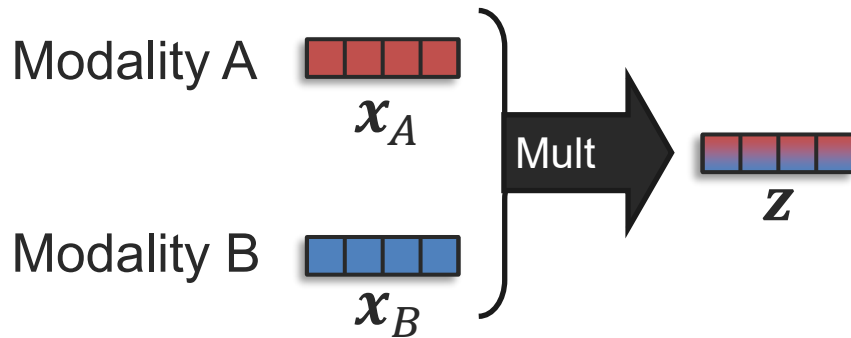


Additive fusion:

$$z = f_A(\triangle) + f_B(\circ)$$

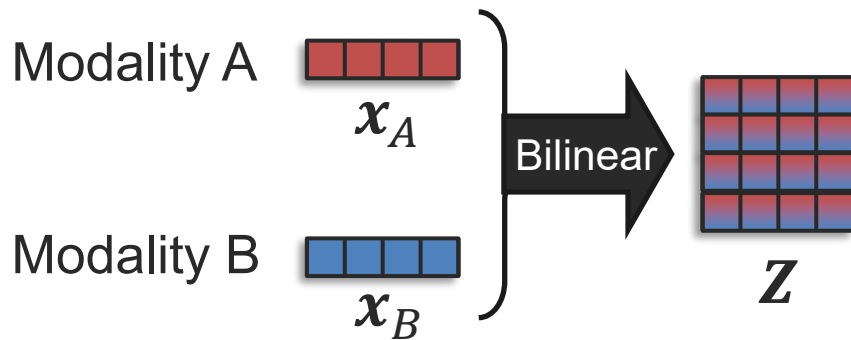
→ It could be seen as an
ensemble approach
(late fusion)

Multiplicative Fusion



Simple multiplicative fusion:

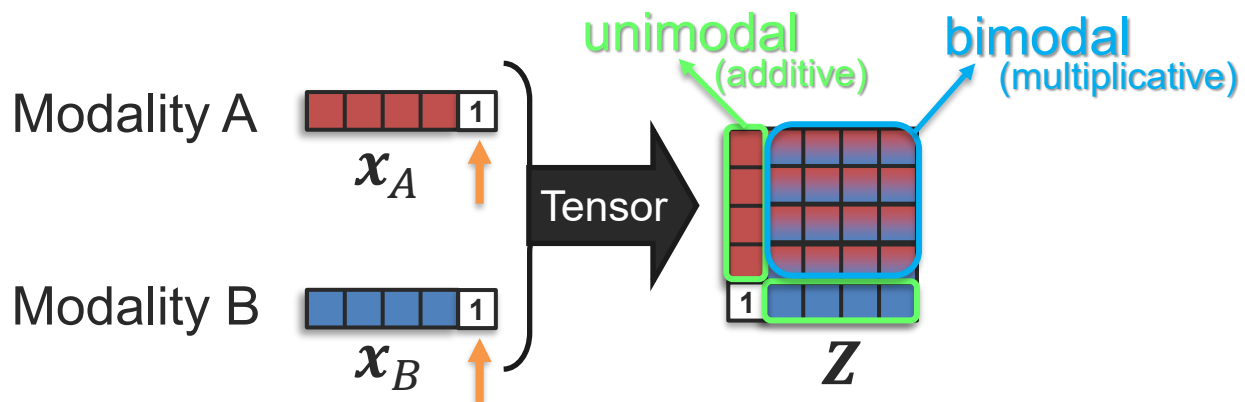
$$z = w(x_A \times x_B)$$



Bilinear Fusion:

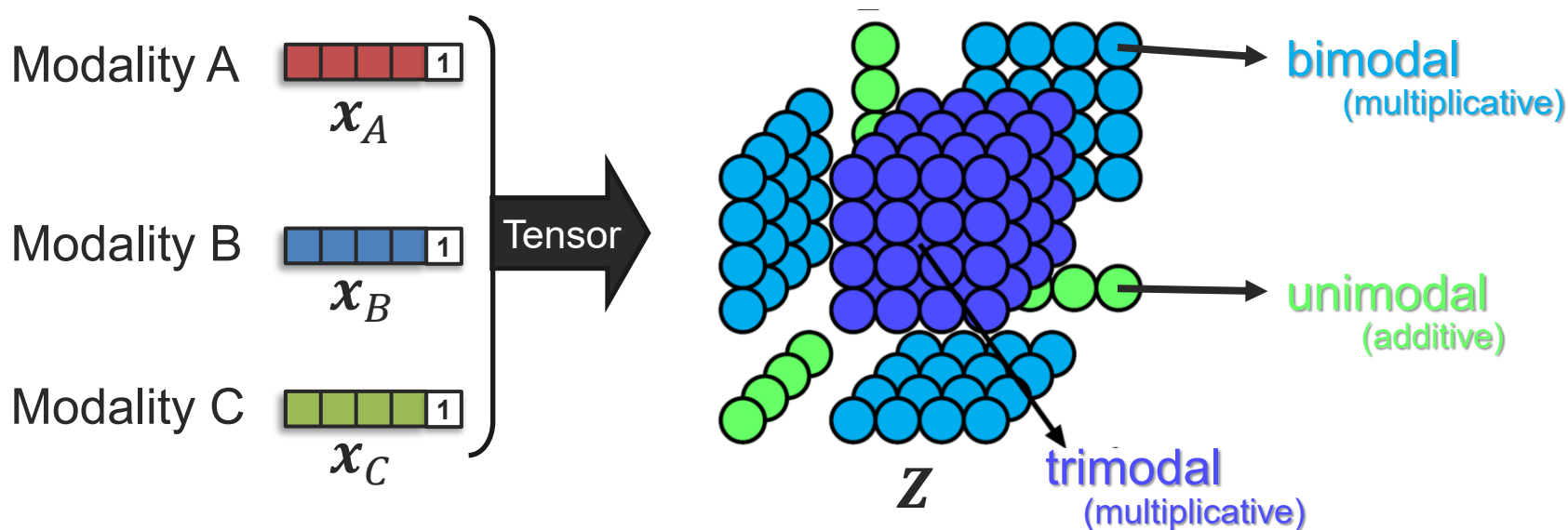
$$Z = W(x_A^T \cdot x_B)$$

Tensor Fusion



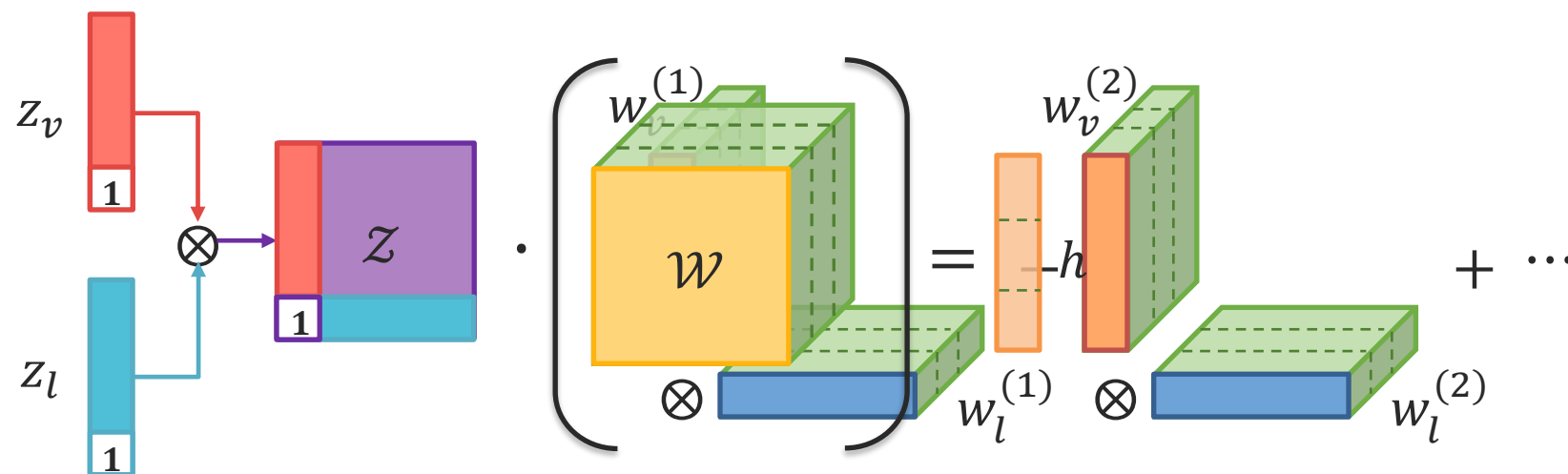
Tensor Fusion (bimodal):

$$\mathbf{Z} = \mathbf{w}([\mathbf{x}_A \ 1]^T \cdot [\mathbf{x}_B \ 1])$$

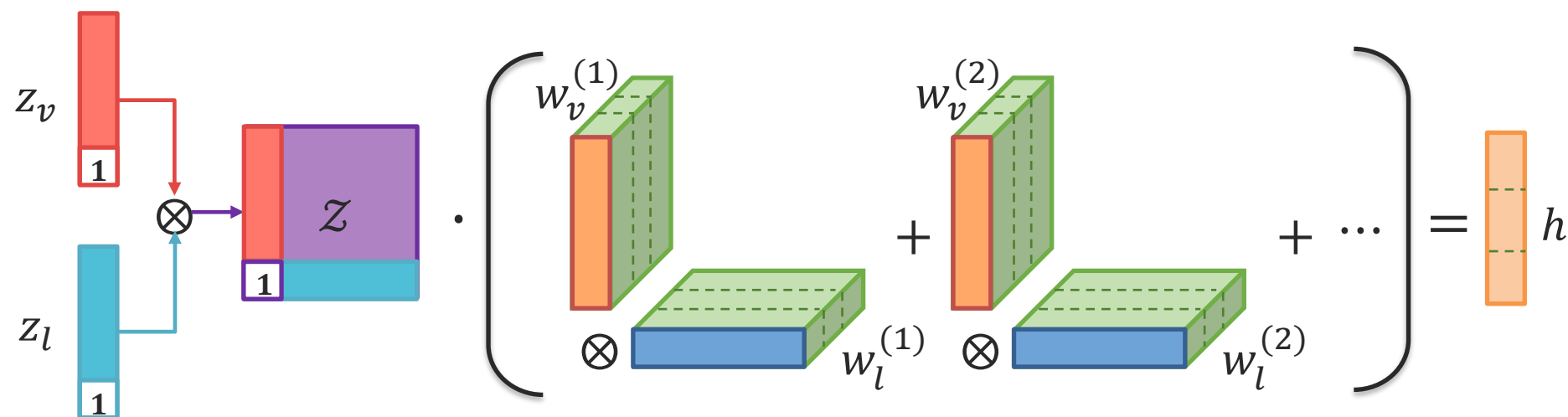


... but the weight matrix may end up quite large!

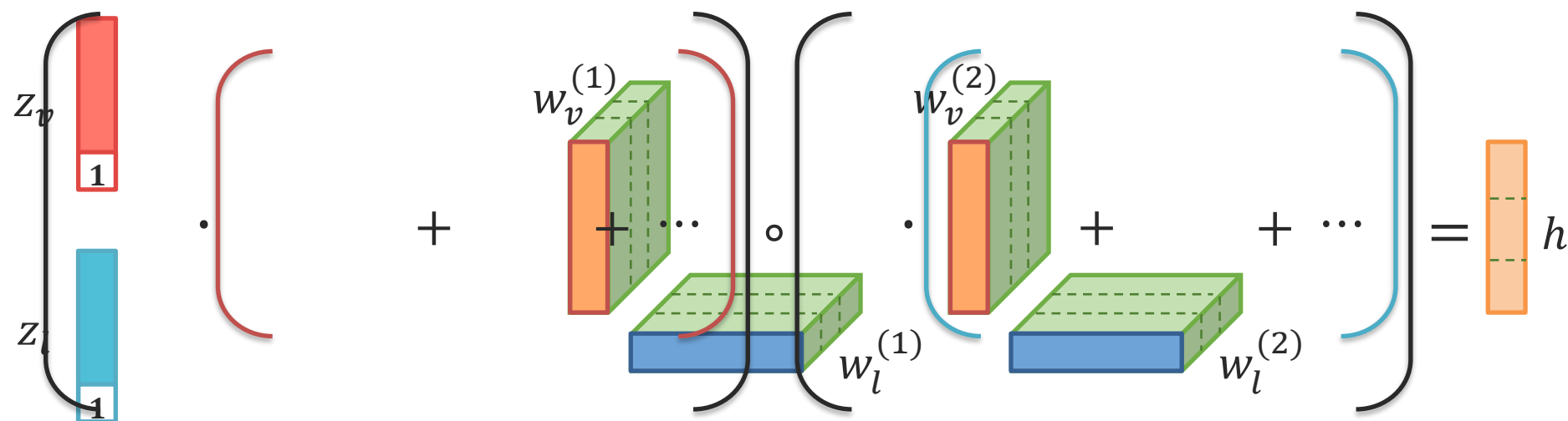
Low-rank Fusion



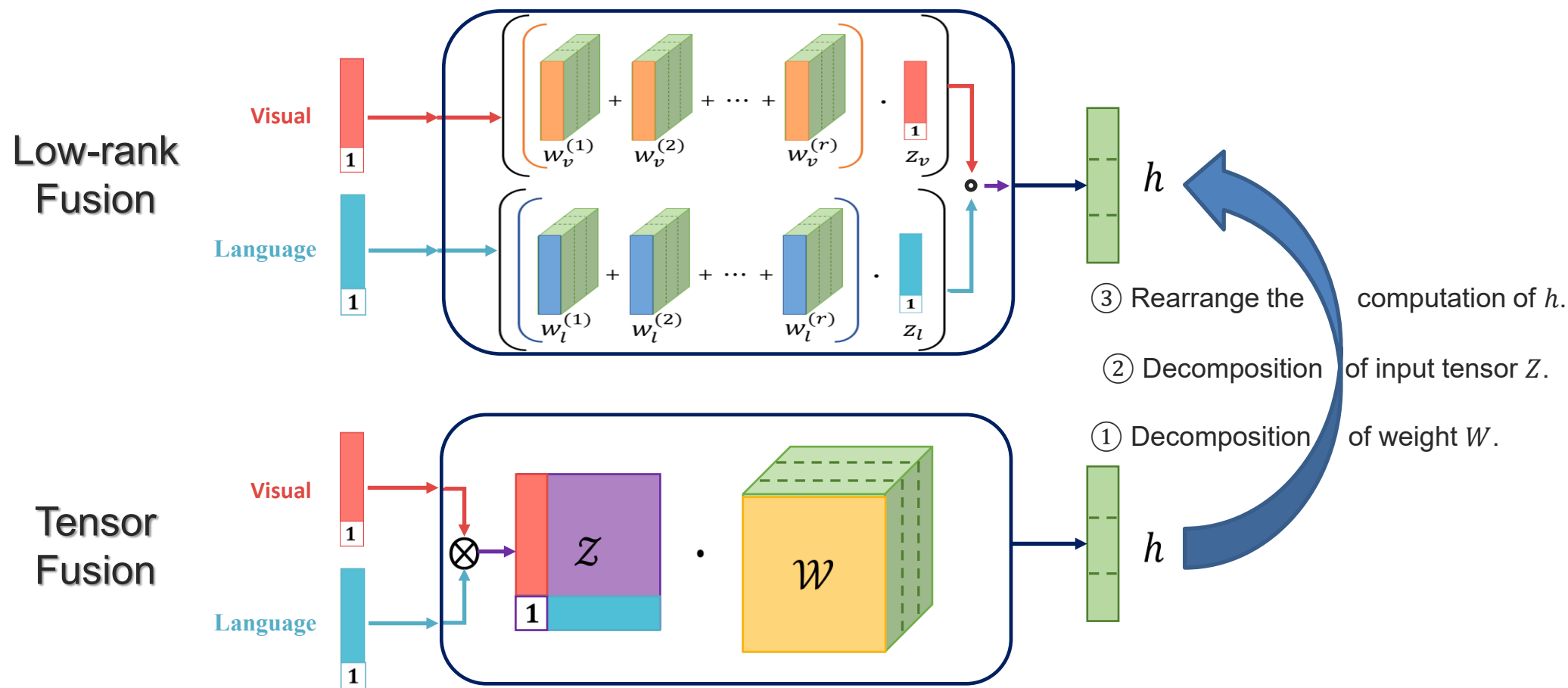
Low-rank Fusion



Low-rank Fusion

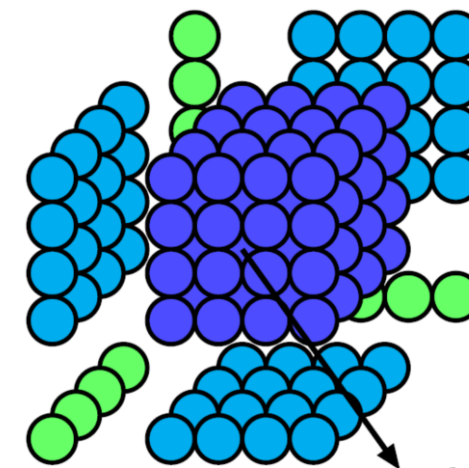


Low-rank Fusion

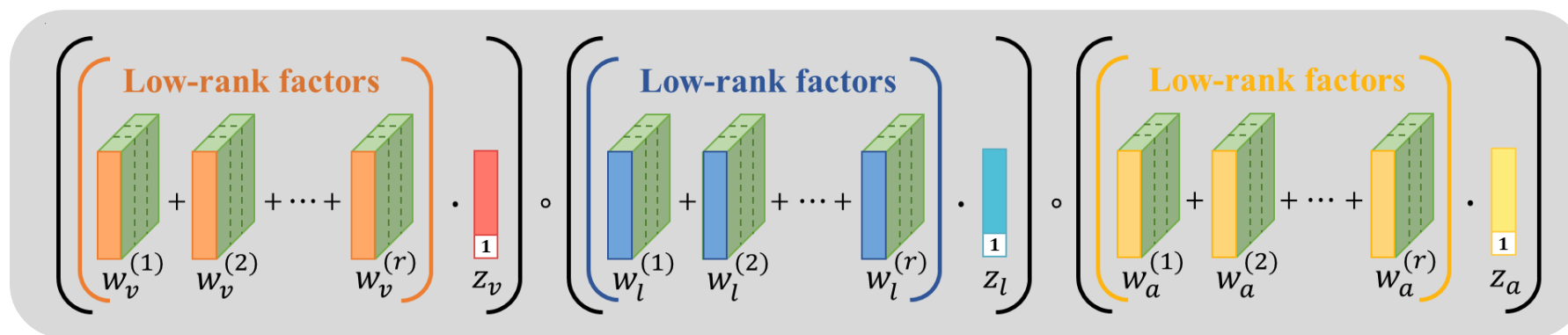


Low-rank Fusion with Trimodal Input

Tensor Fusion



Low-rank Fusion :



Canonical
Polyadic
Decomposition

Going Beyond Additive and Multiplicative Fusion

Additive interaction:

$$z = w_1 x_A + w_2 x_B$$

← First-order polynomial

Additive and multiplicative interaction:

$$z = w_1 x_A + w_2 x_B + w_3 (x_A \times x_B)$$

← Second-order polynomial

Trimodal fusion (e.g., tensor fusion):

$$z = \underbrace{w_1 x_A + w_2 x_B + w_3 x_C}_{\text{Unimodal terms (first-order)}} + \underbrace{w_4 (x_A \times x_C) + w_5 (x_A \times x_B) + w_6 (x_B \times x_C)}_{\text{Bimodal terms (second-order)}} + \underbrace{w_7 (x_A \times x_B \times x_C)}_{\text{Trimodal terms (third-order)}}$$

Can we add
higher-order
interaction terms?

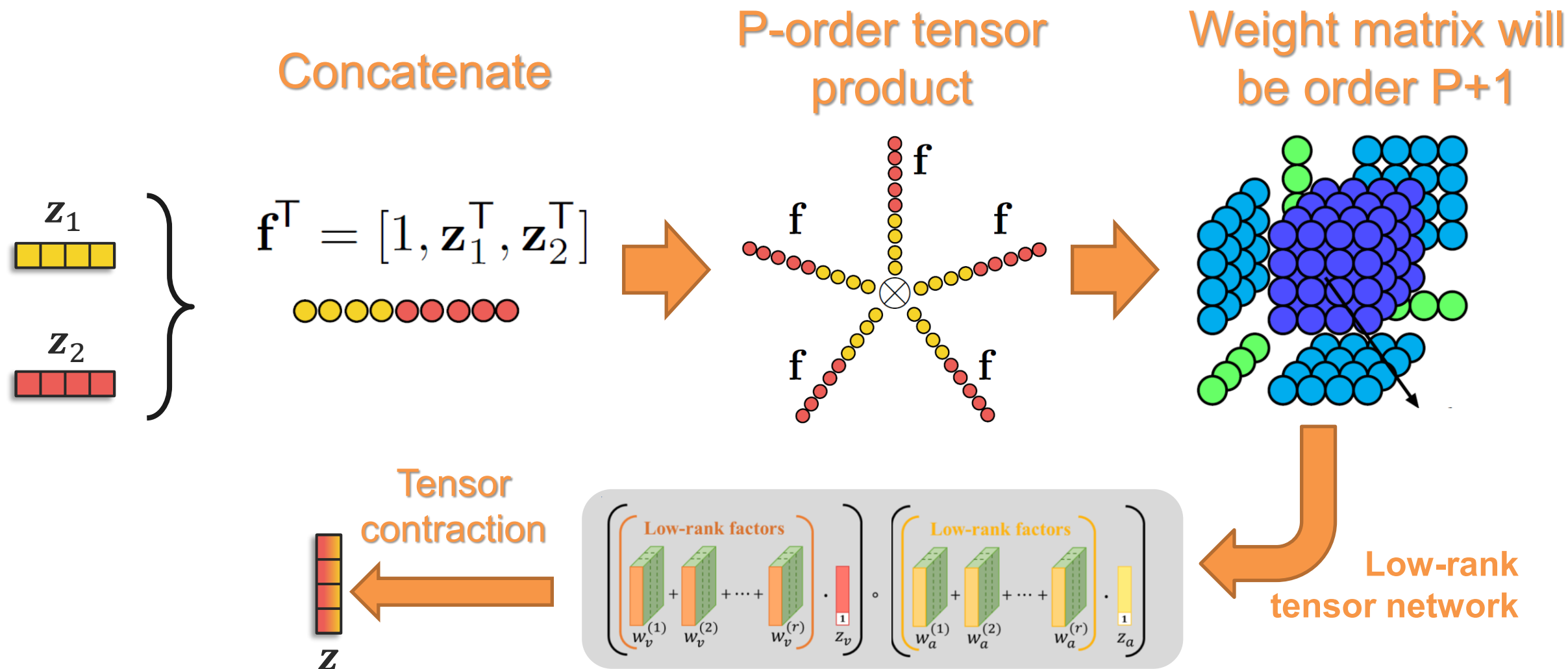
For example:

$$+w_8 (x_A^2 \times x_B^2 \times x_C^2)$$

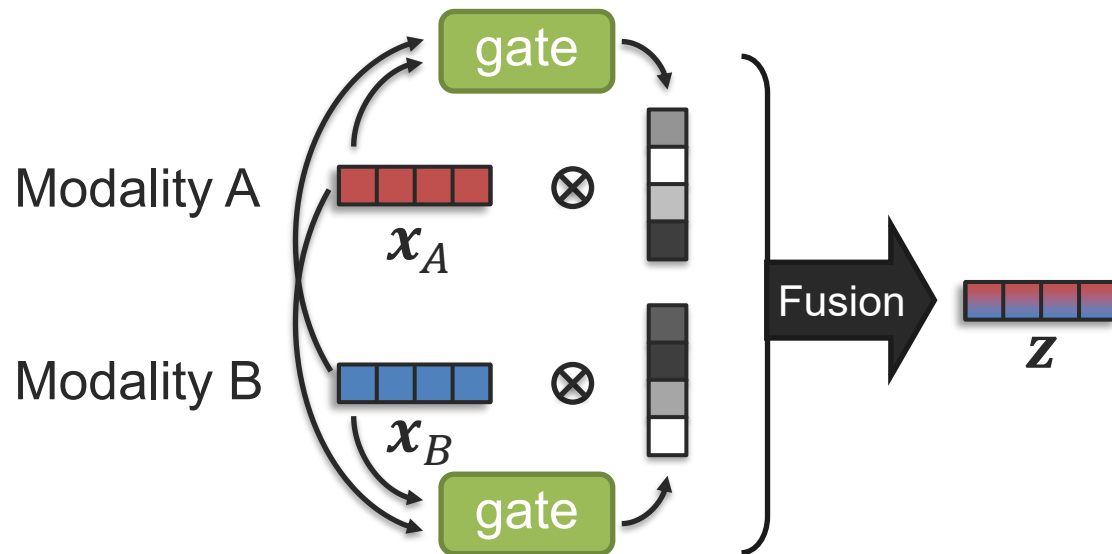
$$+w_9 (x_A^3 \times x_B)$$

$$+w_{10} (x_B^3 \times x_C^3)$$

High-Order Polynomial Fusion



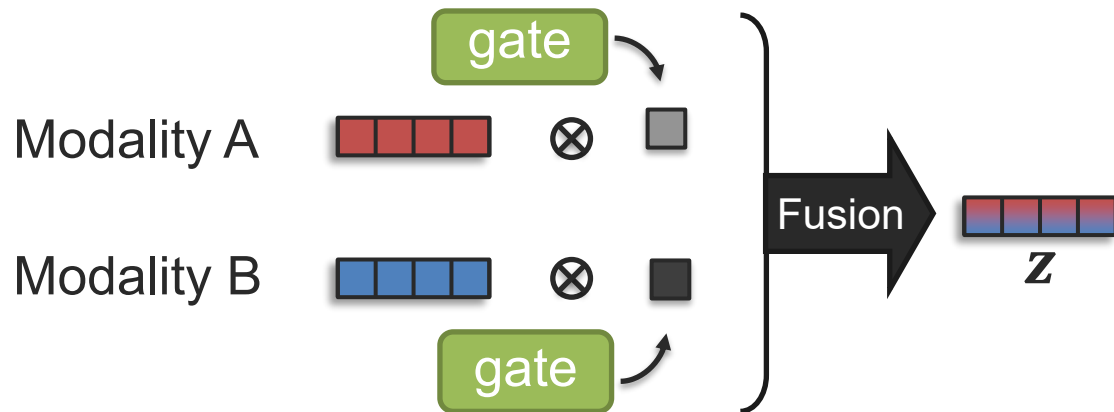
Gated Fusion



Example with additive fusion:

$$z = g_A(x_A, x_B) \cdot x_A + g_B(x_A, x_B) \cdot x_B$$

→ g_A and g_B can be seen as attention functions



→ Gating output can be one weight for the whole modality

Gating Module (aka, attention module)



What should it be?

Target modality []

Other modality []

All modality []



“Neural network designed to mask unwanted signal from propagating forward” (gating)

...or with a more positive view:

“Neural network designed to select preferable signal to move forward” (attention)

Soft attention



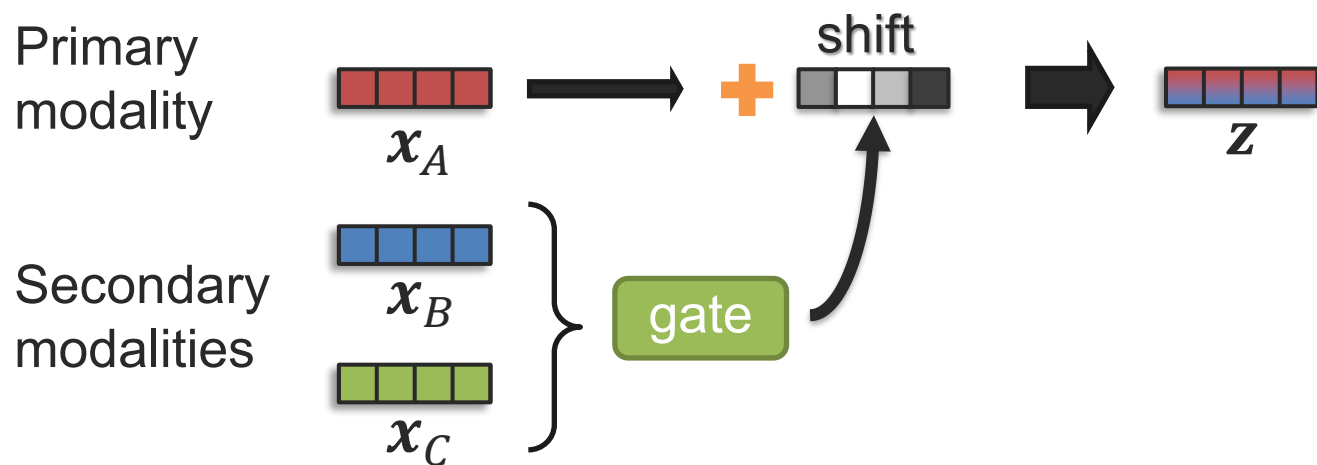
Easier to compute derivative (gradient)

Hard attention



Derivative is harder (e.g., use reinforcement learning)

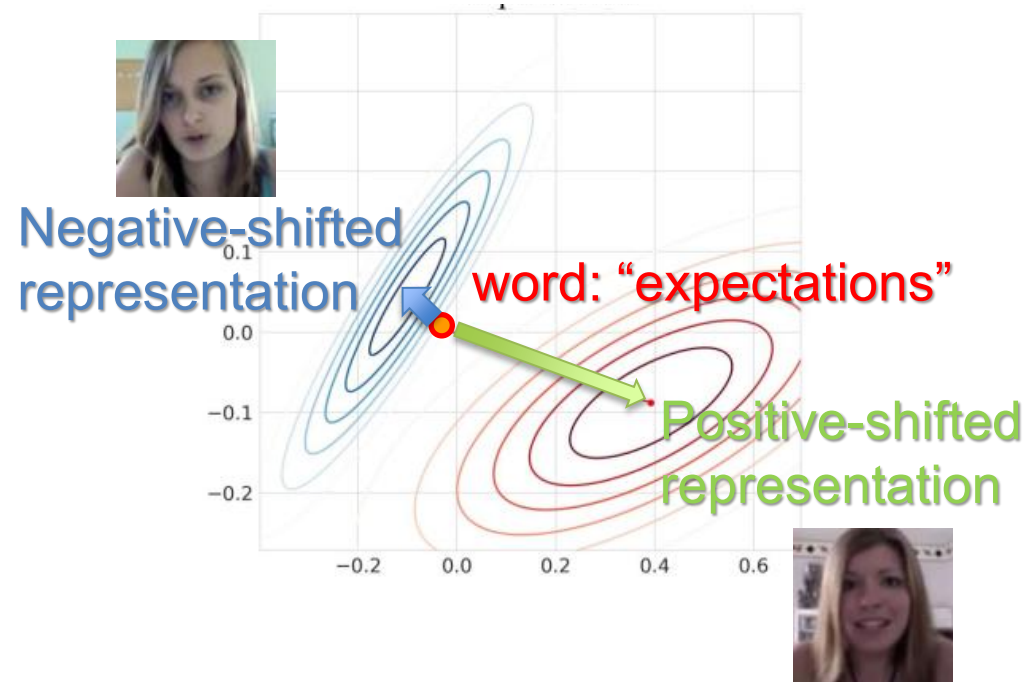
Modality-Shifting Fusion



Example with language modality:

Primary modality: language

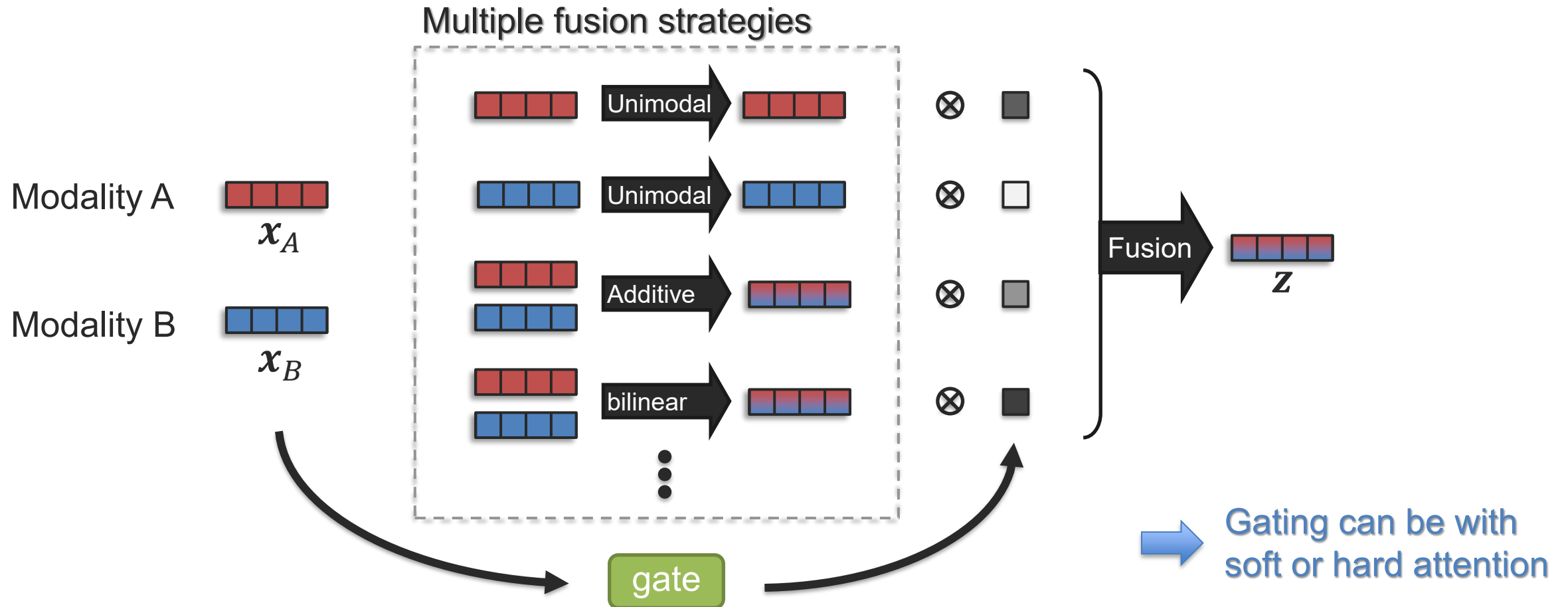
Secondary modalities: acoustic and visual



[Wang et al., Words Can Shift: Dynamically Adjusting Word Representations Using Nonverbal Behaviors, AAAI 2019]

[Rahman et al., Integrating Multimodal Information in Large Pretrained Transformers, ACL 2020]

Mixture of Fusions



[Zadeh et al., Multimodal Language Analysis in the Wild: CMU-MOSEI Dataset and Interpretable Dynamic Fusion Graph, ACL 2018]

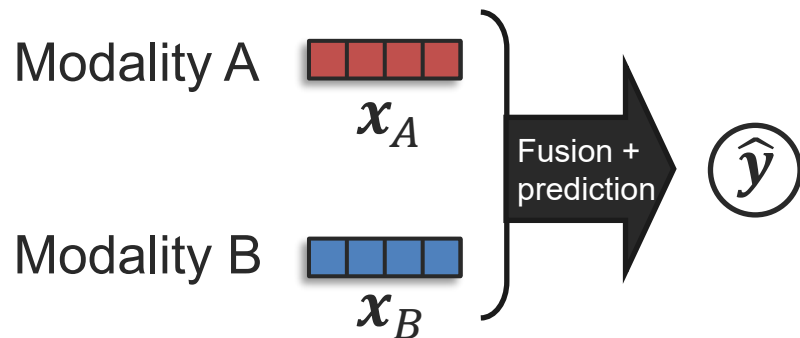
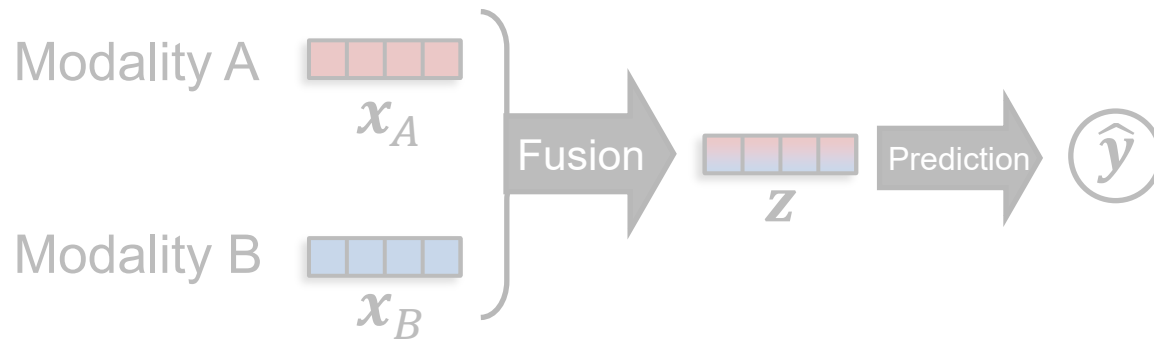
[Xu et al., MUFASA: Multimodal Fusion Architecture Search for Electronic Health Records, AAAI 2021]

Nonlinear Fusion

Nonlinear fusion:

$$\hat{y} = f(x_A, x_B) \in \mathbb{R}^d$$

where f could be a multi-layer perceptron or any nonlinear model

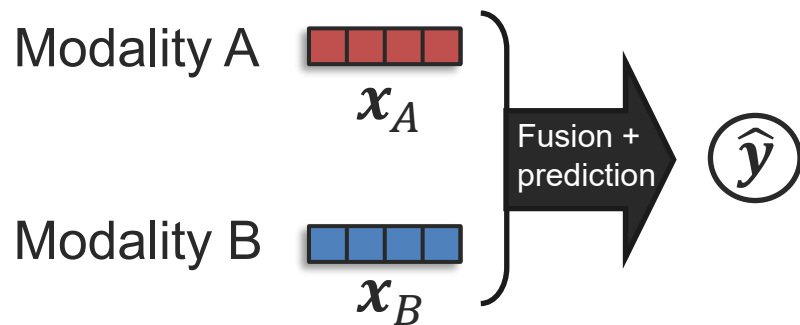


➡ This could be seen as *early fusion*:

$$\hat{y} = f([x_A, x_B])$$

... but will our neural network learn the nonlinear interactions?

Measuring Non-Additive Interactions



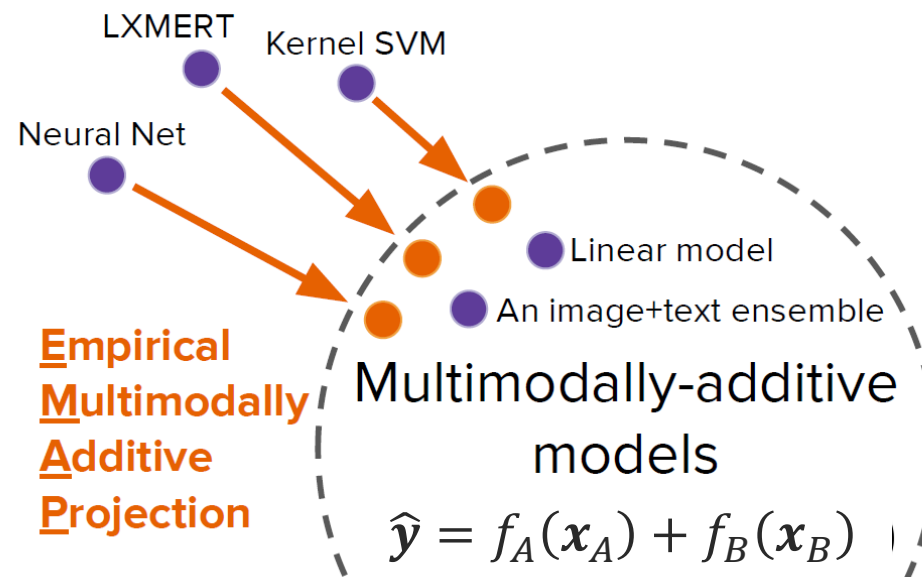
Nonlinear fusion:

$$\hat{\mathbf{y}} = f(\mathbf{x}_A, \mathbf{x}_B)$$

Projection?

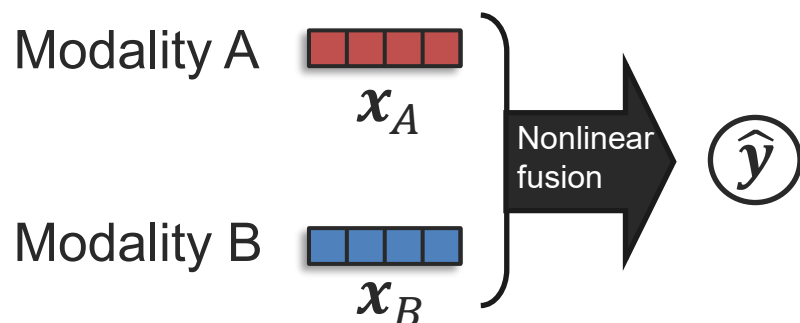
Additive fusion:

$$\hat{\mathbf{y}} = f_A(\mathbf{x}_A) + f_B(\mathbf{x}_B)$$



Hessel and Lee, Does my multimodal model learn cross-modal interactions? It's harder to tell than you might think!, EMNLP 2020 → introduced the EMAP method

Measuring Non-Additive Interactions



Nonlinear fusion:

$$\hat{\mathbf{y}} = f(\mathbf{x}_A, \mathbf{x}_B)$$

Projection?

Additive fusion:

$$\hat{\mathbf{y}}' = f_A(\mathbf{x}_A) + f_B(\mathbf{x}_B)$$

Projection from nonlinear to additive (using EMAP):

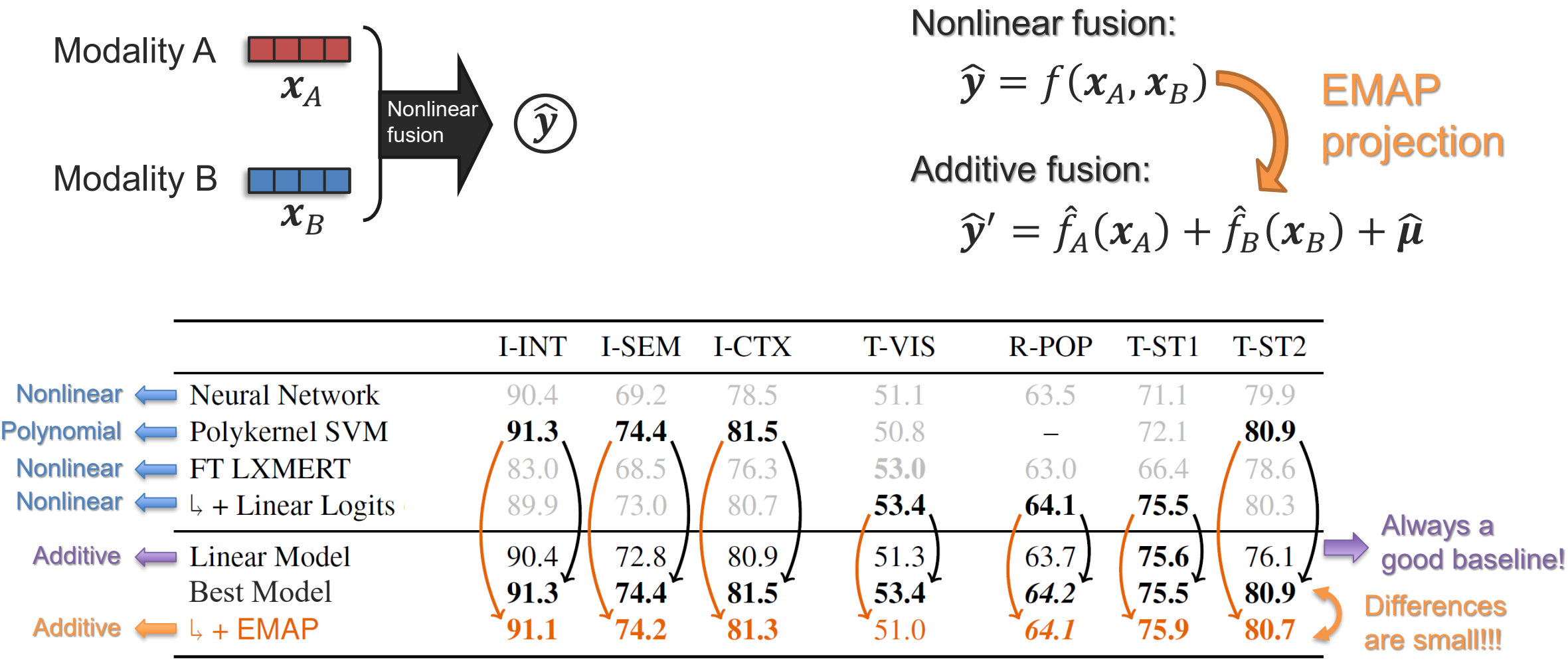
$$\tilde{f}(\mathbf{x}_A, \mathbf{x}_B) = \underbrace{\mathbb{E}_{\mathbf{x}_B} [f(\mathbf{x}_A, \mathbf{x}_B)]}_{f_A(\mathbf{x}_A)} + \underbrace{\mathbb{E}_{\mathbf{x}_A} [f(\mathbf{x}_A, \mathbf{x}_B)]}_{f_B(\mathbf{x}_B)}$$

Modality A Modality B

Additive fusion
(approximation)

[Hessel and Lee, Does my multimodal model learn cross-modal interactions? It's harder to tell than you might think!, EMNLP 2020]

Measuring Non-Additive Interactions



[Hessel and Lee, Does my multimodal model learn cross-modal interactions? It's harder to tell than you might think!, EMNLP 2020]

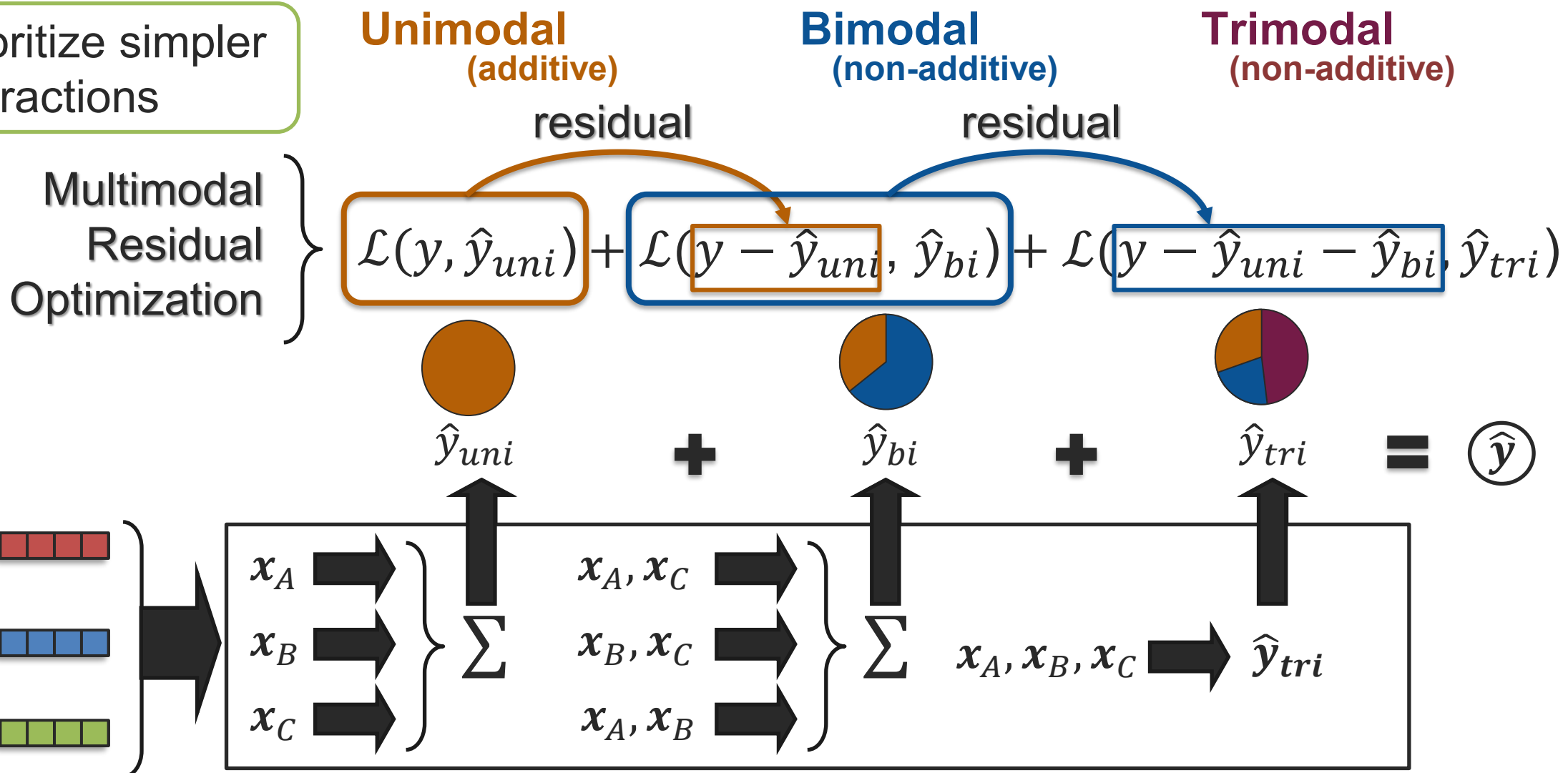
Language Technologies Institute

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Carnegie Mellon University

Learning Non-additive Bimodal and Trimodal Interactions

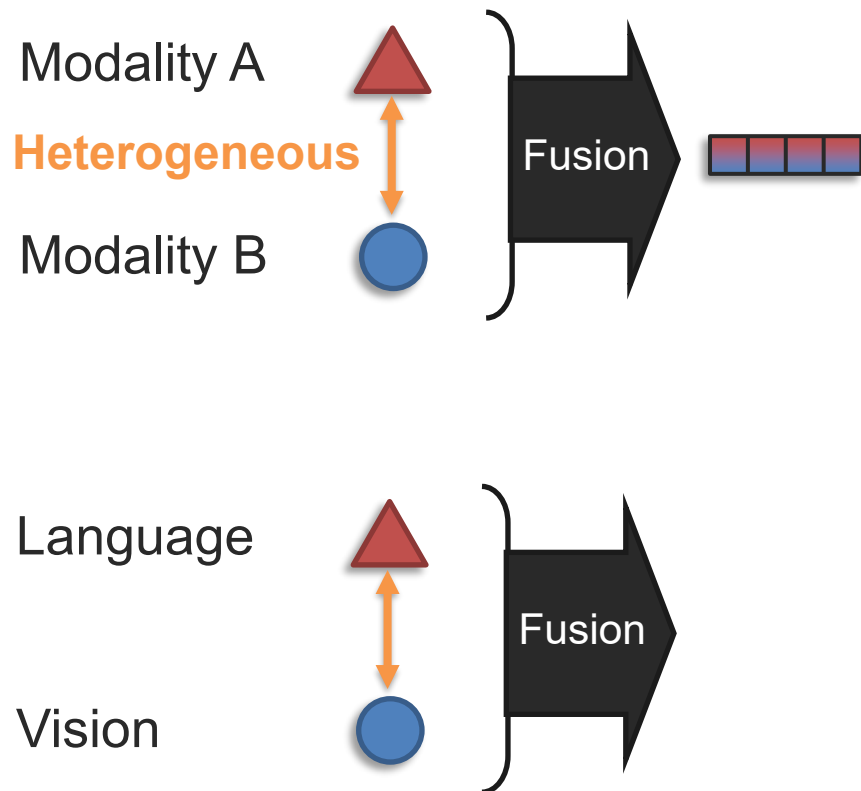
Idea: prioritize simpler interactions



[Wortwein et al., Beyond Additive Fusion: Learning Non-Additive Multimodal Interactions, Findings-EMNLP 2022]

Fusion with Heterogeneous Modalities

Goal: Fusion with raw modalities



Can the same fusion algorithm handle raw heterogeneous modalities?

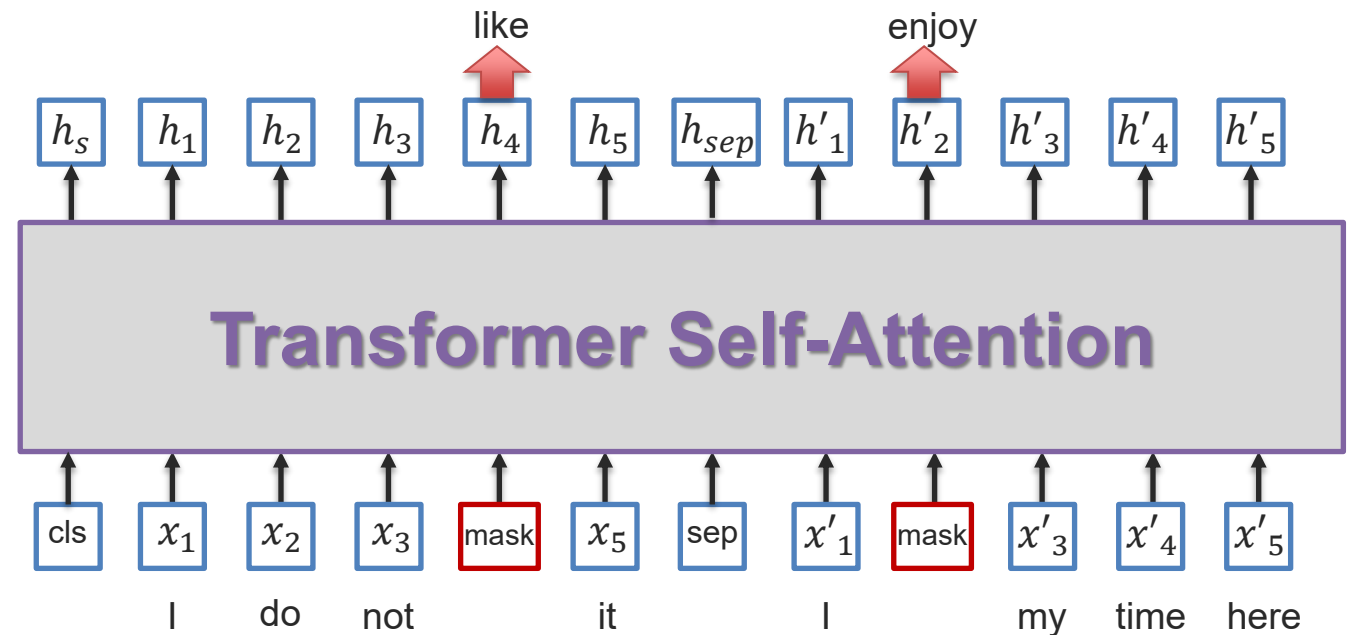
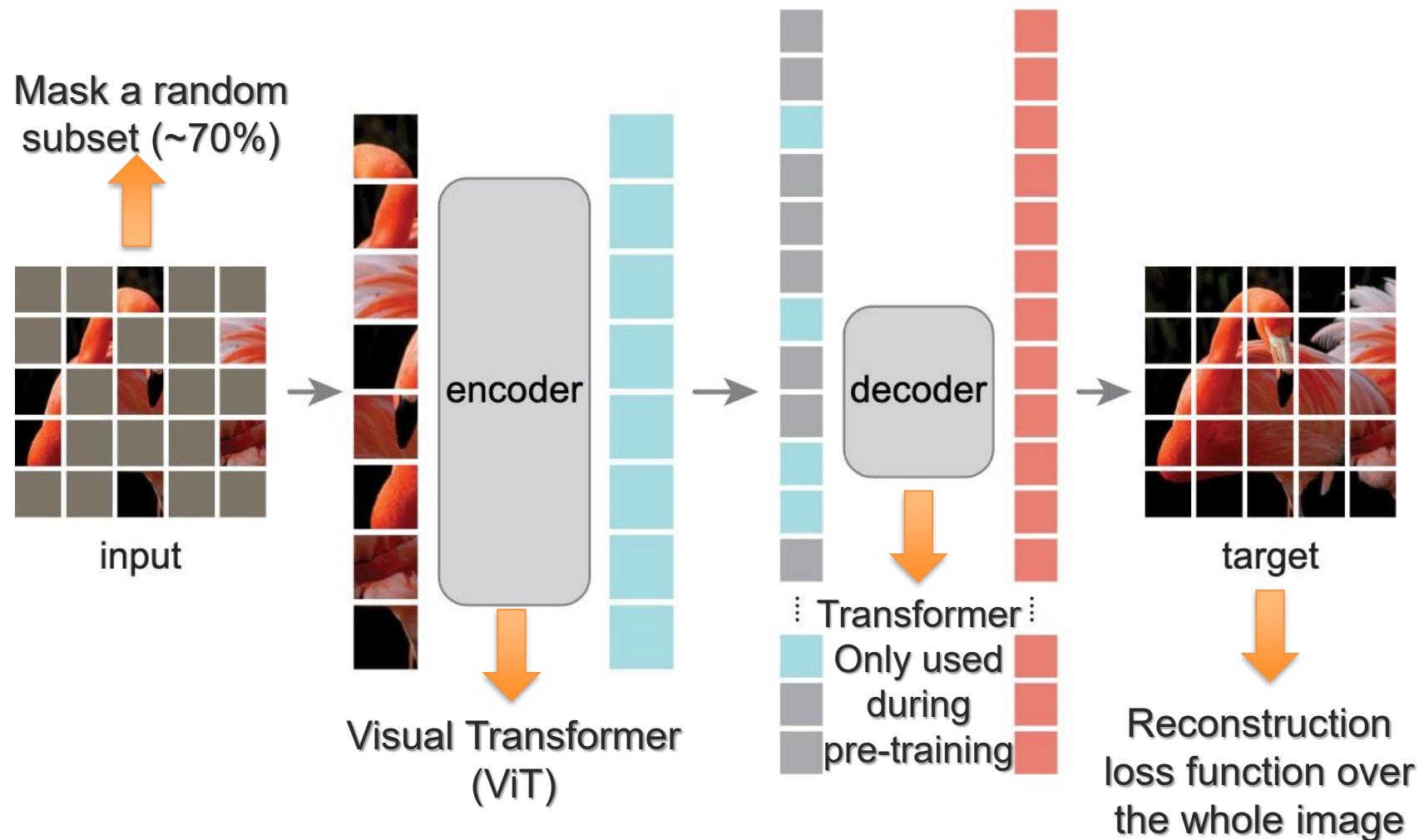
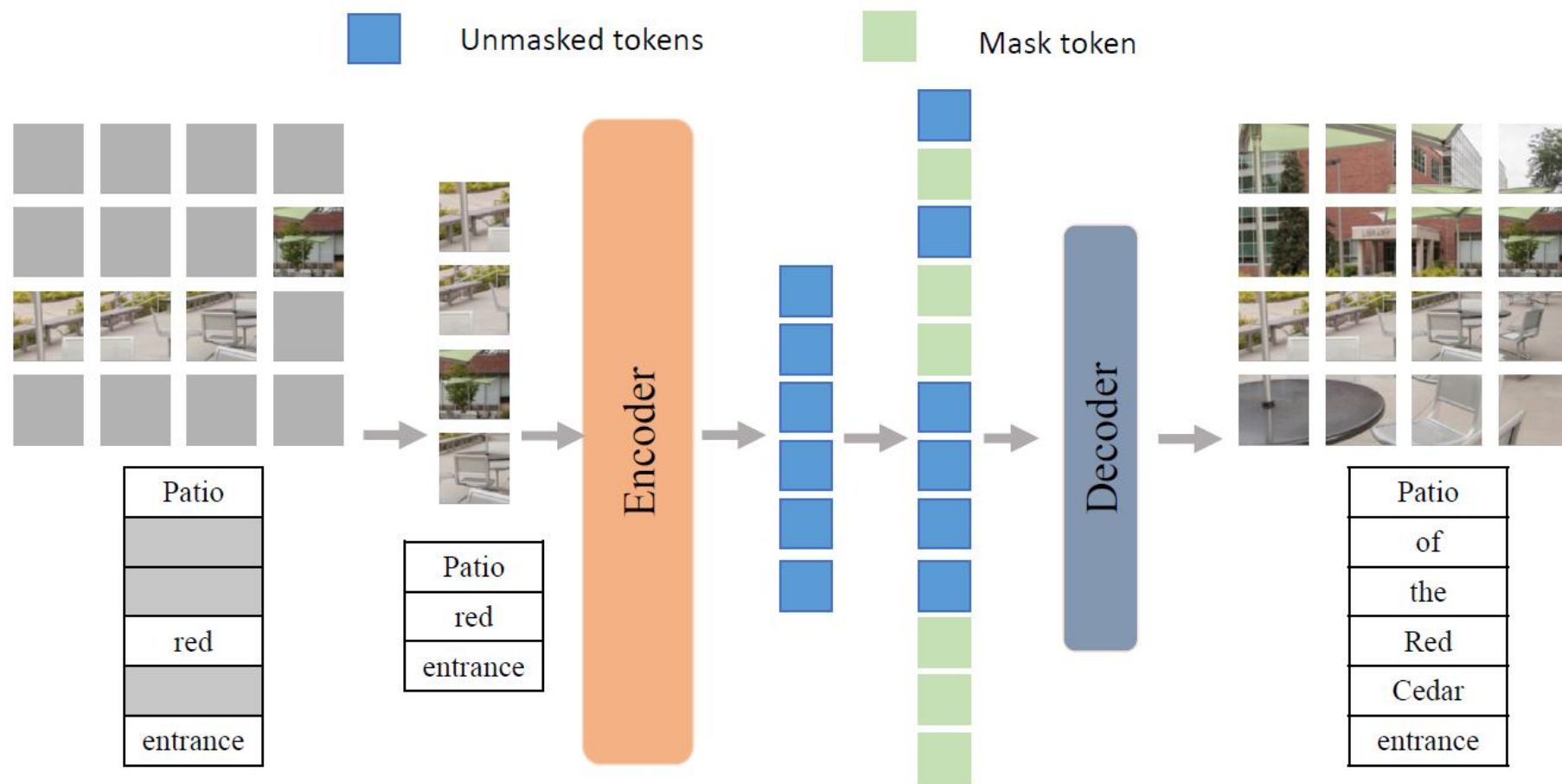


Image Representation Learning: Masked Auto-Encoder (MAE)



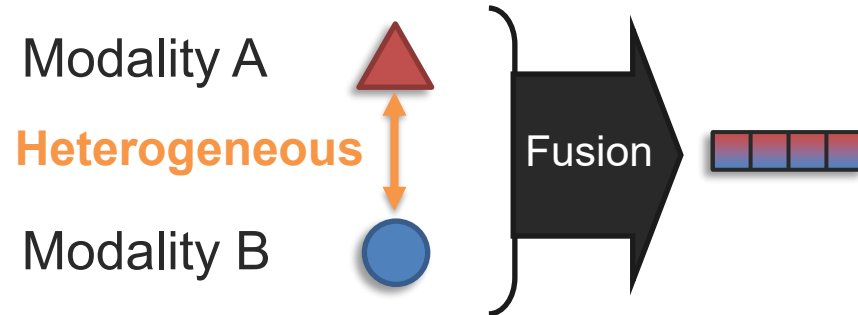
[He et al., Masked Autoencoders Are Scalable Vision Learners, CVPR 2022]

Multimodal Masked Autoencoder

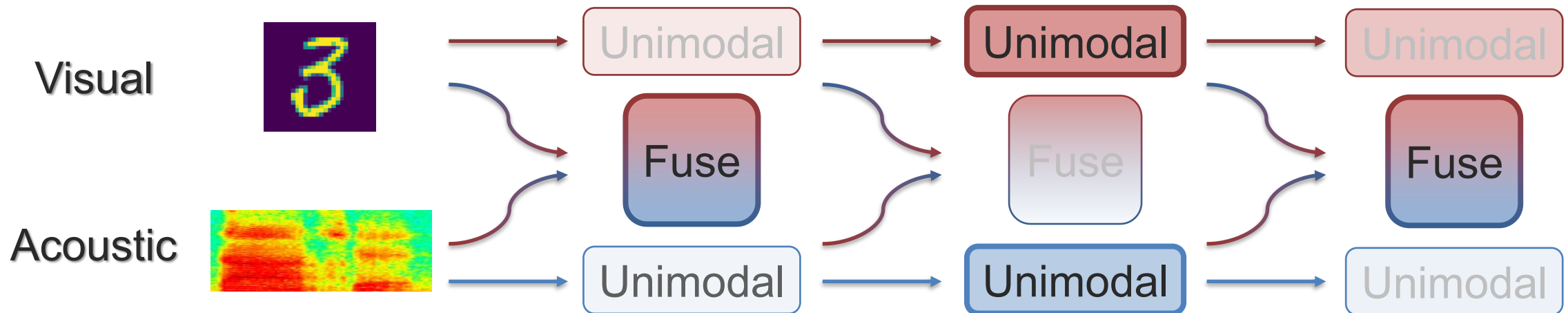


[Geng et al., Multimodal Masked Autoencoders Learn Transferable Representations, 2022]

Dynamic Early Fusion



Idea: Deciding when to fuse in early fusion



[Xue and Marculescu, Dynamic Multimodal Fusion, arxiv 2022]

Dynamic Early Fusion

Fusion fully learned from optimization and data

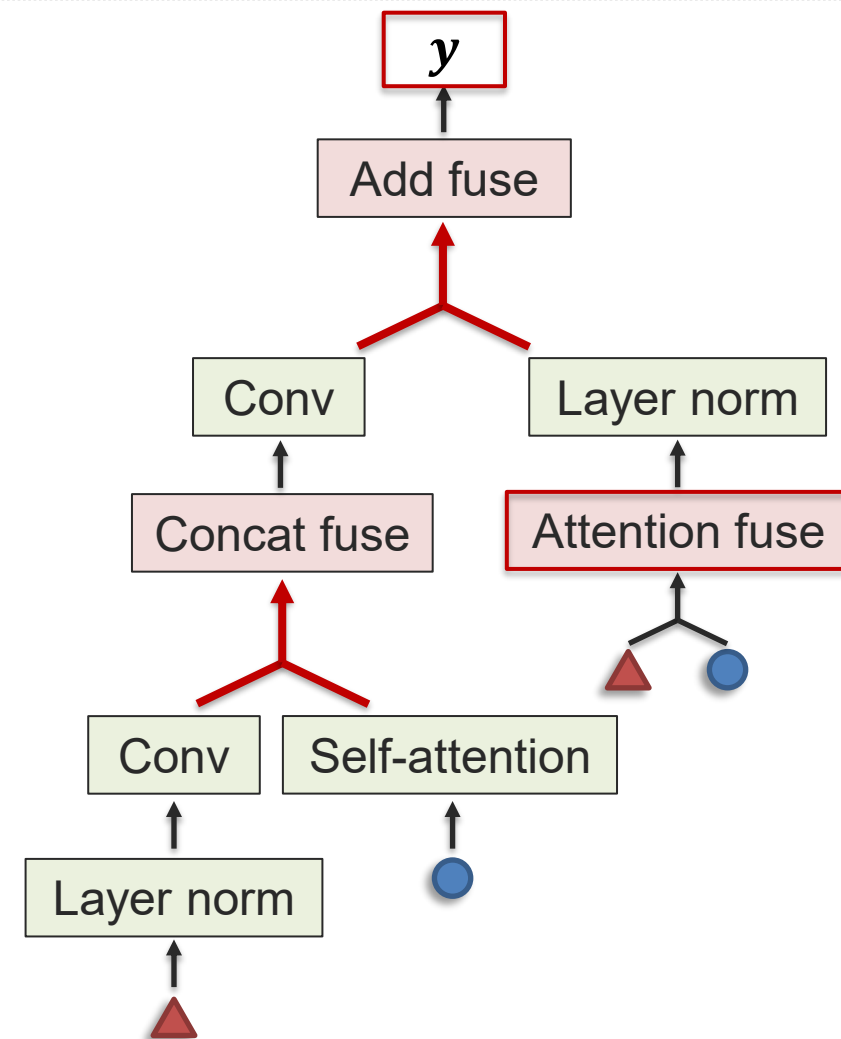
1. Define basic representation building blocks



2. Define basic fusion building blocks



3. Automatically search for composition using neural architecture search



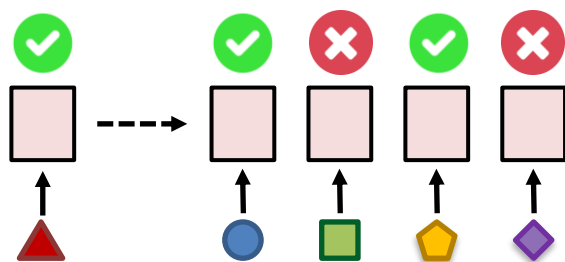
[Xu et al., MUFASA: Multimodal Fusion Architecture Search for Electronic Health Records. AAAI 2021]

[Liu et al., DARTS: Differentiable Architecture Search. ICLR 2019]

Heterogeneity-aware Fusion

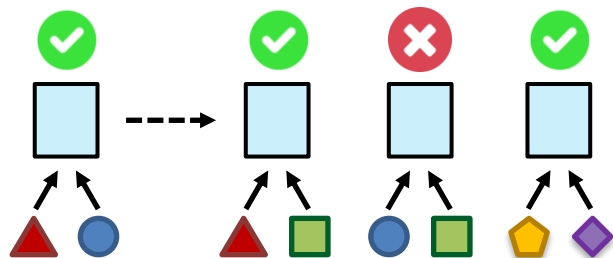
Information transfer, transfer learning perspective

1a. Estimate modality heterogeneity via transfer



(Implicitly captures heterogeneity)

1b. Estimate interaction heterogeneity via transfer



2a. Compute modality heterogeneity matrix

	△	●	■	⬠	◆
△	0				
●	1	0			
■	3	2	0		
⬠	1	2	3	0	
◆	5	4	6	3	0

2b. Compute interaction heterogeneity matrix

	{△, ●}	{△, ■}	{●, ■}	{⬠, ◆}
{△, ●}	0			
{△, ■}	1	0		
{●, ■}	3	2	0	
{⬠, ◆}	1	2	4	0

3. Determine parameter clustering

$$\mathcal{U}_1 = \{U_1, U_2, U_4\}$$

$$\mathcal{U}_2 = \{U_3\}$$

$$\mathcal{U}_3 = \{U_5\}$$

$$\mathcal{C}_1 = \{C_{12}, C_{13}, C_{45}\}$$

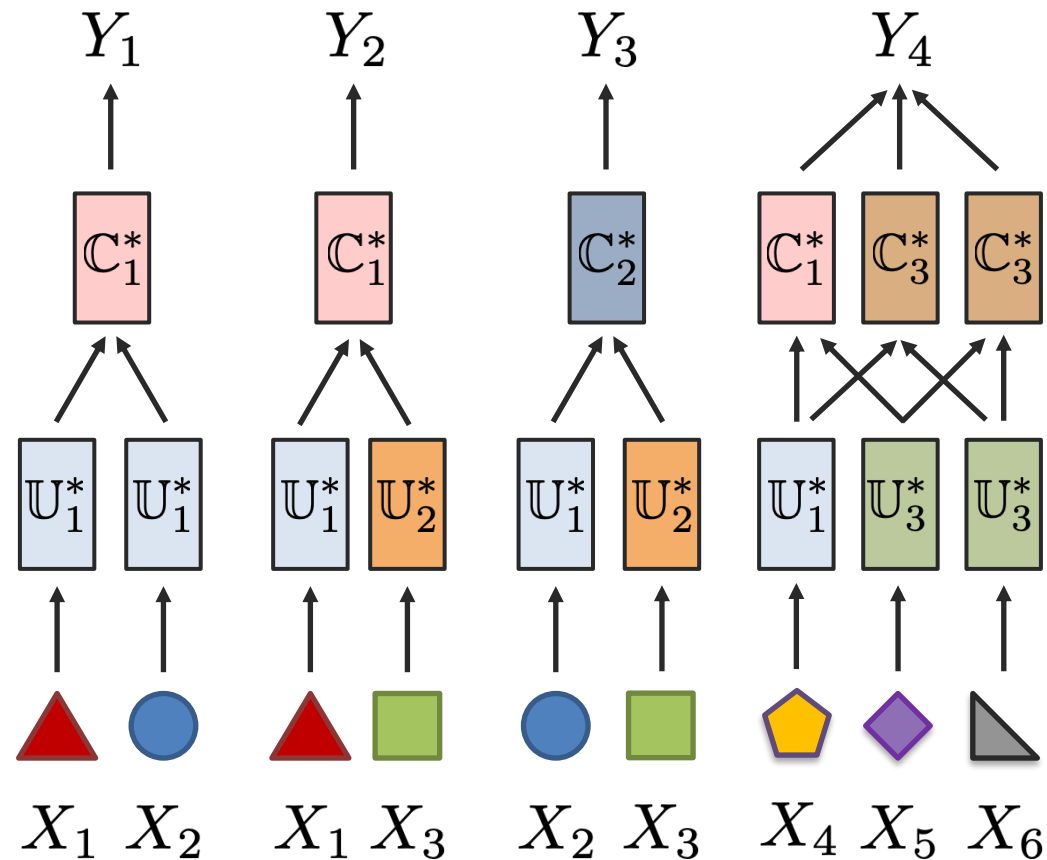
$$\mathcal{C}_2 = \{C_{23}\}$$

[Zamir et al., Taskonomy: Disentangling Task Transfer Learning. CVPR 2018]

[Liang et al., HighMMT: Quantifying Modality & Interaction Heterogeneity for High-Modality Learning. TMLR 2022]

Heterogeneity-aware Fusion

Information transfer, transfer learning perspective



[Liang et al., HighMMT: Quantifying Modality & Interaction Heterogeneity for High-Modality Learning. TMLR 2022]

Improving Optimization

Kinetics dataset



(a) headbanging



(c) shaking hands



(e) robot dancing



(g) riding a bike



Adding more modalities should always help?

Modalities: RGB (video clips)

A (Audio features)

OF (optical flow - motion)

Dataset	Multi-modal	V@1	Best Uni	V@1	Drop
Kinetics	A + RGB	71.4	RGB	72.6	-1.2
	RGB + OF	71.3	RGB	72.6	-1.3
	A + OF	58.3	OF	62.1	-3.8
	A + RGB + OF	70.0	RGB	72.6	-2.6

But sometimes multimodal doesn't help! **Why?**

[Wang et al., What Makes Training Multi-modal Classification Networks Hard? CVPR 2020]

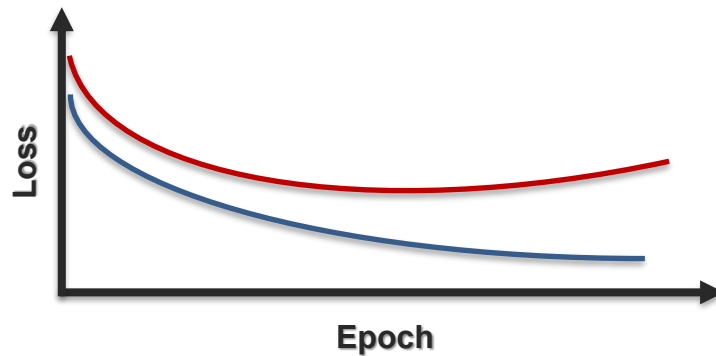
[Wu et al., Characterizing and Overcoming the Greedy Nature of Learning in Multi-modal Deep Neural Networks. ICML 2022]

Improving Optimization

Relevance heterogeneity

2 explanations for drop in performance:

1. Multimodal networks are more prone to overfitting due to **increased complexity**
2. Different modalities overfit and generalize at **different rates**



Key idea 1: compute overfitting-to-generalization ratio (OGR)



Gap between training and valid loss

OGR wrt each modality tells us
how much to train that modality

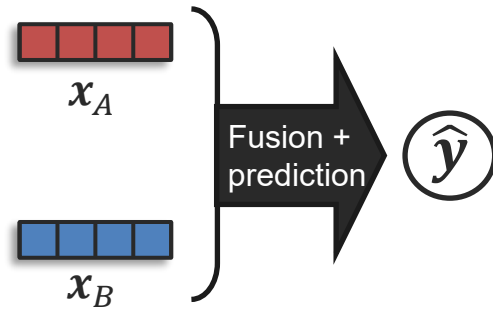
[Wang et al., What Makes Training Multi-modal Classification Networks Hard? CVPR 2020]

[Wu et al., Characterizing and Overcoming the Greedy Nature of Learning in Multi-modal Deep Neural Networks. ICML 2022]

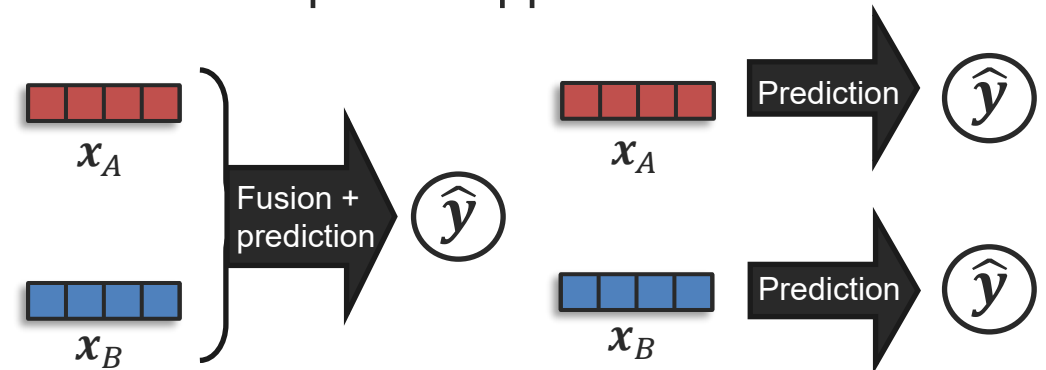
Improving Optimization

Relevance heterogeneity

Conventional approach



Proposed approach



Key idea 2: Simultaneously train unimodal networks to estimate OGR wrt each modality



Reweight multimodal loss using unimodal OGR values



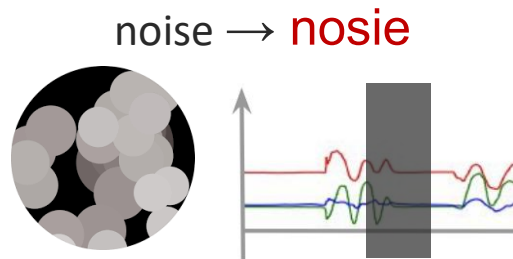
Allows to better balance generalization & overfitting rate of different modalities

[Wang et al., What Makes Training Multi-modal Classification Networks Hard? CVPR 2020]

[Wu et al., Characterizing and Overcoming the Greedy Nature of Learning in Multi-modal Deep Neural Networks. ICML 2022]

Heterogeneity in Noise: Studying Robustness

Noise within Modality



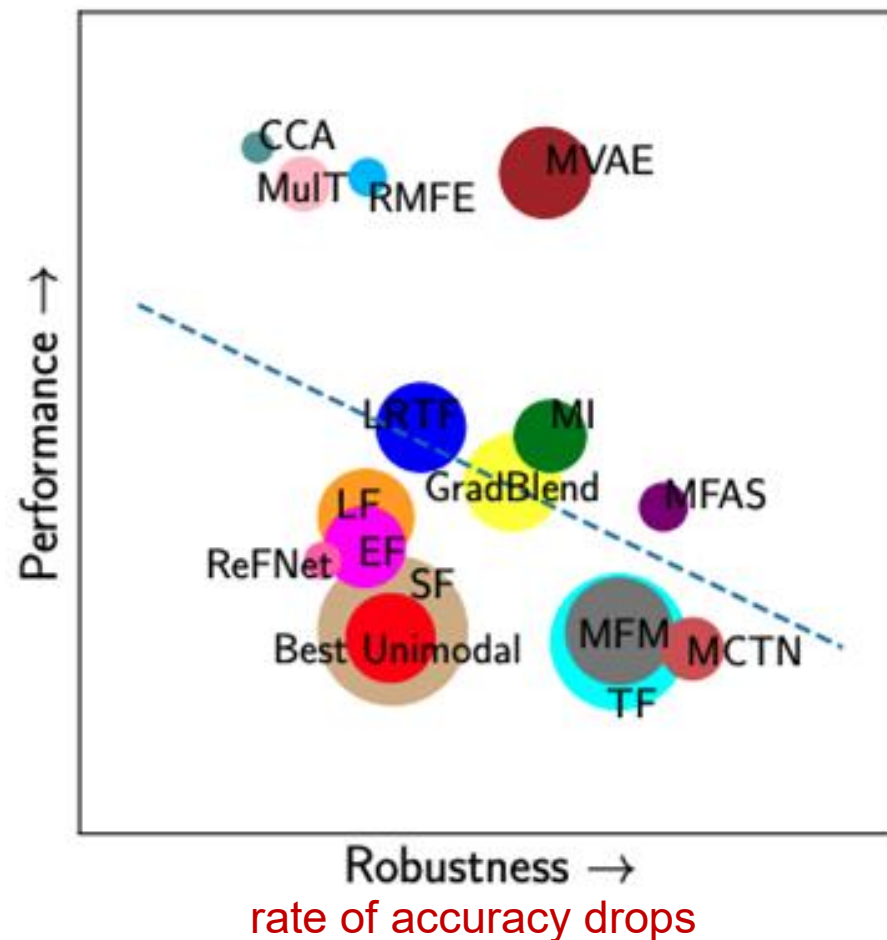
[Belinkov & Bisk, 2018; Subramaniam et al., 2009; Boyat & Joshi, 2015]

Missing Modalities



[Zadeh et al., 2020]

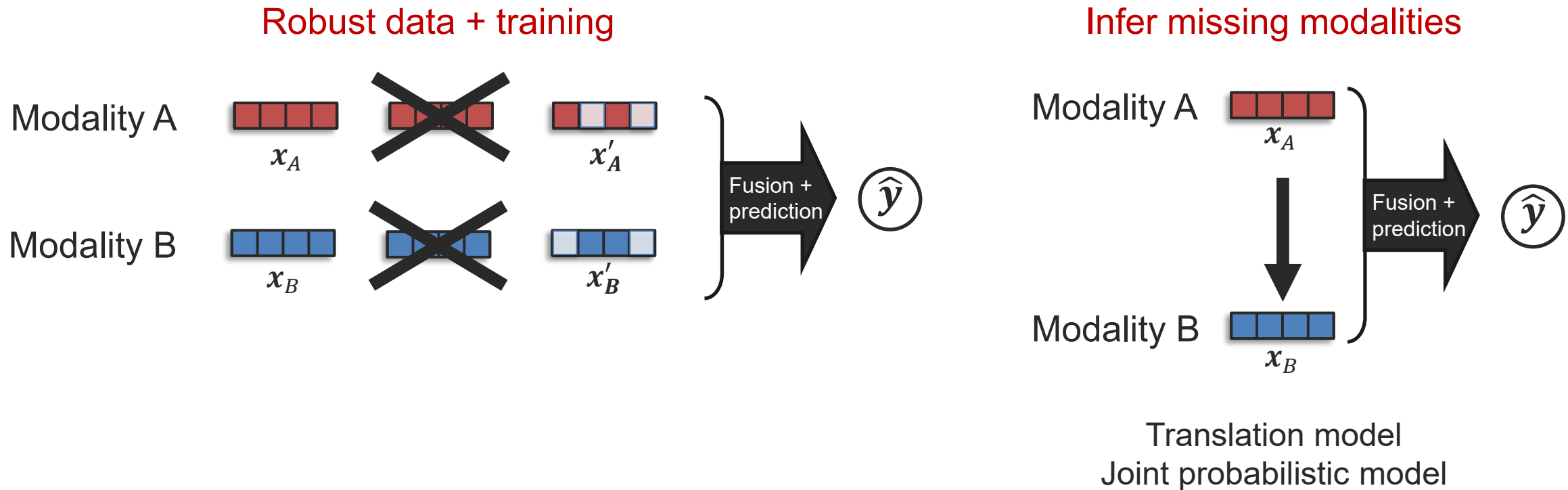
Strong tradeoffs between performance and robustness



[Liang et al., MultiBench: Multiscale Benchmarks for Multimodal Representation Learning. NeurIPS 2021]

Heterogeneity in Noise: Studying Robustness

Several approaches towards more robust models



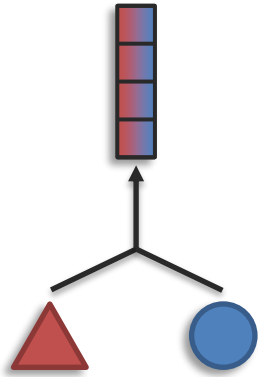
[Ngiam et al., Multimodal Deep Learning. ICML 2011]

[Srivastava and Salakhutdinov, Multimodal Learning with Deep Boltzmann Machines. JMLR 2014]

[Tran et al., Missing Modalities Imputation via Cascaded Residual Autoencoder. CVPR 2017]

[Pham et al., Found in Translation: Learning Robust Joint Representations by Cyclic Translations Between Modalities. AAAI 2019]

Sub-Challenge 1a: Representation Fusion



Definition: Learn a joint representation that models cross-modal interactions between individual elements of different modalities

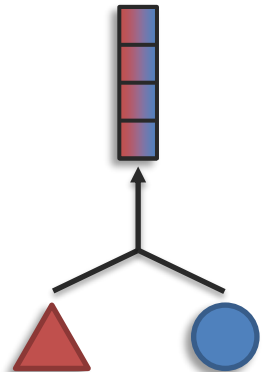


Challenge 1: Representation

Definition: Learning representations that reflect cross-modal interactions between individual elements, across different modalities

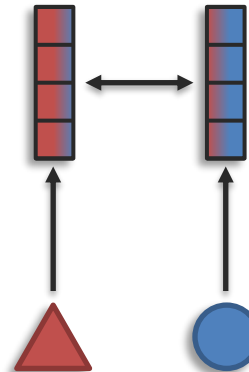
Sub-challenges:

Fusion



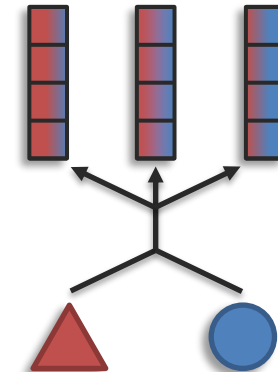
modalities $\gt$ # representations

Coordination



modalities = # representations

Fission



modalities $\lt$ # representations

Next week