Utility Maximization Problem

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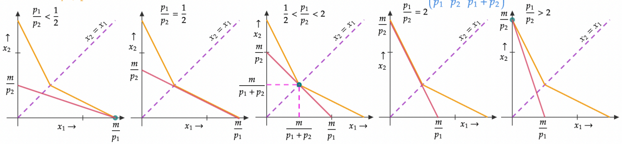
$\max_{x_1 \geq 0,\, x_2 \geq 0} \min(2x_1 + x_2, x_1 + 2x_2)$ subject to $p_1x_1 + p_2x_2 \leq m$ *Indirect Utility* $= \max\left(\frac{m}{p_1}, \frac{m}{p_2}, \frac{3m}{p_1 + p_2}\right)$



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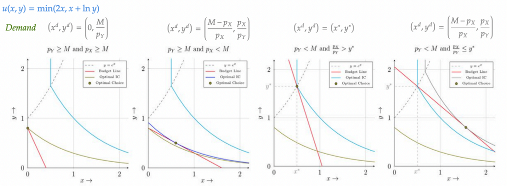

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Discontinuous preference with utility representation

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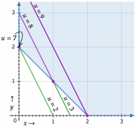
$\succsim$ on $\mathbb{R}^2_+$ is said to be continuous if for any pair of convergent sequences (x'_n, y'_n) and (x''_n, y''_n) in $\mathbb{R}^2_+$,
$(x'_n, y'_n) \succsim (x''_n, y''_n)$ for all $n \in \mathbb{N}$
implies
$\lim_{n\to\infty}(x'_n, y'_n) \succsim \lim_{n\to\infty}(x''_n, y''_n)$

A function $u : \mathbb{R}^2_+ \to \mathbb{R}$ represents $\succsim$ if the following holds:
$(x', y') \succsim (x'', y'')$
if and only if
$u(x', y') \geq u(x'', y'')$

$$u(x,y) = \begin{cases} 2x + y & \text{if } x + y < 2 \\ 2x + y + 5 & \text{if } x + y \geq 2 \end{cases}$$

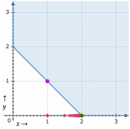
Indifference map of u



Preference $\succsim$ represented by u is discontinuous

$(x'_n, y'_n) = (1,1) \succsim \left(2 - \frac{1}{n}, 0\right) = (x''_n, y''_n)$ for all $n \in \mathbb{N}$

but $\lim_{n\to\infty}(x'_n, y'_n) = (1,1) \not\succsim (2,0) = \lim_{n\to\infty}(x''_n, y''_n)$



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Finding demand

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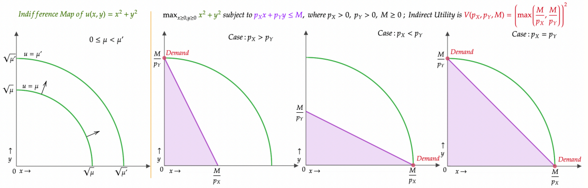

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Better-off sets of a discontinuous preference

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In the Metric Space $\left(\mathbb{R}_+^2, d\right)$, *where* $d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

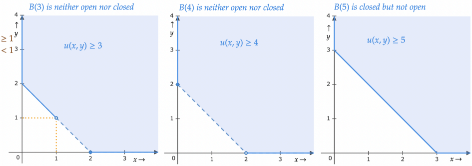
Consider the utility function $u : \mathbb{R}_+^2 \to \mathbb{R}$ *defined as follows:*

$$u(x, y) = \begin{cases} x + y & \text{if } x + y < 2 \\ 1 + x & \text{if } x + y = 2 \text{ and } x \geq 1 \\ 4 - x & \text{if } x + y = 2 \text{ and } x < 1 \\ x + y + 2 & \text{if } x + y > 2 \end{cases}$$

We'll observe the sets

$B(a) = \left\{(x, y) \in \mathbb{R}_+^2 | u(x, y) \geq a\right\}$

for $a \in \{3, 4, 5\}$.



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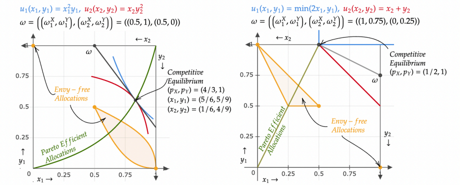

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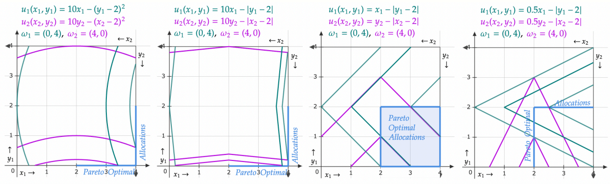

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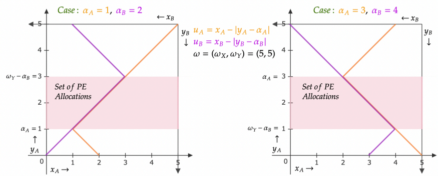

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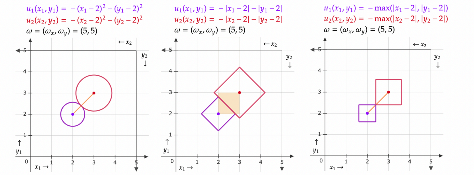

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Economy :

$\omega = (\omega_A, \omega_B) = ((1,0),(0,1))$

$u_A(x_A, y_A) = x_A y_A$

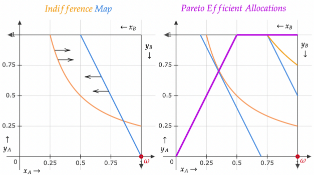
$u_B(x_B, y_B) = 2x_B + y_B$

Feasible Allocations :

$\mathcal{F} = \left\{((x_A, y_A),(x_B, y_B)) \in \mathbb{R}^2_+ \times \mathbb{R}^2_+ \mid x_A + x_B = 1 \wedge y_A + y_B = 1\right\}$

Efficient Allocations :

$\mathcal{E} = \{((x_A, y_A),(x_B, y_B)) \in \mathcal{F} \mid y_A = \min(2x_A, 1)\}$



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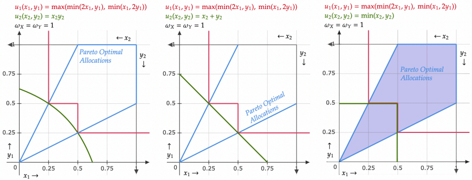

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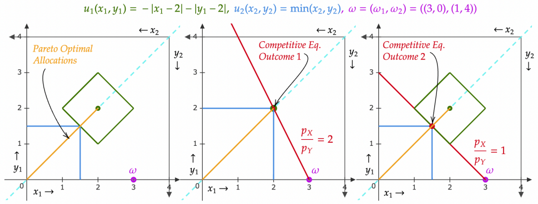

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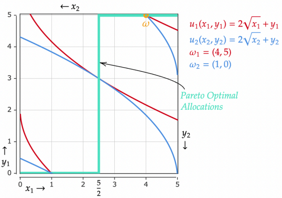

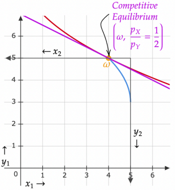

Economies with No Pareto Optimal Allocations

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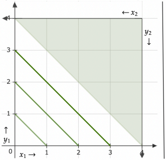
Here are a couple of examples of exchange economies in which no Pareto Optimal allocations exist.

Economy 1 :

$$u_1(x_1, y_1) = \begin{cases} x_1 + y_1 & \text{if } x_1 + y_1 < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$u_2(x_2, y_2) = 0$$

$$\omega = (\omega_X, \omega_Y) = (4, 4)$$

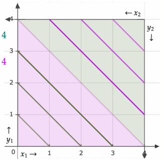


Economy 2 :

$$u_1(x_1, y_1) = \begin{cases} x_1 + y_1 & \text{if } x_1 + y_1 < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$u_2(x_2, y_2) = \begin{cases} x_2 + y_2 & \text{if } x_2 + y_2 < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\omega = (\omega_X, \omega_Y) = (4, 4)$$



Equilibrium in an Exchange Economy may not exist

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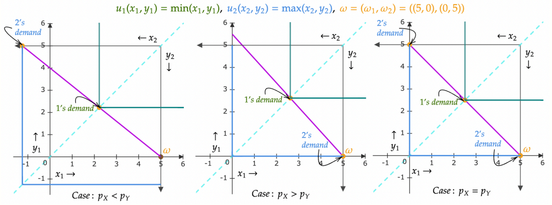

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First Welfare Property Fails to hold

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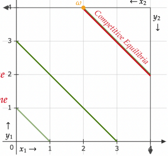
Here are a couple of examples of exchange economies, each of which has a Competitive Equilibrium that is not Pareto efficient.

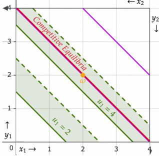
Economy 1:
$u_1(x_1, y_1) = x_1 + y_1$
$u_2(x_2, y_2) = 0$
$\omega = (\omega_1, \omega_2) = ((2,4),(2,0))$

$(p_X, p_Y) = (1,1)$ *supports all the feasible allocations satisfying* $x_1 + y_1 = 6$ *as equilibrium, none of which is Pareto efficient.*



Economy 2:
$u_1(x_1, y_1) = \lfloor x_1 + y_1 + 0.5 \rfloor$
$u_2(x_2, y_2) = x_2 + y_2$
$\omega = (\omega_X, \omega_Y) = ((2,2),(2,2))$

$(p_X, p_Y) = (1,1)$ *supports all the feasible allocations satisfying* $x_1 + y_1 = 4$ *as equilibrium, none of which is Pareto efficient.*



Second Welfare Property Fails to hold

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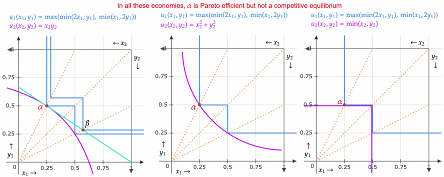

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Cost Minimization Problem of a firm

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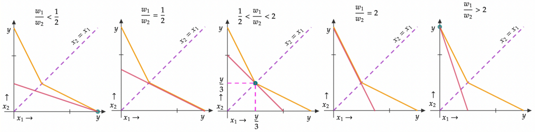
$\min_{x_1\geq 0,\, x_2\geq 0} w_1x_1 + w_2x_2$ subject to $\min(2x_1 + x_2, x_1 + 2x_2) = y$ Optimal cost $= \min\left(w_1, w_2, \frac{w_1+w_2}{3}\right)y$



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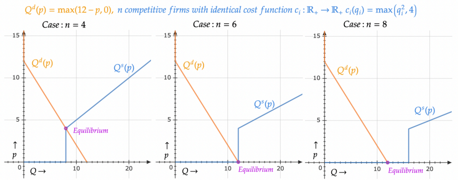

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Optimal locations for the public facility

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$Square - City: \; [0,1] \times [0,1]$

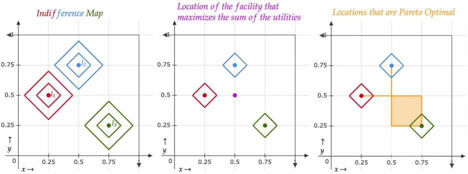
The objective is to establish a public facility at a specific location $(x,\ y)$, *such as a library within the region* $[0,\ 1] \times [0,\ 1]$. *This facility will be accessible to three individuals residing at the following locations:*

$l_1 = (x_1, y_1) = (0.25, 0.5)$,
$l_2 = (x_2, y_2) = (0.5, 0.75)$,
$l_3 = (x_3, y_3) = (0.75, 0.25)$

respectively.

Utility function of $i \in \{1,2,3\}$ *is* $u_i : [0,1]^2 \to \mathbb{R}$ *defined as*

$u_i(x, y) = -|x - x_i| - |y - y_i|$



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Pareto Optimal locations for the public facility

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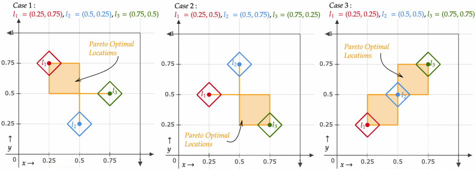
Square – City : $[0,1] \times [0,1]$

The objective is to establish a public facility at a specific location $(x,\ y)$, *such as a library within the region* $[0,\ 1] \times [0,\ 1]$. *This facility will be accessible to three individuals residing at the following locations* : $l_1 = (x_1, y_1)$, $l_2 = (x_2, y_2)$, $l_3 = (x_3, y_3)$ *respectively.*

Utility function of $i \in \{1,2,3\}$ *is* $u_i : [0,1]^2 \to \mathbb{R}$ *defined as*

$u_i(x,y) = -|x - x_i| - |y - y_i|$



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Pareto Optimal Locations for the public facility

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Suppose a square city $[0,1]\times[0,1]$ *has a road network such that the applicable measure of distance between any two points is either the taxi – cab metric or the Euclidean metric. A public facility is to be constructed somewhere in the city. There are two types of legislators who will vote for the location:*
Type 1: Those who favor the location that is closer to the centered location $(0.5, 0.5)$
Type 2: Others who favor the location that is closer to the boundary of the city, i. e., $x=0,\ y=0,\ x=1,\ or\ y=1.$

Proposition. Set of Pareto optimal locations consists of all the locations on the shortest paths from the center $(0.5, 0.5)$ *to the boundary.*

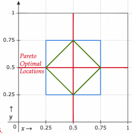
If (x, y) *denotes the proposed location of the public facilty:*

Case 1: Taxi – cab Metric
$d((x,y),(x',y')) = |x-x'| + |y-y'|$

Utility of Type 1:
$u_1(x,y) = -|x-0.5| - |y-0.5|$
(minus of the distance of the proposed location from the center)

Utility of Type 2:
$u_2(x,y) = -\min(x, y, 1-x, 1-y)$
(minus of the distance of the proposed location from the nearest point on the boundary)

Set of Pareto optimal locations consists of set of locations satisfying $x = 0.5$ *or* $y = 0.5$.

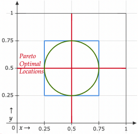


Case 2: Euclidean Metric
$d((x,y),(x',y')) = \sqrt{(x-x')^2 + (y-y')^2}$

Utility of Type 1:
$u_1(x,y) = -\sqrt{(x-0.5)^2 + (y-0.5)^2}$
(minus of the distance of the proposed location from the center)

Utility of Type 2:
$u_2(x,y) = -\min(x, y, 1-x, 1-y)$
(minus of the distance of the proposed location from the nearest point on the boundary)

Set of Pareto optimal locations consists of set of locations satisfying $x = 0.5$ *or* $y = 0.5$.



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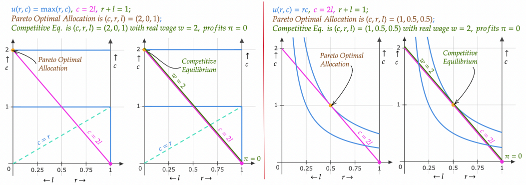

Equilibrium and Efficiency in Crusoe Economies

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$u(r,c) = rc$, $c = \min\left(l, \frac{1}{4} + \frac{l}{2}\right)$, $r + l = 1$;

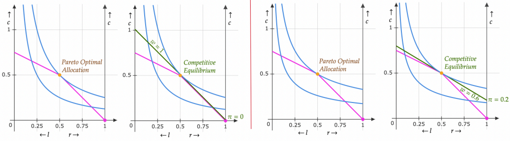
Pareto Optimal Allocation is $(c, r, l) = (0.5, 0.5, 0.5)$;

Competitive Eq. is $(c, r, l) = (0.5, 0.5, 0.5)$ *with real wage* $w = 1$, *profits* $\pi = 0$

$u(r,c) = r^3 c^5$, $c = \min\left(l, \frac{1}{4} + \frac{l}{2}\right)$, $r + l = 1$;

Pareto Optimal Allocation is $(c, r, l) = (0.5, 0.5, 0.5)$;

Competitive Eq. is $(c, r, l) = (0.5, 0.5, 0.5)$ *with real wage* $w = 0.6$, *profits* $\pi = 0.2$



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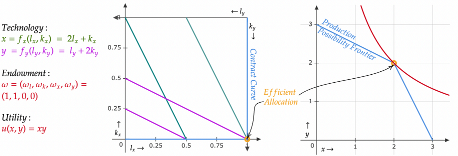

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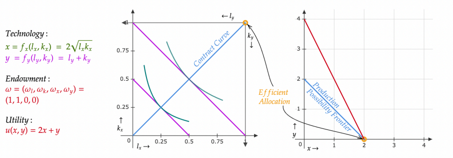

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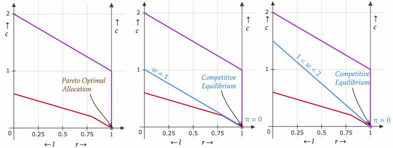
$u(r, c) = \max(2r, r + c)$, $c = \min\left(l, \frac{l}{2} + \frac{1}{10}\right)$, $r + l = 1$;

Pareto Optimal Allocation is $(c, r, l) = (0, 1, 0)$; *Competitive Eq. is* $(c, r, l) = (0, 1, 0)$ *with real wage* $w \in [1, 2]$



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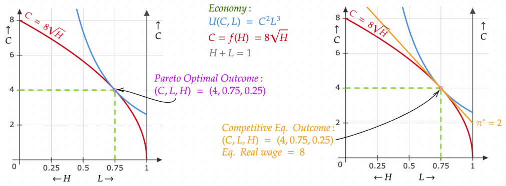

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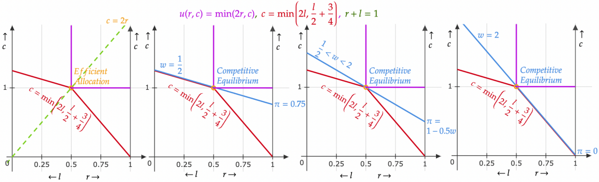

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Single-firm Competitive Equilibrium vs Monopoly

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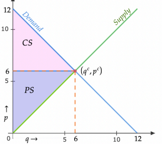
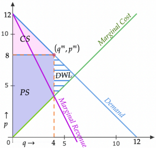
Single competitive (price – taker) firm facing:
– *Inverse Demand*: $p = \max(12 - q, 0)$
– *Cost function is* $c(q) = \frac{q^2}{2}$
So, Inverse supply is $p = q$



A monopoly firm facing:
– *Inverse Demand*: $p = \max(12 - q, 0)$
– *Cost function is* $c(q) = \frac{q^2}{2}$



Here we explore and compare the effects of two different behavioral assumptions about the single firm operating in a market for a good. In one case, we assume that the firm is competitive or a price – taker. In the other case, we assume that the firm acts as a monopolist, choosing a point on the demand curve rather than acting as a price – taker. Everything else about the two situations is assumed to be the same.

As we can observe in the example, moving from competitive equilibrium to monopoly causes the equilibrium price to rise, the equilibrium quantity to fall, and there is also a loss of efficiency with the emergence of deadweight loss.

Monopoly

Econschool

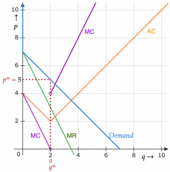
Given :
Demand : $q = \max(7 - p, 0)$
Cost : $c(q) = (|q - 2| + 2)q$

Marginal Revenue is
$\text{MR} = 7 - 2q$ for $0 < q < 7$

Average Cost is
$\text{AC} = |q - 2| + 2$

Marginal Cost is
$\text{MC} = \begin{cases} 4 - 2q & \text{if } q < 2 \\ 2q & \text{if } q > 2 \end{cases}$

Monopoly Eq is
$(q^m, p^m) = (2, 5)$



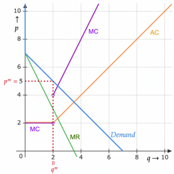
Given :
Demand : $q = \max(7 - p, 0)$
Cost : $c(q) = (\max(q, 2))q$

Marginal Revenue is
$\text{MR} = 7 - 2q$ for $0 < q < 7$

Average Cost is
$\text{AC} = \max(q, 2)$

Marginal Cost is
$\text{MC} = \begin{cases} 2 & \text{if } q < 2 \\ 2q & \text{if } q > 2 \end{cases}$

Monopoly Eq is
$(q^m, p^m) = (2, 5)$



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Monopoly

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Single consumer with valuation $V \sim \text{Unif}(a, b)$*, where* $0 \le a < b$*, for exactly one unit of the good. Consumer knows her valuation, but the monopoly firm doesn't. Monopolist chooses price p to maximize its expected profit. Cost of supplying a unit of the good for the monopolist is* $c \in [0, b]$.

Demand :

$$Q^d(p) = \begin{cases} 1 & \text{if } V \ge p \\ 0 & \text{if } V < p \end{cases}$$

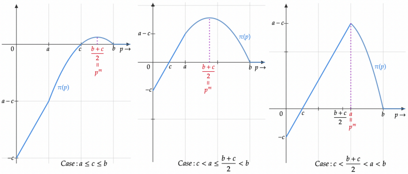
Monopolist chooses p to maximize its Expected Profit :

$$\max_{p \ge 0} \pi(p) = (p - c)\Pr(V \ge p)$$

Observe that the optimal choice of p will always belong to the closed interval $[\max(a, c), b]$.

Solving the problem, we get the optimal p^m *as :*

$$p^m = \max\left(a, \frac{b+c}{2}\right)$$



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Monopoly in Edgeworth Box

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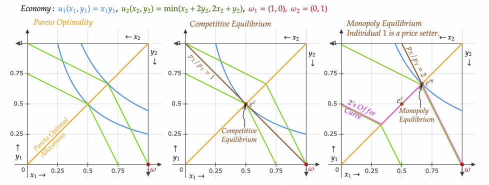

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Finding Nash Equilibrium in a Strategic Game

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Game :

– *Set of Players* : $\{1, 2\}$

–*Action Sets* :

Action Set of Player 1 *is* $A_1 = [0, 1]$

Action Set of Player 2 *is* $A_2 = [0, 1]$

– *Utility functions* :

Utility of Player 1 *is* $u_1 : A_1 \times A_2 \to \mathbb{R}$ *defined as* $u_1(a_1, a_2) = a_1(1 - 2a_2)$

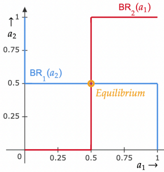
Utility of Player 2 *is* $u_2 : A_1 \times A_2 \to \mathbb{R}$ *defined as* $u_2(a_1, a_2) = a_2(2a_1 - 1)$

$\mathrm{BR}_1(a_2)$ *is the set of solutions to* $\max_{a_1 \in A_1} u_1(a_1, a_2)$

$$\mathrm{BR}_1(a_2) = \begin{cases} \{1\} & \text{if } a_2 < 0.5 \\ \{0\} & \text{if } a_2 > 0.5 \\ [0, 1] & \text{if } a_2 = 0.5 \end{cases}$$

$\mathrm{BR}_2(a_1)$ *is the set of solutions to* $\max_{a_2 \in A_2} u_2(a_1, a_2)$

$$\mathrm{BR}_2(a_1) = \begin{cases} \{0\} & \text{if } a_1 < 0.5 \\ \{1\} & \text{if } a_1 > 0.5 \\ [0, 1] & \text{if } a_1 = 0.5 \end{cases}$$



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Game with No Equilibrium

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Econ School Logo Game :

Action Set of Player 1 is $A_1 = \mathbb{R}_+$

$u_1 : A_1 \times A_2 \to \mathbb{R}$ *is defined as*

$$u_1(a_1, a_2) = \begin{cases} a_1 & \text{if } a_2 < 1 \\ -|a_1 - 9| & \text{if } a_2 \in (1,5) \\ -|a_1 - 6| - |a_1 - 9| & \text{if } a_2 \in \{1,5,9\} \\ -|a_1 - 6| & \text{if } a_2 \in (5,9) \\ a_1 & \text{if } a_2 > 9 \end{cases}$$

$\mathrm{BR}_1(a_2)$ *denotes the set of solutions to*

$\max_{a_1 \in A_1} u_1(a_1, a_2)$

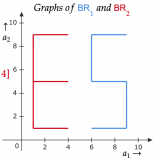
Action Set of Player 2 is $A_2 = \mathbb{R}_+$

$u_2 : A_1 \times A_2 \to \mathbb{R}$ *is defined as*

$$u_2(a_1, a_2) = \begin{cases} a_2 & \text{if } a_1 < 1 \\ -|a_2 - 1| - |a_2 - 9| & \text{if } a_1 = 1 \\ \max(-|a_2 - 1|, -|a_2 - 5|, -|a_2 - 9|) & \text{if } a_1 \in (1,4] \\ a_2 & \text{if } a_1 > 4 \end{cases}$$

$\mathrm{BR}_2(a_1)$ *denotes the set of solutions to*

$\max_{a_2 \in A_2} u_2(a_1, a_2)$



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Cournot Duopoly with Fixed Costs

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Demand :
$P^d(Q) = \max(12 - Q, 0)$

Cost functions :
$C_i(q_i) = \begin{cases} F & \text{if } q_i > 0 \\ 0 & \text{if } q_i = 0 \end{cases}$
for $i \in \{1, 2\}$

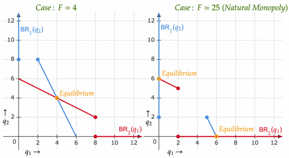
Profits :
$\pi_i(q_1, q_2) = q_i P^d(q_1 + q_2) - C_i(q_i)$
for $i \in \{1, 2\}$

Firm 1 *chooses* q_1
Firm 2 *chooses* q_2

$\mathrm{BR}_1(q_2)$ *is the set of solutions to*
$\max_{q_1 \in \mathbb{R}_+} \pi_1(q_1, q_2)$
$\mathrm{BR}_2(q_1)$ *is the set of solutions to*
$\max_{q_2 \in \mathbb{R}_+} \pi_2(q_1, q_2)$

$$\mathrm{BR}_i(q_j) = \begin{cases} \left\{\dfrac{12 - q_j}{2}\right\} & \text{if } q_j < 12 - 2\sqrt{F} \\ \{0\} & \text{if } q_j > 12 - 2\sqrt{F} \\ \left\{0, \dfrac{12 - q_j}{2}\right\} & \text{if } q_j = 12 - 2\sqrt{F} \end{cases}$$

for $i, j \in \{1, 2\}$ *and* $i \neq j$



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Pay your own bill vs Split the bill

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Pay your own bill:

2 *friends*: $\{1, 2\}$ *simultaneously choose the amount of food* $f_1, f_2 \in \mathbb{R}_+$ *they want to order and consume at a restaurant.*

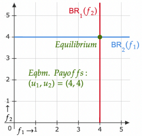
Quasi – linear Utility of $i \in \{1, 2\}$ *is* $u_i : \mathbb{R}_+^2 \to \mathbb{R}$ *defined as*

$u_i(f_i, p_i) = 4\sqrt{f_i} - p_i$

where $p_i = f_i$ *is the payment made by i, which is equal to i's own food bill.*

$\text{BR}_i(f_j)$ *is the set of solutions to*

$\max_{f_i \geq 0} 4\sqrt{f_i} - f_i$



Split the bill:

2 *friends*: $\{1, 2\}$ *simultaneously choose the amount of food* $f_1, f_2 \in \mathbb{R}_+$ *they want to order and consume at a restaurant.*

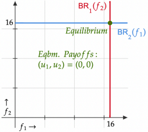
Quasi – linear Utility of $i \in \{1, 2\}$ *is* $u_i : \mathbb{R}_+^2 \to \mathbb{R}$ *defined as*

$u_i(f_i, p_i) = 4\sqrt{f_i} - p_i$

where $p_i = \dfrac{f_1 + f_2}{2}$ *is the payment made by i, which is equal to half of the total food bill.*

$\text{BR}_i(f_j)$ *is the set of solutions to*

$\max_{f_i \geq 0} 4\sqrt{f_i} - \dfrac{f_i + f_j}{2}\ ; j \neq i$



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First-price sealed-bid auction

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First Price Auction (2-players' Perfect Information Game)

There are two bidders, 1 and 2. Players simultaneously choose their bids b_1 and b_2 respectively. Player 1's valuation for the object is 4, and player 2's valuation for the object is 3. The winner of the object is the highest bidder. In case of a tie, the object goes to Player 1.

Preferences are represented by:

$u_1 : \mathbb{R}^2_+ \to \mathbb{R}$

$$u_1(b_1, b_2) = \begin{cases} 4 - b_1 & \text{if } b_1 \geq b_2 \\ 0 & \text{if } b_1 < b_2 \end{cases}$$

$u_2 : \mathbb{R}^2_+ \to \mathbb{R}$

$$u_2(b_1, b_2) = \begin{cases} 0 & \text{if } b_1 \geq b_2 \\ 3 - b_2 & \text{if } b_1 < b_2 \end{cases}$$

$\mathrm{BR}_1(b_2)$ *is the set of solutions to* $\max_{b_1 \geq 0} u_1(b_1, b_2)$

$\mathrm{BR}_2(b_1)$ *is the set of solutions to* $\max_{b_2 \geq 0} u_2(b_1, b_2)$

$$\mathrm{BR}_1(b_2) = \begin{cases} [0, b_2) & \text{if } b_2 > 4 \\ [0, 4] & \text{if } b_2 = 4 \\ \{b_2\} & \text{if } b_2 < 4 \end{cases}$$

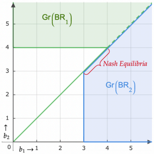
$$\mathrm{BR}_2(b_1) = \begin{cases} [0, b_1] & \text{if } b_1 \geq 3 \\ \varnothing & \text{if } b_1 < 3 \end{cases}$$

for $i, j \in \{1, 2\}$, *and* $i \neq j$, *define*

$$\mathrm{Gr}\left(\mathrm{BR}_i\right) = \left\{(b_1, b_2) \in \mathbb{R}^2_+ | b_i \in \mathrm{BR}_i(b_j)\right\}$$

Set of Nash Equilibria $= \mathrm{Gr}\left(\mathrm{BR}_1\right) \cap \mathrm{Gr}\left(\mathrm{BR}_2\right)$

$$= \left\{(b_1, b_2) \in \mathbb{R}^2_+ | 3 \leq b_1 = b_2 \leq 4\right\}$$



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Maintaining Cleanliness in the Apartment

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There are two roommates, 1 and 2. They simultaneously choose their effort levels $e_1 \in \mathbb{R}_+$ *and* $e_2 \in \mathbb{R}_+$ *respectively to keep their apartment clean. Also, Individual 1 values cleanliness more than Individual 2.*

Preferences are represented by :

$u_1 : \mathbb{R}^2_+ \to \mathbb{R}$

$u_1(e_1, e_2) = 4\sqrt{e_1 + e_2} - e_1$

$u_2 : \mathbb{R}^2_+ \to \mathbb{R}$

$u_2(e_1, e_2) = 2\sqrt{e_1 + e_2} - e_2$

$\mathrm{BR}_1(e_2)$ *denotes the set of solutions to* $\max_{e_1 \geq 0} u_1(e_1, e_2)$

$\mathrm{BR}_2(e_1)$ *denotes the set of solutions to* $\max_{e_2 \geq 0} u_2(e_1, e_2)$

In this game,

$$\mathrm{BR}_1(e_2) = \begin{cases} \{4 - e_2\} & \text{if } e_2 \leq 4 \\ \{0\} & \text{if } e_2 > 4 \end{cases}$$

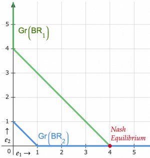
$$\mathrm{BR}_2(e_1) = \begin{cases} \{1 - e_1\} & \text{if } e_1 \leq 1 \\ \{0\} & \text{if } e_1 > 1 \end{cases}$$

for $i,\ j \in \{1, 2\}$*, and* $i \neq j$*, define*

$$\mathrm{Gr}\left(\mathrm{BR}_i\right) = \left\{(e_1, e_2) \in \mathbb{R}^2_+ | e_i \in \mathrm{BR}_i(e_j)\right\}$$

Set of Nash Equilibria $= \mathrm{Gr}\left(\mathrm{BR}_1\right) \cap \mathrm{Gr}\left(\mathrm{BR}_2\right)$

$= \left\{(e_1, e_2) \in \mathbb{R}^2_+ | e_1 = 4, e_2 = 0\right\}$



Note that the equilibrium $(e_1, e_2) = (4, 0)$ *is not Pareto optimal, and there is an under – provision of effort levels, as* $(e_1, e_2) = (7.5, 1.5)$ *yields higher utility for both roommates.*

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Stackelberg Duopoly with Fixed Costs

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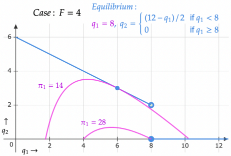
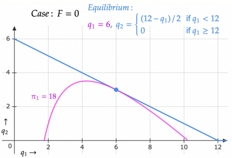
Demand :
$P^d(Q) = \max(12 - Q, 0)$

Cost functions :
$C_i(q_i) = \begin{cases} F & \text{if } q_i > 0 \\ 0 & \text{if } q_i = 0 \end{cases}$
for $i \in \{1, 2\}$

$\pi_i = q_i P^d(q_1 + q_2) - C_i(q_i)$

Leader : *Firm* 1
Follower : *Firm* 2





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Efficiency in Exchange Economies with Externalities

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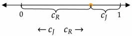
$\mathcal{F} = \left\{(c_R, c_J) \in \mathbb{R}^2_+ | c_R + c_J = 1\right\}$

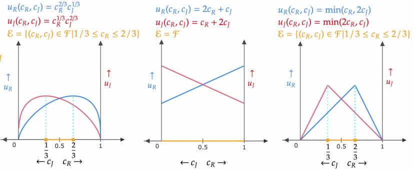
$u_R : \mathcal{F} \to \mathbb{R}$

$u_J : \mathcal{F} \to \mathbb{R}$

Set of efficient allocations is denoted by $\mathcal{E}$

Feasible Set Representation :





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Economy:

$\omega = (\omega_x, \omega_y) = (1, 1)$

$u_A(x_A, y_A, y_B) = x_A(1 + \max(y_A - y_B, 0))$

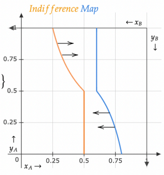
$u_B(x_B, y_B, y_A) = x_B(1 + \max(y_B - y_A, 0))$

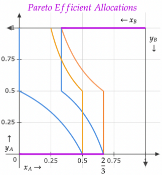
Feasible Allocations:

$\mathcal{F} = \left\{((x_A, y_A), (x_B, y_B)) \in \mathbb{R}^2_+ \times \mathbb{R}^2_+ \mid x_A + x_B = 1 \wedge y_A + y_B = 1\right\}$

Efficient Allocations:

$\mathcal{E} = \{((x_A, y_A), (x_B, y_B)) \in \mathcal{F} \mid y_A = 0 \wedge 0 \leq x_A \leq 2/3\} \cup$
$\{((x_A, y_A), (x_B, y_B)) \in \mathcal{F} \mid y_A = 1 \wedge 1/3 \leq x_A \leq 1\}$





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Efficiency in an Exchange Economy with Externalities

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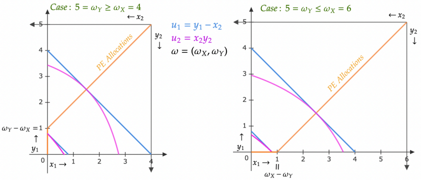

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Efficiency in Exchange Economies with Externalities

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Economy 1 :

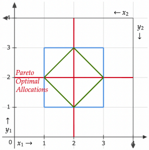
$u_1(x_1, y_1, x_2, y_2) = \max(x_1, y_1, x_2, y_2)$

$u_2(x_2, y_2) = -|x_2 - 2| - |y_2 - 2|$

$\omega = (\omega_X, \omega_Y) = (4, 4)$

Set of Pareto optimal Allocations consists of set of feasible allocations satisfying

$x_1 = 2$ *or* $y_1 = 2$



Economy 2 :

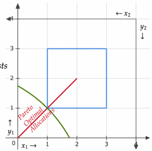
$u_1(x_1, y_1, x_2, y_2) = \min(x_1, y_1, x_2, y_2)$

$u_2(x_2, y_2) = x_2 y_2$

$\omega = (\omega_X, \omega_Y) = (4, 4)$

Set of Pareto optimal Allocations consists of set of feasible allocations satisfying

$0 \leq y_1 = x_1 \leq 2$



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Pareto Efficiency in a Public Good Economy

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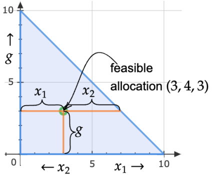
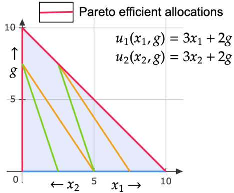
$\mathcal{F} = \left\{(x_1, x_2, g) \in \mathbb{R}^3_+ | x_1 + x_2 + g = 10\right\}$





Pareto Efficiency in a Public Good Economy

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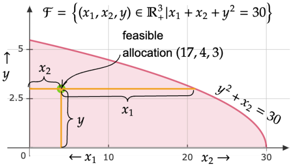

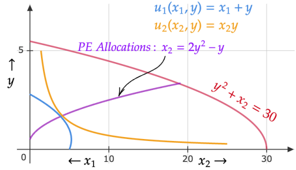

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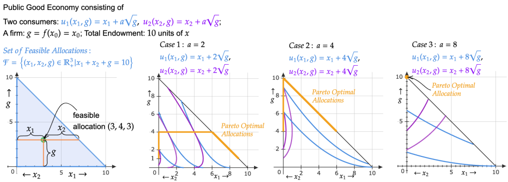

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Welfare Rank Reversal due to International Trade

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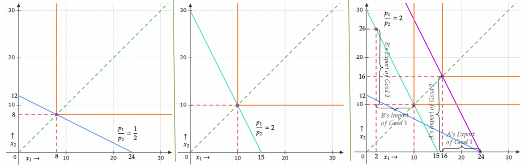
Country A ("Small" Country)
Given the production possibility frontier : $x_1 + 2x_2 = 24$
Representative consumer with utility : $u(x_1, x_2) = \min(x_1, x_2)$
In Autarky, A's Production = A's Consumption = $(8, 8)$

Country B ("Large" Country)
Given the production possibility frontier : $2x_1 + x_2 = 30$
Representative consumer with utility : $u(x_1, x_2) = \min(x_1, x_2)$
In Autarky, B's Production = B's Consumption = $(10, 10)$

With Trade of goods between Country A & B
In equilibrium,
A's Production = $(24, 0)$; A's Consumption = $(16, 16)$
B's Production = $(2, 26)$; B's Consumption = $(10, 10)$



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Y-Equilibrium in Pure Exchange Economies

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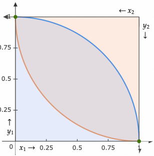
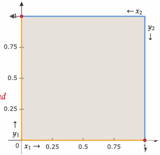
Pure Exchange Economy 1 :
$u_1(x_1, y_1) = x_1^2 + y_1^2$
$u_2(x_2, y_2) = x_2^2 + y_2^2$
$\omega = (\omega_X, \omega_Y) = (1, 1)$

Y – Equilibrium is given by
$\langle Y, ((x_1, y_1), (x_2, y_2)) \rangle$ *where*
$\cdot\ Y = \left\{(x, y) \in \mathbb{R}_+^2 | x^2 + y^2 \leq 1\right\}$ *and*
$\cdot\ ((x_1, y_1), (x_2, y_2)) = ((1, 0), (0, 1))$ *or*
$((x_1, y_1), (x_2, y_2)) = ((0, 1), (1, 0))$



Pure Exchange Economy 2 :
$u_1(x_1, y_1) = \max(x_1, y_1)$
$u_2(x_2, y_2) = \max(x_2, y_2)$
$\omega = (\omega_X, \omega_Y) = (1, 1)$

Y – Equilibrium is given by
$\langle Y, ((x_1, y_1), (x_2, y_2)) \rangle$ *where*
$\cdot\ Y = \left\{(x, y) \in \mathbb{R}_+^2 | \max(x, y) \leq 1\right\}$ *and*
$\cdot\ ((x_1, y_1), (x_2, y_2)) = ((1, 0), (0, 1))$ *or*
$((x_1, y_1), (x_2, y_2)) = ((0, 1), (1, 0))$



To read about Y-equilibrium, refer: The permissible and the forbidden by M. Richter and A. Rubinstein

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Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u: \mathbb{R}^3_+ \to \mathbb{R}$
$u(x, y, z) = (x + y)z$

Is u concave?	Is u quasi – concave?
No	Yes

Is u convex?	Is u quasi – convex?
No	No

$\max_{x,y,z} (x+y)z$
s.t. $p_X x + p_Y y + p_Z z \le M$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M \ge 0$

Demand

$$\left(x^d, y^d, z^d\right)(p_X, p_Y, p_Z, M) \in \begin{cases} \left\{\left(\frac{M}{2p_X}, 0, \frac{M}{2p_Z}\right)\right\} & \text{if } p_X < p_Y \\ \left\{\left(0, \frac{M}{2p_Y}, \frac{M}{2p_Z}\right)\right\} & \text{if } p_X > p_Y \\ \left\{\left(\frac{\alpha M}{2p_X}, \frac{(1-\alpha)M}{2p_Y}, \frac{M}{2p_Z}\right) \middle| 0 \le \alpha \le 1\right\} & \text{if } p_X = p_Y \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $(x+y)z \ge \mu$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \ge 0$

Hicksian Demand

$$\left(x^h, y^h, z^h\right)(p_X, p_Y, p_Z, \mu) \in \begin{cases} \left\{\left(\sqrt{\frac{p_Z \mu}{p_X}}, 0, \sqrt{\frac{p_X \mu}{p_Z}}\right)\right\} & \text{if } p_X < p_Y \\ \left\{\left(0, \sqrt{\frac{p_Z \mu}{p_Y}}, \sqrt{\frac{p_Y \mu}{p_Z}}\right)\right\} & \text{if } p_X > p_Y \\ \left\{\left(\alpha\sqrt{\frac{p_Z \mu}{p_X}}, (1-\alpha)\sqrt{\frac{p_Z \mu}{p_Y}}, \sqrt{\frac{p_Y \mu}{p_Z}}\right) \middle| 0 \le \alpha \le 1\right\} & \text{if } p_X = p_Y \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u : \mathbb{R}^3_+ \to \mathbb{R}$
$u(x, y, z) = 2\sqrt{\min(x, y)} + z$

Is u concave? Yes

Is u quasi – concave? Yes

Is u convex? No

Is u quasi – convex? No

$\max_{x,y,z} 2\sqrt{\min(x, y)} + z$
s.t. $p_X x + p_Y y + p_Z z \le M$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M \ge 0$

Demand
$(x^d, y^d, z^d)(p_X, p_Y, p_Z, M)$

$$= \begin{cases} \left(\frac{M}{p_X + p_Y}, \frac{M}{p_X + p_Y}, 0\right) & \text{if } M(p_X + p_Y) < p_Z^2 \\ \left(\frac{p_Z^2}{(p_X + p_Y)^2}, \frac{p_Z^2}{(p_X + p_Y)^2}, \frac{M}{p_Z} - \frac{p_Z}{p_X + p_Y}\right) & \text{if } M(p_X + p_Y) \ge p_Z^2 \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $2\sqrt{\min(x, y)} + z \ge \mu$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \ge 0$

Hicksian Demand
$(x^h, y^h, z^h)(p_X, p_Y, p_Z, \mu)$

$$= \begin{cases} \left(\frac{\mu^2}{4}, \frac{\mu^2}{4}, 0\right) & \text{if } \mu - \frac{2p_Z}{p_X + p_Y} < 0 \\ \left(\frac{p_Z^2}{(p_X + p_Y)^2}, \frac{p_Z^2}{(p_X + p_Y)^2}, \mu - \frac{2p_Z}{p_X + p_Y}\right) & \text{if } \mu - \frac{2p_Z}{p_X + p_Y} \ge 0 \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u : \mathbb{R}^3_+ \to \mathbb{R}$
$u(x, y, z) = xy + z$

Is u concave? No

Is u quasi-concave? No

Is u convex? No

Is u quasi-convex? No

$\max_{x,y,z} xy + z$
s.t. $p_X x + p_Y y + p_Z z \le M$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M \ge 0$

Demand

$$\left(x^d, y^d, z^d\right)(p_X, p_Y, p_Z, M) \in \begin{cases} \left\{\left(\frac{M}{2p_X}, \frac{M}{2p_Y}, 0\right)\right\} & \text{if } \frac{M^2}{4p_Xp_Y} > \frac{M}{p_Z} \\ \left\{\left(0, 0, \frac{M}{p_Z}\right)\right\} & \text{if } \frac{M^2}{4p_Xp_Y} < \frac{M}{p_Z} \\ \left\{\left(\frac{M}{2p_X}, \frac{M}{2p_Y}, 0\right), \left(0, 0, \frac{M}{p_Z}\right)\right\} & \text{if } \frac{M^2}{4p_Xp_Y} = \frac{M}{p_Z} \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $xy + z \ge \mu$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \ge 0$

Hicksian Demand

$$\left(x^h, y^h, z^h\right)(p_X, p_Y, p_Z, \mu) \in \begin{cases} \left\{\left(\sqrt{\frac{p_Y\mu}{p_X}}, \sqrt{\frac{p_X\mu}{p_Y}}, 0\right)\right\} & \text{if } 2\sqrt{p_Xp_Y\mu} < p_Z\mu \\ \{(0, 0, \mu)\} & \text{if } 2\sqrt{p_Xp_Y\mu} > p_Z\mu \\ \left\{\left(\sqrt{\frac{p_Y\mu}{p_X}}, \sqrt{\frac{p_X\mu}{p_Y}}, 0\right), (0, 0, \mu)\right\} & \text{if } 2\sqrt{p_Xp_Y\mu} = p_Z\mu \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u : \mathbb{R}^3_+ \to \mathbb{R}$
$u(x, y, z) = \max(\min(x, y), z)$

Is u concave? *No*

Is u quasi – concave? *No*

Is u convex? *No*

Is u quasi – convex? *No*

$\max_{x,y,z} \max(\min(x, y), z)$
s.t. $p_X x + p_Y y + p_Z z \le M$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M \ge 0$

Demand
$(x^d, y^d, z^d)(p_X, p_Y, p_Z, M)$

$$\in \begin{cases} \left\{\left(\frac{M}{p_X+p_Y}, \frac{M}{p_X+p_Y}, 0\right)\right\} & \text{if } p_X + p_Y < p_Z \\ \left\{\left(0, 0, \frac{M}{p_Z}\right)\right\} & \text{if } p_X + p_Y > p_Z \\ \left\{\left(\frac{M}{p_X+p_Y}, \frac{M}{p_X+p_Y}, 0\right), \left(0, 0, \frac{M}{p_Z}\right)\right\} & \text{if } p_X + p_Y = p_Z \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $\max(\min(x, y), z) \ge \mu$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \ge 0$

Hicksian Demand
$(x^h, y^h, z^h)(p_X, p_Y, p_Z, \mu)$

$$\in \begin{cases} \{(\mu, \mu, 0)\} & \text{if } p_X + p_Y < p_Z \\ \{(0, 0, \mu)\} & \text{if } p_X + p_Y > p_Z \\ \{(\mu, \mu, 0), (0, 0, \mu)\} & \text{if } p_X + p_Y = p_Z \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u : \mathbb{R}^3_+ \to \mathbb{R}$
$u(x, y, z) = \sqrt{\max(x, y, z)}$

Is u concave?	Is u quasi – concave?
No	No

Is u convex?	Is u quasi – convex?
No	Yes

$\max_{x,y,z} \sqrt{\max(x, y, z)}$
s.t. $p_X x + p_Y y + p_Z z \leq M$
and $x \geq 0,\ y \geq 0,\ z \geq 0$
where $0 < p_X \leq p_Y \leq p_Z,\ M \geq 0$

Demand

$$(x^d, y^d, z^d)(p_X, p_Y, p_Z, M) \in \begin{cases} \left\{\left(\frac{M}{p_X}, 0, 0\right)\right\} & \text{if } p_X < p_Y \leq p_Z \\ \left\{\left(\frac{M}{p_X}, 0, 0\right), \left(0, \frac{M}{p_Y}, 0\right)\right\} & \text{if } p_X = p_Y < p_Z \\ \left\{\left(\frac{M}{p_X}, 0, 0\right), \left(0, \frac{M}{p_Y}, 0\right), \left(0, 0, \frac{M}{p_Z}\right)\right\} & \text{if } p_X = p_Y = p_Z \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $\sqrt{\max(x, y, z)} \geq \mu$
and $x \geq 0,\ y \geq 0,\ z \geq 0$
where $0 < p_X \leq p_Y \leq p_Z,\ \mu \geq 0$

Hicksian Demand

$$(x^h, y^h, z^h)(p_X, p_Y, p_Z, \mu) \in \begin{cases} \left\{(\mu^2, 0, 0)\right\} & \text{if } p_X < p_Y \leq p_Z \\ \left\{(\mu^2, 0, 0), (0, \mu^2, 0)\right\} & \text{if } p_X = p_Y < p_Z \\ \left\{(\mu^2, 0, 0), (0, \mu^2, 0), (0, 0, \mu^2)\right\} & \text{if } p_X = p_Y = p_Z \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u : \mathbb{R}_{++} \times \mathbb{R}_{++} \times \mathbb{R}_{+} \to \mathbb{R}$
$u(x, y, z) = \alpha \ln x + \beta \ln y + z,$
where $\alpha > 0,\ \beta > 0,\ \alpha + \beta = 1$

Is u concave?
Yes

Is u quasi − concave?
Yes

Is u convex?
No

Is u quasi − convex?
No

$\max_{x,y,z} \alpha \ln x + \beta \ln y + z$
s.t. $p_X x + p_Y y + p_Z z \leq M$
and $x > 0,\ y > 0,\ z \geq 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M > 0$

Demand

$$\left(x^d, y^d, z^d\right)(p_X, p_Y, p_Z, M) = \begin{cases} \left(\frac{\alpha M}{p_X}, \frac{\beta M}{p_Y}, 0\right) & \text{if } M < p_Z \\ \left(\frac{\alpha p_Z}{p_X}, \frac{\beta p_Z}{p_Y}, \frac{M - p_Z}{p_Z}\right) & \text{if } M \geq p_Z \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $\alpha \ln x + \beta \ln y + z \geq \mu$
and $x > 0,\ y > 0,\ z \geq 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \in \mathbb{R}$

Hicksian Demand

$$\left(x^h, y^h, z^h\right)(p_X, p_Y, p_Z, \mu) = \begin{cases} \left(e^{\mu - \beta(\ln(\beta p_X) - \ln(\alpha p_Y))}, e^{\mu + \alpha(\ln(\beta p_X) - \ln(\alpha p_Y))}, 0\right) & \text{if } \mu - \alpha \ln\left(\frac{\alpha p_Z}{p_X}\right) - \beta \ln\left(\frac{\beta p_Z}{p_Y}\right) < 0 \\ \left(\frac{\alpha p_Z}{p_X}, \frac{\beta p_Z}{p_Y}, \mu - \alpha \ln\left(\frac{\alpha p_Z}{p_X}\right) - \beta \ln\left(\frac{\beta p_Z}{p_Y}\right)\right) & \text{if } \mu - \alpha \ln\left(\frac{\alpha p_Z}{p_X}\right) - \beta \ln\left(\frac{\beta p_Z}{p_Y}\right) \geq 0 \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u : \mathbb{R}^2_+ \to \mathbb{R}$
$u(x, y) = \max(\min(x, 2y),\ \min(2x, y))$

Is u concave?	No
Is u quasi – concave?	No
Is u convex?	No
Is u quasi – convex?	No

$\max_{x,y} \max(\min(x, 2y),\ \min(2x, y))$
s.t. $p_X x + p_Y y \le M$
and $x \ge 0,\ y \ge 0$
where $p_X > 0,\ p_Y > 0,\ M \ge 0$

Demand

$$(x^d, y^d)(p_X, p_Y, M) \in \begin{cases} \left\{\left(\frac{M}{p_X + 2p_Y}, \frac{2M}{p_X + 2p_Y}\right)\right\} & \text{if } \frac{p_X}{p_Y} > 1 \\ \left\{\left(\frac{M}{3p_X}, \frac{2M}{3p_X}\right), \left(\frac{2M}{3p_X}, \frac{M}{3p_X}\right)\right\} & \text{if } \frac{p_X}{p_Y} = 1 \\ \left\{\left(\frac{2M}{2p_X + p_Y}, \frac{M}{2p_X + p_Y}\right)\right\} & \text{if } \frac{p_X}{p_Y} < 1 \end{cases}$$

Indirect Utility

$$v(p_X, p_Y, M) = \max\left(\frac{2M}{p_X + 2p_Y}, \frac{2M}{2p_X + p_Y}\right)$$

$\min_{x,y} p_X x + p_Y y$
s.t. $\max(\min(x, 2y),\ \min(2x, y)) \ge \mu$
and $x \ge 0,\ y \ge 0$
where $p_X > 0,\ p_Y > 0,\ \mu \ge 0$

Hicksian Demand

$$(x^h, y^h)(p_X, p_Y, \mu) \in \begin{cases} \left\{\left(\frac{\mu}{2}, \mu\right)\right\} & \text{if } \frac{p_X}{p_Y} > 1 \\ \left\{\left(\frac{\mu}{2}, \mu\right), \left(\mu, \frac{\mu}{2}\right)\right\} & \text{if } \frac{p_X}{p_Y} = 1 \\ \left\{\left(\mu, \frac{\mu}{2}\right)\right\} & \text{if } \frac{p_X}{p_Y} < 1 \end{cases}$$

Expenditure Function

$$e(p_X, p_Y, \mu) = \mu\left(\min\left(\frac{p_X}{2} + p_Y, p_X + \frac{p_Y}{2}\right)\right)$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u : \mathbb{R}^2_+ \to \mathbb{R}$
$u(x, y) = \min\left(\sqrt{xy}, y\right)$

Is u concave? Yes

Is u quasi – concave? Yes

Is u convex? No

Is u quasi – convex? No

$\max_{x,y} \min\left(\sqrt{xy}, y\right)$
s.t. $p_X x + p_Y y \leq M$
and $x \geq 0,\ y \geq 0$
where $p_X > 0,\ p_Y > 0,\ M \geq 0$

Demand

$$(x^d, y^d)(p_X, p_Y, M) = \begin{cases} \left(\frac{M}{2p_X}, \frac{M}{2p_Y}\right) & \text{if } \frac{p_X}{p_Y} \geq 1 \\ \left(\frac{M}{p_X + p_Y}, \frac{M}{p_X + p_Y}\right) & \text{if } \frac{p_X}{p_Y} < 1 \end{cases}$$

Indirect Utility

$$v(p_X, p_Y, M) = \begin{cases} \frac{M}{2\sqrt{p_X p_Y}} & \text{if } \frac{p_X}{p_Y} \geq 1 \\ \frac{M}{p_X + p_Y} & \text{if } \frac{p_X}{p_Y} < 1 \end{cases}$$

$\min_{x,y} p_X x + p_Y y$
s.t. $\min\left(\sqrt{xy}, y\right) \geq \mu$
and $x \geq 0,\ y \geq 0$
where $p_X > 0,\ p_Y > 0,\ \mu \geq 0$

Hicksian Demand

$$(x^h, y^h)(p_X, p_Y, \mu) = \begin{cases} \left(\mu\sqrt{\frac{p_Y}{p_X}}, \mu\sqrt{\frac{p_X}{p_Y}}\right) & \text{if } \frac{p_X}{p_Y} \geq 1 \\ (\mu, \mu) & \text{if } \frac{p_X}{p_Y} < 1 \end{cases}$$

Expenditure Function

$$e(p_X, p_Y, \mu) = \begin{cases} 2\mu\sqrt{p_X p_Y} & \text{if } \frac{p_X}{p_Y} \geq 1 \\ (p_X + p_Y)\mu & \text{if } \frac{p_X}{p_Y} < 1 \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:

$u : \mathbb{R}^2_+ \to \mathbb{R}$

$u(x, y) = \min\left(3x^2, xy, 3y^2\right)$

Is u concave? No

Is u quasi – concave? Yes

Is u convex? No

Is u quasi – convex? No

$\max_{x,y} \min\left(3x^2, xy, 3y^2\right)$
s.t. $p_X x + p_Y y \le M$
and $x \ge 0,\ y \ge 0$
where $p_X > 0,\ p_Y > 0,\ M \ge 0$

Demand

$$\left(x^d, y^d\right)(p_X, p_Y, M) = \begin{cases} \left(\dfrac{M}{p_X + 3p_Y}, \dfrac{3M}{p_X + 3p_Y}\right) & \text{if } \dfrac{p_X}{p_Y} \ge 3 \\ \left(\dfrac{M}{2p_X}, \dfrac{M}{2p_Y}\right) & \text{if } \dfrac{1}{3} < \dfrac{p_X}{p_Y} < 3 \\ \left(\dfrac{3M}{3p_X + p_Y}, \dfrac{M}{3p_X + p_Y}\right) & \text{if } \dfrac{p_X}{p_Y} \le \dfrac{1}{3} \end{cases}$$

$\min_{x,y} p_X x + p_Y y$
s.t. $\min\left(3x^2, xy, 3y^2\right) \ge \mu$
and $x \ge 0,\ y \ge 0$
where $p_X > 0,\ p_Y > 0,\ \mu \ge 0$

Hicksian Demand

$$\left(x^h, y^h\right)(p_X, p_Y, \mu) = \begin{cases} \left(\sqrt{\dfrac{\mu}{3}}, \sqrt{3\mu}\right) & \text{if } \dfrac{p_X}{p_Y} \ge 3 \\ \left(\sqrt{\dfrac{\mu p_Y}{p_X}}, \sqrt{\dfrac{\mu p_X}{p_Y}}\right) & \text{if } \dfrac{1}{3} < \dfrac{p_X}{p_Y} < 3 \\ \left(\sqrt{3\mu}, \sqrt{\dfrac{\mu}{3}}\right) & \text{if } \dfrac{p_X}{p_Y} \le \dfrac{1}{3} \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u : \mathbb{R}^4_+ \to \mathbb{R}$
$u(x, y, w, z) = (\min(x, y))(w + z)$

Is u concave?	No
Is u quasi – concave?	Yes
Is u convex?	No
Is u quasi – convex?	No

$\max_{x,y,w,z} (\min(x, y))(w + z)$
s.t. $p_X x + p_Y y + p_W w + p_Z z \le M$
and $x \ge 0,\ y \ge 0, w \ge 0, z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_W > 0,\ p_Z > 0,\ M \ge 0$

Demand

$$(x^d, y^d, w^d, z^d)(p_X, p_Y, p_W, p_Z, M) \in \begin{cases} \left\{\left(\frac{M}{2(p_X+p_Y)}, \frac{M}{2(p_X+p_Y)}, \frac{\theta M}{2p_W}, \frac{(1-\theta)M}{2p_Z}\right) \middle| \theta \in [0,1]\right\} & \text{if } \frac{p_W}{p_Z} = 1 \\ \left\{\left(\frac{M}{2(p_X+p_Y)}, \frac{M}{2(p_X+p_Y)}, \frac{M}{2p_W}, 0\right)\right\} & \text{if } \frac{p_W}{p_Z} < 1 \\ \left\{\left(\frac{M}{2(p_X+p_Y)}, \frac{M}{2(p_X+p_Y)}, 0, \frac{M}{2p_Z}\right)\right\} & \text{if } \frac{p_W}{p_Z} > 1 \end{cases}$$

Indirect Utility function:

$$v(p_X, p_Y, p_W, p_Z, M) = \frac{M^2}{4(p_X+p_Y)\min(p_W, p_Z)}$$

$\min_{x,y,w,z} p_X x + p_Y y + p_W w + p_Z z$
s.t. $(\min(x, y))(w + z) \ge \mu$
and $x \ge 0,\ y \ge 0,\ w \ge 0, z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_W > 0,\ p_Z > 0,\ \mu \ge 0$

Hicksian Demand

$$(x^h, y^h, w^h, z^h)(p_X, p_Y, p_W, p_Z, \mu) \in \begin{cases} \left\{\left(\sqrt{\frac{p_W\mu}{p_X+p_Y}}, \sqrt{\frac{p_W\mu}{p_X+p_Y}}, \theta\sqrt{\frac{(p_X+p_Y)\mu}{p_W}}, (1-\theta)\sqrt{\frac{(p_X+p_Y)\mu}{p_W}}\right) \middle| \theta \in [0,1]\right\} & \text{if } \frac{p_W}{p_Z} = 1 \\ \left\{\left(\sqrt{\frac{p_W\mu}{p_X+p_Y}}, \sqrt{\frac{p_W\mu}{p_X+p_Y}}, \sqrt{\frac{(p_X+p_Y)\mu}{p_W}}, 0\right)\right\} & \text{if } \frac{p_W}{p_Z} < 1 \\ \left\{\left(\sqrt{\frac{p_Z\mu}{p_X+p_Y}}, \sqrt{\frac{p_Z\mu}{p_X+p_Y}}, 0, \sqrt{\frac{(p_X+p_Y)\mu}{p_Z}}\right)\right\} & \text{if } \frac{p_W}{p_Z} > 1 \end{cases}$$

Expenditure function:

$$e(p_X, p_Y, p_W, p_Z, \mu) = 2\sqrt{(p_X+p_Y)\min(p_W, p_Z)\mu}$$

Preferences over Necessities and Luxuries

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Utility, $u:\mathbb{R}_+^L \times \mathbb{R}_+^2 \to \mathbb{R}$ *defined as* $u(x_1, \dots, x_L, y, z) = \left(2\sum_{i=1}^{L} \alpha_i \sqrt{x_i}\right) + \min(y, z)$, *where* $\alpha_i > 0$ *for all* $i \in \{1, 2, \dots, L\}$.

Is u concave?	*Yes*
Is u quasiconcave?	*Yes*
Is u convex?	*No*
Is u quasiconvex?	*No*

$$\max_{x_1,\dots,x_L,y,z} \left(2\sum_{i=1}^{L} \alpha_i \sqrt{x_i}\right) + \min(y, z)$$

s.t. $p_1 x_1 + \cdots p_L x_L + p_Y y + p_Z z \le M$

and $x_1 \ge 0, \dots, x_L \ge 0, y \ge 0, z \ge 0$

where $L \in \mathbb{N}$, $p_1 > 0, \dots, p_L > 0, p_Y \in (0,1), p_Y + p_Z = 1, M \ge 0$

Demand ⟹

$$x_i^d(p_1, \dots, p_L, p_Y, p_Z = 1 - p_Y, M) = \frac{\alpha_i^2}{p_i^2} \min\left(1, \frac{M}{\sum_{j=1}^{L} \frac{\alpha_j^2}{p_j}}\right)$$

$$y^d(p_1, \dots, p_L, p_Y, p_Z = 1 - p_Y, M) = \max\left(0, M - \sum_{j=1}^{L} \frac{\alpha_j^2}{p_j}\right)$$

$$z^d(p_1, \dots, p_L, p_Y, p_Z = 1 - p_Y, M) = \max\left(0, M - \sum_{j=1}^{L} \frac{\alpha_j^2}{p_j}\right)$$

Demand Description: *Up to a threshold income, the consumer spends* $\frac{\left(\alpha_i^2 / p_i\right)}{\sum_{j=1}^{L} \left(\alpha_j^2 / p_j\right)}$ *proportion of their income on commodity* x_i *and nothing on y and z. Beyond that threshold, the consumer spends the amount* $\left(\alpha_i^2 / p_i\right)$ *on commodity* x_i, *and the remaining amount* $M - \sum_{j=1}^{L} \frac{\alpha_j^2}{p_j}$ *on y and z in such a way that* $y = z$.

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Preferences over Necessities and Luxuries

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Utility, $u : \mathbb{R}^L_{++} \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ defined as $u(x_1, \ldots, x_L, y, z) = \left(\sum_{i=1}^{L} \alpha_i \ln x_i\right) + y^\beta z^{1-\beta}$, where $\sum_{i=1}^{L} \alpha_i = 1$ and $\alpha_i > 0$ for all $i \in \{1, 2, \ldots, L\}$, and $\beta \in (0, 1)$

Is u concave? Yes

Is u quasiconcave? Yes

Is u convex? No

Is u quasiconvex? No

$$\max_{x_1,\ldots,x_L,y,z} \left(\sum_{i=1}^{L} \alpha_i \ln x_i\right) + y^\beta z^{1-\beta}$$

s.t. $p_1 x_1 + \cdots p_L x_L + p_Y y + p_Z z \le M$

and $x_1 > 0, \ \ldots, x_L > 0, y \ge 0, \ z \ge 0$

where $L \in \mathbb{N}$, $p_1 > 0, \ldots, p_L > 0$, $p_Y > 0$, $p_Z > 0$, $M > 0$

Demand

$$x_i^d(p_1, \ldots, p_L, p_Y, p_Z, M) = \frac{\alpha_i}{p_i} \min\left(M, \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}\right)$$

$$y^d(p_1, \ldots, p_L, p_Y, p_Z, M) = \frac{\beta}{p_Y} \max\left(0, \ M - \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}\right)$$

$$z^d(p_1, \ldots, p_L, p_Y, p_Z, M) = \frac{1-\beta}{p_Z} \max\left(0, \ M - \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}\right)$$

Demand Description: Up to a threshold income, the consumer spends α_i proportion of their income on commodity x_i and nothing on y and z. Beyond that threshold, the consumer spends α_i proportion of the threshold amount: $\frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}$ on commodity x_i, and β proportion of the remaining amount $M - \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}$ on y, and $(1-\beta)$ proportion of the remaining amount $M - \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}$ on z.

R^2_+ is a closed set, open ball is an open set

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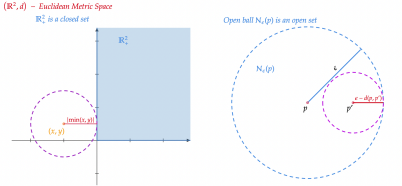

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Open Neighborhoods are Open Sets

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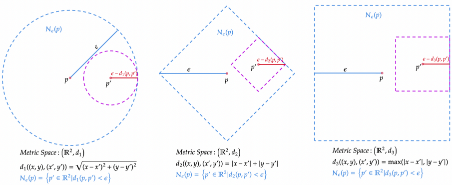

Examples of Open Sets and Closed Sets

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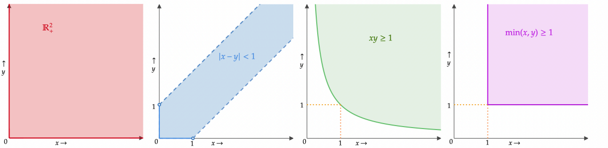
In the Metric Space $\left(\mathbb{R}^2_+, d\right)$, *where* $d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\mathbb{R}^2_+$ *is both Closed & Open*

$\{(x, y) \mid |x - y| < 1\}$ *is Open but not Closed*

$\{(x, y) \mid xy \geq 1\}$ *is Closed but not Open*

$\{(x, y) \mid \min(x, y) \geq 1\}$ *is Closed but not Open*



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Examples of Open Sets and Closed Sets

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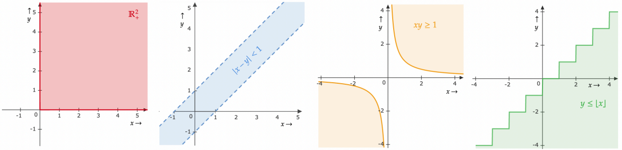
In the Metric Space $(\mathbb{R}^2, d)$, *where* $d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\mathbb{R}^2_+$ *is Closed but not Open* | $\{(x, y) \mid |x - y| < 1\}$ *is Open but not Closed* | $\{(x, y) \mid xy \geq 1\}$ *is Closed but not Open* | $\{(x, y) \mid y \leq \lfloor x \rfloor\}$ *is Closed but not Open*



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Examples of Open Sets and Closed Sets

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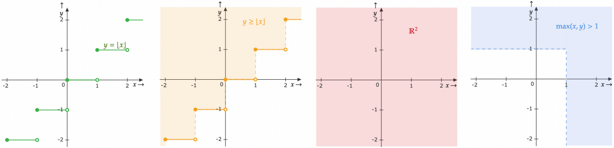
In the Metric Space $(\mathbb{R}^2, d)$, *where* $d((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2|$

$\{(x, y) | y = \lfloor x \rfloor\}$ *is neither open nor closed*

$\{(x, y) | y \geq \lfloor x \rfloor\}$ *is neither open nor closed*

$\mathbb{R}^2$ *is both Open and Closed*

$\{(x, y) | \max(x, y) > 1\}$ *is Open but not Closed*



Concave/Convex/Quasi-concave/Quasi-convex

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$u: \mathbb{R}^4_+ \to \mathbb{R}$	Is u concave?	Is u convex?	Is u quasi-concave?	Is u quasi-convex?
$u(x,y,w,z) = x + y + \min(w,z)$	☑	☐	☑	☐
$u(x,y,w,z) = x^{0.5}y^{0.5} + w^{0.5}z^{0.5}$	☑	☐	☑	☐
$u(x,y,w,z) = x + y + w + z$	☑	☑	☑	☑
$u(x,y,w,z) = xy + wz$	☐	☐	☐	☐
$u(x,y,w,z) = \min(xy, wz)$	☐	☐	☑	☐
$u(x,y,w,z) = \sqrt{\min(xy, wz)}$	☑	☐	☑	☐
$u(x,y,w,z) = x + y + \max(w,z)$	☐	☑	☐	☑
$u(x,y,w,z) = xy + \min(w,z)$	☐	☐	☐	☐

Kakutani's Fixed Point Theorem

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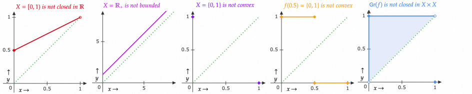
Kakutani's Fixed Point Theorem :
Given a set $X \subset \mathbb{R}^n$ *that is convex, compact and a correspondence (set – valued function)* $f : X \rightrightarrows X$ *satisfying* :
– $\forall x \in X$, *set* $f(x) \subset X$ *is non – empty, convex.*
– *Graph of f i. e.* $\mathrm{Gr}(f) := \{(x, y) \in X \times X | y \in f(x)\}$ *is closed in* $X \times X$.
Then there exists $x^* \in X$ *such that* $x^* \in f(x^*)$. *This* x^* *is known as the fixed point.*

We'll observe that, even if we relax exactly one of these assumptions at a time while keeping all the others satisfied, f may not have a fixed point. :
[1] X *is closed in* $\mathbb{R}^n$
[2] X *is bounded*
[3] X *is convex*
[4] $f(x)$ *is convex for each* $x \in X$
[5] f *has a closed graph in* $X \times X$

Examples :
[1] : $X = [0, 1)$, $f(x) = \{0.5 + 0.5x\}$
[2] : $X = \mathbb{R}_+$, $f(x) = \{1 + x\}$
[3] : $X = \{0, 1\}$, $f(x) = \{1 - x\}$
[4] : $X = [0, 1]$, $f(x) = \begin{cases} \{1\} & \text{if } x < 0.5 \\ \{0\} & \text{if } x > 0.5 \\ \{0, 1\} & \text{if } x = 0.5 \end{cases}$
[5] : $X = [0, 1]$, $f(x) = \begin{cases} (x, 1] & \text{if } x < 1 \\ \{0\} & \text{if } x = 1 \end{cases}$



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Distribution of max and min of i.i.d Uniform RVs

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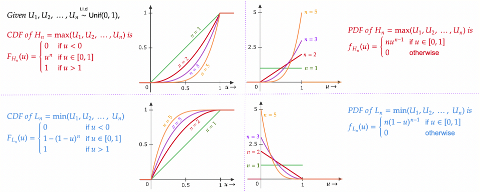

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Joint Distributions

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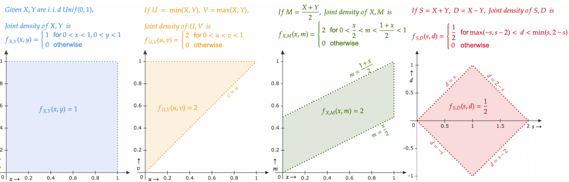

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Joint Distributions

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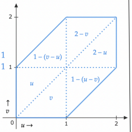
Given X, Y, Z *are i. i. d* $Unif(0,1)$.

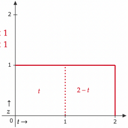
Joint density of
$U = X + Z,\ V = Y + Z$ *is*

$$f_{U,V}(u,v) = \begin{cases} \min(u,v) & \text{for } 0 < u, v < 1 \\ 2 - \max(u,v) & \text{for } 1 < u, v < 2 \\ 1 - (u - v) & \text{for } 1 < u,\ 0 < v < 1,\ u - v < 1 \\ 1 - (v - u) & \text{for } 1 < v,\ 0 < u < 1,\ v - u < 1 \\ 0 & \text{otherwise} \end{cases}$$



Joint density of
$T = X + Y,\ Z$ *is*

$$f_{T,Z}(t,z) = \begin{cases} t & \text{for } 0 < t < 1,\ 0 < z < 1 \\ 2 - t & \text{for } 1 \le t < 2,\ 0 < z < 1 \\ 0 & \text{otherwise} \end{cases}$$



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