Preferences over Necessities and Luxuries

Econschool

Utility, $u : \mathbb{R}^L_{++} \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ defined as $u(x_1, \ldots, x_L, y, z) = \left(\sum_{i=1}^{L} \alpha_i \ln x_i\right) + y^\beta z^{1-\beta}$, where $\sum_{i=1}^{L} \alpha_i = 1$ and $\alpha_i > 0$ for all $i \in \{1, 2, \ldots, L\}$, and $\beta \in (0, 1)$

Is u concave? Yes

Is u quasiconcave? Yes

Is u convex? No

Is u quasiconvex? No

$$\max_{x_1,\ldots,x_L,y,z} \left(\sum_{i=1}^{L} \alpha_i \ln x_i\right) + y^\beta z^{1-\beta}$$

s.t. $p_1 x_1 + \cdots p_L x_L + p_Y y + p_Z z \leq M$

and $x_1 > 0, \ \ldots, x_L > 0, y \geq 0, \ z \geq 0$

where $L \in \mathbb{N}$, $p_1 > 0, \ldots, p_L > 0$, $p_Y > 0$, $p_Z > 0$, $M > 0$

Demand ⇨

$$x_i^d(p_1, \ldots, p_L, p_Y, p_Z, M) = \frac{\alpha_i}{p_i} \min\left(M, \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}\right)$$

$$y^d(p_1, \ldots, p_L, p_Y, p_Z, M) = \frac{\beta}{p_Y} \max\left(0, \ M - \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}\right)$$

$$z^d(p_1, \ldots, p_L, p_Y, p_Z, M) = \frac{1-\beta}{p_Z} \max\left(0, \ M - \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}\right)$$

Demand Description: *Up to a threshold income, the consumer spends α_i proportion of their income on commodity x_i and nothing on y and z. Beyond that threshold, the consumer spends α_i proportion of the threshold amount: $\frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}$ on commodity x_i, and β proportion of the remaining amount $M - \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}$ on y, and $(1-\beta)$ proportion of the remaining amount $M - \frac{p_Y^\beta p_Z^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}}$ on z.*

Utility Maximization and Expenditure Minimization

Econschool

Utility function :
$u : \mathbb{R}^3_+ \to \mathbb{R}$
$u(x, y, z) = (x + y)z$

Is u concave? No	*Is u quasi – concave*? Yes
Is u convex? No	*Is u quasi – convex*? No

$\max_{x,y,z} (x+y)z$
s.t. $p_X x + p_Y y + p_Z z \le M$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M \ge 0$

Demand

$$\left(x^d, y^d, z^d\right)(p_X, p_Y, p_Z, M) \in \begin{cases} \left\{\left(\frac{M}{2p_X}, 0, \frac{M}{2p_Z}\right)\right\} & \text{if } p_X < p_Y \\ \left\{\left(0, \frac{M}{2p_Y}, \frac{M}{2p_Z}\right)\right\} & \text{if } p_X > p_Y \\ \left\{\left(\frac{\alpha M}{2p_X}, \frac{(1-\alpha)M}{2p_Y}, \frac{M}{2p_Z}\right) \middle| 0 \le \alpha \le 1\right\} & \text{if } p_X = p_Y \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $(x+y)z \ge \mu$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \ge 0$

Hicksian Demand

$$\left(x^h, y^h, z^h\right)(p_X, p_Y, p_Z, \mu) \in \begin{cases} \left\{\left(\sqrt{\frac{p_Z \mu}{p_X}}, 0, \sqrt{\frac{p_X \mu}{p_Z}}\right)\right\} & \text{if } p_X < p_Y \\ \left\{\left(0, \sqrt{\frac{p_Z \mu}{p_Y}}, \sqrt{\frac{p_Y \mu}{p_Z}}\right)\right\} & \text{if } p_X > p_Y \\ \left\{\left(\alpha \sqrt{\frac{p_Z \mu}{p_X}}, (1-\alpha)\sqrt{\frac{p_Z \mu}{p_Y}}, \sqrt{\frac{p_Y \mu}{p_Z}}\right) \middle| 0 \le \alpha \le 1\right\} & \text{if } p_X = p_Y \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function :
$u : \mathbb{R}_{++} \times \mathbb{R}_{++} \times \mathbb{R}_{+} \to \mathbb{R}$
$u(x, y, z) = \alpha \ln x + \beta \ln y + z,$
where $\alpha > 0,\ \beta > 0,\ \alpha + \beta = 1$

Is u concave? Yes

Is u quasi − concave? Yes

Is u convex? No

Is u quasi − convex? No

$\max_{x,y,z} \alpha \ln x + \beta \ln y + z$
s.t. $p_X x + p_Y y + p_Z z \le M$
and $x > 0,\ y > 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M > 0$

Demand

$$(x^d, y^d, z^d)(p_X, p_Y, p_Z, M) = \begin{cases} \left(\dfrac{\alpha M}{p_X}, \dfrac{\beta M}{p_Y}, 0\right) & \text{if } M < p_Z \\ \left(\dfrac{\alpha p_Z}{p_X}, \dfrac{\beta p_Z}{p_Y}, \dfrac{M - p_Z}{p_Z}\right) & \text{if } M \ge p_Z \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $\alpha \ln x + \beta \ln y + z \ge \mu$
and $x > 0,\ y > 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \in \mathbb{R}$

Hicksian Demand

$$(x^h, y^h, z^h)(p_X, p_Y, p_Z, \mu) = \begin{cases} \left(e^{\mu - \beta(\ln(\beta p_X) - \ln(\alpha p_Y))}, e^{\mu + \alpha(\ln(\beta p_X) - \ln(\alpha p_Y))}, 0\right) & \text{if } \mu - \alpha \ln\left(\dfrac{\alpha p_Z}{p_X}\right) - \beta \ln\left(\dfrac{\beta p_Z}{p_Y}\right) < 0 \\ \left(\dfrac{\alpha p_Z}{p_X}, \dfrac{\beta p_Z}{p_Y}, \mu - \alpha \ln\left(\dfrac{\alpha p_Z}{p_X}\right) - \beta \ln\left(\dfrac{\beta p_Z}{p_Y}\right)\right) & \text{if } \mu - \alpha \ln\left(\dfrac{\alpha p_Z}{p_X}\right) - \beta \ln\left(\dfrac{\beta p_Z}{p_Y}\right) \ge 0 \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function :
$u: \mathbb{R}^3_+ \to \mathbb{R}$
$u(x,y,z) = 2\sqrt{\min(x,y)} + z$

Is u concave? *Yes*	*Is u quasi – concave?* *Yes*
Is u convex? *No*	*Is u quasi – convex?* *No*

$\max_{x,y,z} 2\sqrt{\min(x,y)} + z$
s.t. $p_X x + p_Y y + p_Z z \le M$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M \ge 0$

Demand

$$(x^d, y^d, z^d)(p_X, p_Y, p_Z, M) = \begin{cases} \left(\frac{M}{p_X + p_Y}, \frac{M}{p_X + p_Y}, 0\right) & \text{if } M(p_X + p_Y) < p_Z^2 \\ \left(\frac{p_Z^2}{(p_X + p_Y)^2}, \frac{p_Z^2}{(p_X + p_Y)^2}, \frac{M}{p_Z} - \frac{p_Z}{p_X + p_Y}\right) & \text{if } M(p_X + p_Y) \ge p_Z^2 \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $2\sqrt{\min(x,y)} + z \ge \mu$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \ge 0$

Hicksian Demand

$$(x^h, y^h, z^h)(p_X, p_Y, p_Z, \mu) = \begin{cases} \left(\frac{\mu^2}{4}, \frac{\mu^2}{4}, 0\right) & \text{if } \mu - \frac{2p_Z}{p_X + p_Y} < 0 \\ \left(\frac{p_Z^2}{(p_X + p_Y)^2}, \frac{p_Z^2}{(p_X + p_Y)^2}, \mu - \frac{2p_Z}{p_X + p_Y}\right) & \text{if } \mu - \frac{2p_Z}{p_X + p_Y} \ge 0 \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function :
$u : \mathbb{R}^3_+ \to \mathbb{R}$
$u(x, y, z) = xy + z$

Is u concave?
No

Is u quasi − concave?
No

Is u convex?
No

Is u quasi − convex?
No

$\max_{x,y,z} xy + z$
s.t. $p_X x + p_Y y + p_Z z \le M$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M \ge 0$

Demand
$(x^d, y^d, z^d)(p_X, p_Y, p_Z, M)$

$$\in \begin{cases} \left\{\left(\frac{M}{2p_X}, \frac{M}{2p_Y}, 0\right)\right\} & \text{if } \frac{M^2}{4p_Xp_Y} > \frac{M}{p_Z} \\ \left\{\left(0, 0, \frac{M}{p_Z}\right)\right\} & \text{if } \frac{M^2}{4p_Xp_Y} < \frac{M}{p_Z} \\ \left\{\left(\frac{M}{2p_X}, \frac{M}{2p_Y}, 0\right), \left(0, 0, \frac{M}{p_Z}\right)\right\} & \text{if } \frac{M^2}{4p_Xp_Y} = \frac{M}{p_Z} \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $xy + z \ge \mu$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \ge 0$

Hicksian Demand
$(x^h, y^h, z^h)(p_X, p_Y, p_Z, \mu)$

$$\in \begin{cases} \left\{\left(\sqrt{\frac{p_Y\mu}{p_X}}, \sqrt{\frac{p_X\mu}{p_Y}}, 0\right)\right\} & \text{if } 2\sqrt{p_Xp_Y\mu} < p_Z\mu \\ \{(0, 0, \mu)\} & \text{if } 2\sqrt{p_Xp_Y\mu} > p_Z\mu \\ \left\{\left(\sqrt{\frac{p_Y\mu}{p_X}}, \sqrt{\frac{p_X\mu}{p_Y}}, 0\right), (0, 0, \mu)\right\} & \text{if } 2\sqrt{p_Xp_Y\mu} = p_Z\mu \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function:
$u : \mathbb{R}^3_+ \to \mathbb{R}$
$u(x, y, z) = \max(\min(x, y), z)$

Is u concave? *No*

Is u quasi – concave? *No*

Is u convex? *No*

Is u quasi – convex? *No*

$\max_{x,y,z} \max(\min(x, y), z)$
s.t. $p_X x + p_Y y + p_Z z \le M$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ M \ge 0$

Demand

$$(x^d, y^d, z^d)(p_X, p_Y, p_Z, M) \in \begin{cases} \left\{\left(\frac{M}{p_X + p_Y}, \frac{M}{p_X + p_Y}, 0\right)\right\} & \text{if } p_X + p_Y < p_Z \\ \left\{\left(0, 0, \frac{M}{p_Z}\right)\right\} & \text{if } p_X + p_Y > p_Z \\ \left\{\left(\frac{M}{p_X + p_Y}, \frac{M}{p_X + p_Y}, 0\right), \left(0, 0, \frac{M}{p_Z}\right)\right\} & \text{if } p_X + p_Y = p_Z \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $\max(\min(x, y), z) \ge \mu$
and $x \ge 0,\ y \ge 0,\ z \ge 0$
where $p_X > 0,\ p_Y > 0,\ p_Z > 0,\ \mu \ge 0$

Hicksian Demand

$$(x^h, y^h, z^h)(p_X, p_Y, p_Z, \mu) \in \begin{cases} \{(\mu, \mu, 0)\} & \text{if } p_X + p_Y < p_Z \\ \{(0, 0, \mu)\} & \text{if } p_X + p_Y > p_Z \\ \{(\mu, \mu, 0), (0, 0, \mu)\} & \text{if } p_X + p_Y = p_Z \end{cases}$$

Utility Maximization and Expenditure Minimization

Econschool

Utility function :
$u : \mathbb{R}^3_+ \to \mathbb{R}$
$u(x,y,z) = \sqrt{\max(x,y,z)}$

Is u concave? No	*Is u quasi – concave?* No
Is u convex? No	*Is u quasi – convex?* Yes

$\max_{x,y,z} \sqrt{\max(x,y,z)}$
s.t. $p_X x + p_Y y + p_Z z \leq M$
and $x \geq 0,\ y \geq 0,\ z \geq 0$
where $0 < p_X \leq p_Y \leq p_Z,\ M \geq 0$

Demand

$$\left(x^d, y^d, z^d\right)(p_X, p_Y, p_Z, M) \in \begin{cases} \left\{\left(\frac{M}{p_X},0,0\right)\right\} & \text{if } p_X < p_Y \leq p_Z \\ \left\{\left(\frac{M}{p_X},0,0\right), \left(0,\frac{M}{p_Y},0\right)\right\} & \text{if } p_X = p_Y < p_Z \\ \left\{\left(\frac{M}{p_X},0,0\right), \left(0,\frac{M}{p_Y},0\right), \left(0,0,\frac{M}{p_Z}\right)\right\} & \text{if } p_X = p_Y = p_Z \end{cases}$$

$\min_{x,y,z} p_X x + p_Y y + p_Z z$
s.t. $\sqrt{\max(x,y,z)} \geq \mu$
and $x \geq 0,\ y \geq 0,\ z \geq 0$
where $0 < p_X \leq p_Y \leq p_Z,\ \mu \geq 0$

Hicksian Demand

$$\left(x^h, y^h, z^h\right)(p_X, p_Y, p_Z, \mu) \in \begin{cases} \left\{\left(\mu^2,0,0\right)\right\} & \text{if } p_X < p_Y \leq p_Z \\ \left\{\left(\mu^2,0,0\right), \left(0,\mu^2,0\right)\right\} & \text{if } p_X = p_Y < p_Z \\ \left\{\left(\mu^2,0,0\right), \left(0,\mu^2,0\right), \left(0,0,\mu^2\right)\right\} & \text{if } p_X = p_Y = p_Z \end{cases}$$